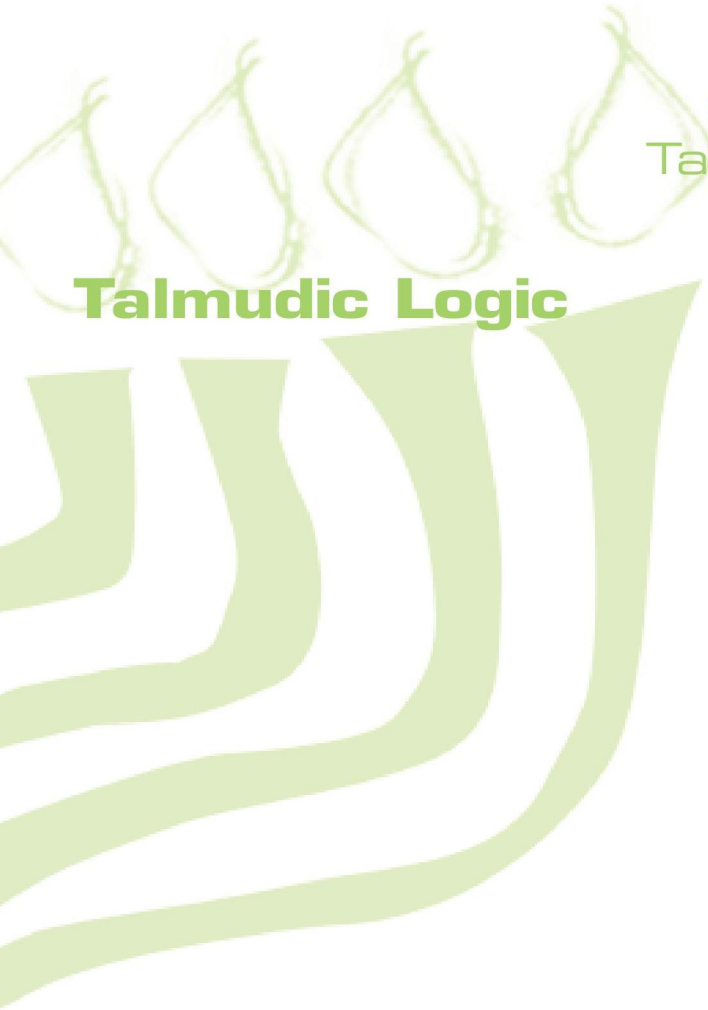

Studies in Talmudic Logic 6

Talmudic Logic

Andrew Schumann

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Volume 6

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Volume 1
Non-Deductive Inferences in the Talmud
Michael Abraham, Dov Gabbay and Uri Schild

Volume 2
The Textual Inference Rules Klal uPrat. How the Talmud Defines Sets
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Andrew Schumann

Studies in Talmudic Logic
Series Editors
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Andrew Schumann

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Preface

The Talmudic exegesis is constructed on special hermeneutic rules which interpret the text of the Torah within the limits of Judaism. The majority of these rules have a logical meaning in fact. On the basis of this meaning it is possible to speak about a special logical culture of the Talmud and to call the logic used there Judaic logic. Comparative researches in Judaic logic have carried out since the Middle Ages, when the Talmudic authorities began to compare the logical rules of the Talmud with Aristotle's dialectic and apodictic syllogisms. In the 19th century, H.S. Hirschfeld became the first scholar and Rabbi who suggested formalizing logical reasoning of the Talmud.

The contemporary interest in Judaic logic is related to the possibility of creating new logical approaches in modern computer science. Currently, computer science has been dealing with the logical systems formalizing multi-agent and interactive computing. Talmudic reasoning is just multi-agent and interactive, since it has been constructed by taking dissonant opinions and conflicting authorities into account. There is a complex system of Talmudic definitions of how and who wins in dispute. If we understood this massive-parallel, concurrent and multi-agent reasoning better, then it would provoke a true revolution in developing and implementing artificial intelligence.

Jewish logic has been generated on the basis of inference rules for expounding the Holy Script. Since the Middle Ages, it has been extended to other tasks, such as philosophical, scientific ones, formulated within the limits of the Jewish culture. The main difference is that this logic has never been Aristotelian from the very beginning. If we take the Aristotelian logic, then it is constructed by assuming that there are higher genera and there are their particular varieties, i.e. species. Thereby any reasoning is being

set up if we know a genus, its properties and transfer this knowledge down onto species. That is we move down from above in our reasoning, vertically. In Judaism, reasoning of this kind is unacceptable as a whole, because Judaic reasoning is constructed not vertically, but horizontally: from some species to other species and from some genera to other genera. As a result, there are assumed many points of view, different families of reasoning agents, i.e. a kind of multi-agentness.

Roots of Jewish logic are undoubtedly religious. Nevertheless, roots of the Western world-outlook and Aristotelian logic are in many respects religious, too. The point is that presuppositions of Jewish logic and Aristotelian one are too different. The assumption of Aristotelian logic consists in that there are many divine entities, gods, axioms. Consequently, from these axioms any particular knowledge is deduced. On the other hand, in Judaism it is supposed that there exists just one divine entity, God, the Lord. Therefore inferring from genera is a prerogative only of God. In their reasoning the Jews should move only horizontally. Now in computer science there is growing heightened interest in reasoning not from generals to particulars, but from some particulars to other particulars, i.e. massive parallelism. Thence knowledge which is entailed from religious premises of Judaism can really be useful in modern computer science.

Why do massive parallelism and interactivity awake such an interest of modern computer science? Let us reflect. Assume that we wish to design a robot that is able to understand semantics. On the one hand, to make so that the robot can understand real values (denotative meanings) is workable in general. It is sufficient to teach him to compare images of real objects with appropriate linguistic expressions. But how can we teach him to detect abstract values (connotative meanings)? That is the question.

In modern logic it is ascertained that abstract values as such are not present. They are a result of convention in this or that group of people. For this reason there are no universal abstract

values. It can easily be proved. For instance, we can ask a number of people to define love. After that we receive a huge family of ill-matched definitions. And if we ask representatives of different cultures to define love, in this case the distinction in results is even more appreciable.

It means that semantics is a part of pragmatics and abstract values are actually pragmatic values and not semantic ones. The robot can understand abstract values just in case he is able to know how to interact with people successfully, i.e. how to be a full member of human society. Pragmatic values appear results of interactions within the limits of massive parallelism of human behaviors.

The universal theory of activity which is while only planed to be obtained will become pure pragmatics. In this theory all pragmatic relations will be formalized. The Platonism in the European culture then will disappear as a whole. Any activity, any human interaction, any human conventions will become completely transparent for mathematicians and programmers.

But what will it cause? First of all radical transformations of all our society! For example, for a long time one has spoken about perspectives of creating an electronic government (e-government) when the role and right of officials and managers begin to be transferred to computer systems. And so, the universal theory of activity will allow us to realize the project of electronic government and electronic management as much as possible for all mankind. Then will an era of libertarian communism occur?

The *life-world* (daily occurrence) is an ultimate source of any objectiveness and any evidences for us. If we admit that there is only the life-world, space of our direct interactions, then automatically we admit that there are only individual agents of activity who cooperate with each other in the various way, i.e. they collaborate, compete or are just engaged in parallel things. Social groups, from family to state, are an outcome of massive-parallel interactions of many and many individual agents. Social groups as

such do not exist, therefore out of everyday contexts (live-world interactions) they are losing any meaning.

For mathematics the assumption of that there is only the life-world (i.e. only our everyday practice) would mean the requirement to construct *pure pragmatics* or *pure praxeology*, i.e. the requirement to clarify not only how game rules in mathematics are born, but also what the ultimate source of these rules is. Pure (formal) pragmatics is a horizon of any possible mathematics, the pragmatic limits of activity of any mathematicians, though ours, though belonging to extraterrestrials. At the same time, pure pragmatics is a logical theory about activity on a broader scale, about pragmatic estimations of any propositions in principle. Therefore it has not only theoretical meaning (as horizon of any mathematical designing), but also a special common sense (as ordering and algorithmization of any game rules of individual agents in any area).

The intuition of pure pragmatics came to mind of the late Wittgenstein. He assumed how this discipline should look. Wittgenstein's major thesis is *meaning as use*, according to which there is no independent sphere of meanings and values, all meanings are contextual and defined by direct aims of our activity. This thesis focuses on how we can consider lexical items (words, propositions) and semantic items (meanings of words and propositions) in pure pragmatics.

Thus, pure pragmatics is *anti-Platonic*. It supposes pure constructivism that there is no independent sphere of propositions, there is no initial integrity before its incarnation (accordingly, the eidetic/Platonic approach is senseless basically).

In Judaic logic we can find out an original approach to pure pragmatics and praxeology. Also notice that in Judaism, anti-Platonic assumptions are so brightly expressed and the existential significance of the life-world is postulated. In order to show inseparability of Judaic logic from the Talmudic tradition, this logic is said to be *Talmudic Logic*. The title of book refers to this philosophical context of pure pragmatics. In the book we will

consider logic of Jews from the standpoint of analytic philosophy and symbolic logic. We will show that philosophical and logical ideas of the Talmud are really actual and after a certain interpretation they can be considered as popular logical introduction to the theory of pure pragmatics and massive-parallel reasoning.

The perceptive reader might ask if Talmudic logic is so specific and esoteric in fact and how far it differs from general logic. My reply is that it is not esoteric, but it presents a unique logical culture that may be interesting from the point of view of analytic philosophy. The purposes of Talmudic logic consist in considering the Torah statements as axioms, initial data, the first premises of our reasoning and this logic aims to entail new data by using traditional Judaic hermeneutics containing some logical inference rules. Some of these rules have analogies in traditional logic, some others do not. However, the combination of these rules is unique indeed, which allows us to claim that there exists Talmudic logic different from other ones.

Notice that in the whole text the English translation of the Old Testament is quoted from

The Holy Bible. King James Version. New York: American Bible Society, 1999.

The English translation of the New Testament is quoted from

Murdock, James. *The Book of the Holy Gospel of Our Lord and Our God, Jesus the Messiah, a Literal Translation from the Syriac Peshito version*. New York: Robert Carter & Brothers; Massachusetts Bible Society, 1858.

The English translation of the Talmud is quoted from

Soncino Babylonian Talmud. Translated into English with Notes, Glossary and Indices under the Editorship of Rabbi Dr. I. Epstein. London: the Soncino Press, 1935—52.

Transliterations of Hebrew letters in the whole text as a rule satisfy the following table:

letter	*name*	*translite ration*	*letter*	*name*	*translite ration*
א	’alef	’	ל	lamed	l
ב	beyt	b, v	מ, ם	mem, *final* mem	m
ג	gimel	g	נ, ן	nun, *final* nun	n
ד	dalet	d	ס	samekh	s
ה	hey	h	ע	‘ayin	‘
ו	waw	w	פ, ף	pey, *final* pey	p, f
ז	zayin	z	צ, ץ	Ẓadi, *final* Ẓadi	Ẓ
ח	ḥet	ḥ	ק	quf	q
ט	ṭet	ṭ	ר	reš	r
י	yud	y	ש	šin, śin	š, ś
כ, ך	kaf, *final* kaf	k,kh	ת	taw	t

All main logical and mathematical ideas of this book were published in peer-reviewed journals such as (1) Journal of Multiple-Valued Logic and Soft Computing, (2) Journal of Cellular Automata, (3) Journal of Applied Non-Classical Logics, (4) Advances in Soft Computing, (5) Bulletin of the Section of Logic, (6) Studies in Logic, Grammar and Rhetoric, (7) Kybernetes, (8) Logic and Logical Philosophy, (9) *p*-Adic Numbers, Ultrametric Analysis, and Applications, (10) New Mathematics and Natural Computations, (11) Acta Analytica, (12) Journal of Uncertain Systems, (13) Argumentation, etc. The first logical books in English devoted to Talmudic logic are as follows:

Moshe Koppel. *Meta-Halakhah: Logic, Intuition, and the Unfolding of Jewish Law*. Jason Aronson, Inc., 1997.

Andrew Schumann (ed.). *Judaic Logic*. Gorgias Press, 2010.

Andrew Schumann (ed.). *Modern Review of Judaic Logic*. Special issue of History and Philosophy of Logic, Taylor & Francis, 2011.

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Chapter 1. The logical nature of revelation

1. Judaism and social constructions

We live in the democratic world where rights and freedom are guaranteed to the people. In this world everyone if (s)he wishes is capable to realize himself (herself). Henry Louis Mencken, the American journalist (1880—1956), claimed that democracy is the theory that the common people know what they want and deserve to get it good and hard.

Democracy assumes the activity of human beings, abilities to act and behave, to build the society so that it is comfortable for everybody. Such an assumption of human activity is based on the social constructivism, a belief that "Nothing is given. All is constructed" (*Bachelard*). The social constructivism allows us to expand the frameworks of human activity beyond all bounds, to transform the settled social frames into absolutely new ones, giving significant opportunities for human actions.

As Ian Hacking (*1999*) shows, the social constructivism assumes the following algorithm of transformation of social frames:

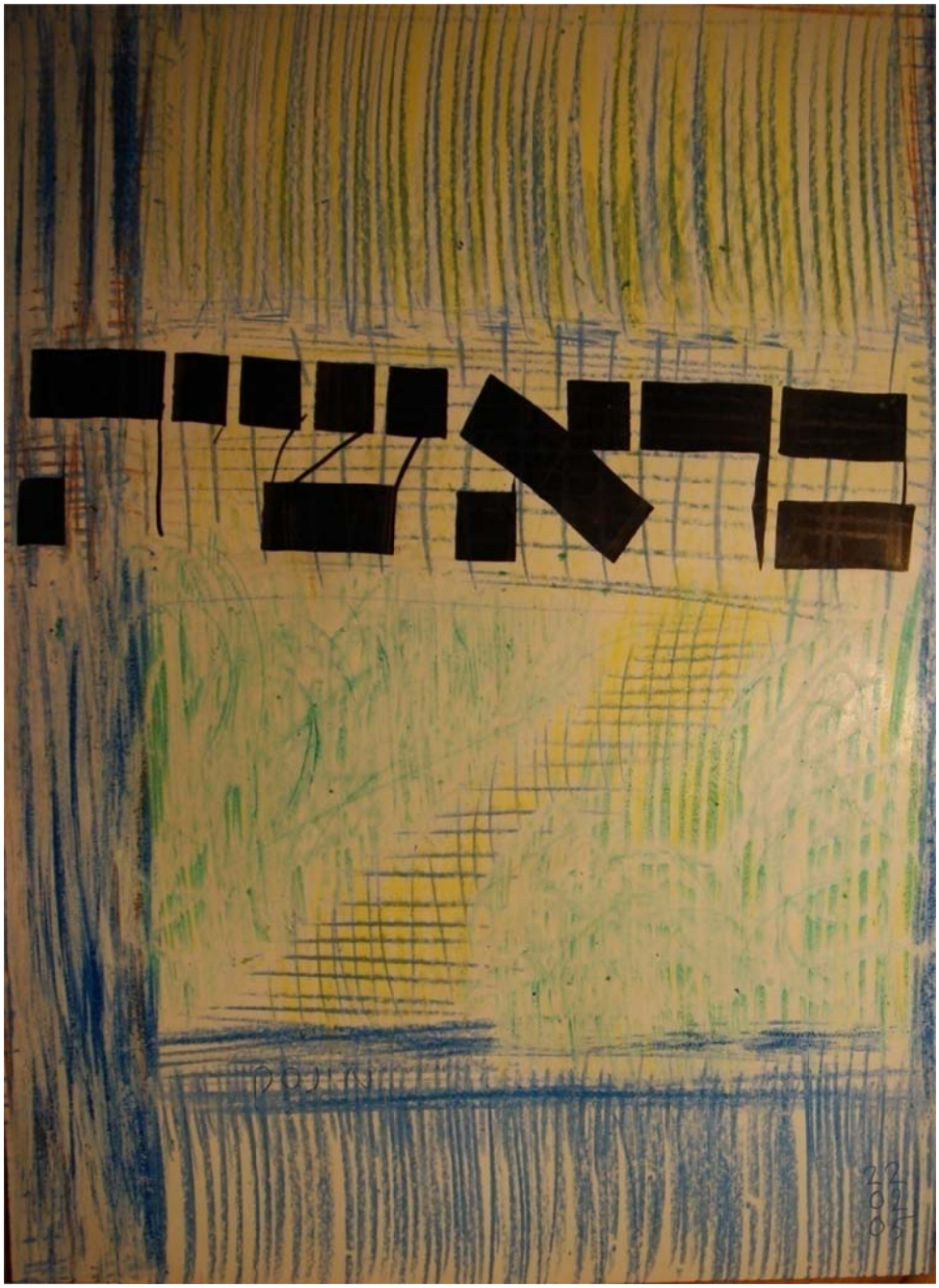
(1) A social construction, *X* (gender, nationality, family, etc.), is taken for granted; it appears to be inevitable.

(2) However, *X*, as it is at present, is not inevitable and is quite bad as it is.

(3) We would be much better off if *X* radically transformed or eliminated.

There is a number of manifestations of social constructivism. Everywhere, where one is talking about overcoming social conventions, about the boldness of decisions, manifestations of this outlook are found out.

According to constructivism, there is nothing eternal. All flows, all varies. As Michel Foucault claims, through history the sexual predilections have varied even. The notion of mental health, legal treatments of crime and so on have changed permanently. Past



Picture 1 The title page of the Book of Genesis. Version 1

Picture 2 The title page of the Book of Genesis. Version 2

generations perceived almost all social notions in the different way. Many things that were considered abnormal are already a norm, and many things that in former times were regarded as a crime are conceived already as innocent prank. Marquis de Sade was estimated by contemporaries as pathological person and for many years he spent in prison. Now he became a prophet of emancipation, the person who outstripped time.

Social constructivism is confronted with ideology of traditional religions, in particular with ideology of Judaism. This ideology consists in supporting the unbounded repetition of past schemes of social behavior; it is directed towards the maintenance of eternal tradition, and plays role of example for *eternal social technologies*. Everything is perishable and mortal, me too, but eternal social technologies will remain in their immutable cleanliness not only after my death, but also after the death of my children and even grandsons. To these technologies nothing is superior. Even time sands in awe of them, because it headlong runs, but eternal social technologies remain.

Social constructivism and eternal social technologies are extreme contrasts. The first type urges us to plunge into everyday vanity, into activity.

The second type enters into the sacred space of eternity where time stops. Such an enormous distinction between two types of human relations refers to that social constructivism and eternal social technologies have ever been irreconcilable enemies. In Judaism it is affirmed that the history of humanity is a history of struggle between two ways of understanding the life.

By Judaism, the ancestor of social constructivism, its patriarch was Amalek, the strongest chief in Canaan (Palestine). Amalek was the first who battled with Israel as it is told about him:

> רֵאשִׁית גּוֹיִם עֲמָלֵק וְאַחֲרִיתוֹ עֲדֵי אֹבֵד
>
> Amalek was the first of the nations; but his latter end shall be that he perish for ever (*Numbers* 24:20).

Thus, Amalek is evaluated as the first and the last from the people. We can remember that about the Lord there is the verse:

> אֲנִי רִאשׁוֹן וַאֲנִי אַחֲרוֹן
>
> I am the first, and I am the last (*Isaiah* 44:6).

Therefore Amalek is not just one of the ordinary tribe chiefs in Canaan, he is a spiritual source, the leader of all seven pagan tribes in Palestine, the root for them, רֵאשִׁית הַגּוֹיִם, the head of the people.

According to the Bible, by time of the military company inspired by Amalek, Israel was concerned in many improbable stories such as their escape from slavery in Egypt, the passage of the Jews through the Red Sea, manna, etc. All the people were afraid of Israel, because they knew that the Lord is the God of Israel. Nevertheless, Amalek, perfectly informed about these facts, dares to act in battle against the Israelites. Undoubtedly, Amalek was convinced of force of social constructivism. Amalek always liked hardiness, resoluteness, fearlessness and strong-willed decisions. Nothing promised him defeat. By time there was nothing more constructivist, than his courageous and deeply thought plan of victory. However, Amalek lost.

In Kabbalistic doctrines, the human evil is called *qelipah* [קְלִיפָּה], shell, capsule, thereby it is meant that evil is a condition when God's light cannot get into our heart in any way. It is having fancy for himself (herself), egoistic attitudes towards others, retreating from open human relations. If, for example, we start to consider moral obligations only as a way of manipulating us, we already becomes *qelipah*, we are already fenced off from the Divine light. In Hasidism, social constructivism is called *qelipat 'amaleq* [קְלִיפַּת עֲמָלֵק], Amalek's shell. It is the lowest *qelipah*, a source for absolutely all *qelipot* [קְלִיפּוֹת]. According to Judaism, any instability in human relations has its roots in social constructivism, strong-willed

self-sufficiency, arrogance; *qelipah* ever contains *toqef wa'oz* [תוֹקֶף וָעוֹז], force and impudence.

Amalek was one of the brightest representatives of social constructivism for the whole Biblical history. But, in spite of his sad end, he had many followers. Aman became one of them. According to the Judaic legend, Aman's sons studied the Torah. However, they had the same plan to take the Jewish people into camp, and this goal was a kind of impudence taking into account their knowledge of the Torah.

Evil cannot exist for a long time, since in *qelipot* [קְלִיפוֹת] there is no essence, there is no *mamašut* [מַמָּשׁוּת], reality of existence; in them there is only an absence of existence [מְצִיאוּת הַהֶעְדֵּר]. Any harm eats from the garbage of holiness [פְּסוֹלֶת דִּקְדוּשָׁה], and sooner or later it will be completely exterminated. Any pure constructivist plan is doomed to failure. Any negation of eternal social technologies is ever fraught with defeat.

By Judaism, therefore, eventually, all *qelipot* will be absolutely eradicated, and there will come a victory of eternal social technologies. The beginning of this time will be marked by the arrival of the Messiah. In the Holy Writ about those future events it is told so:

> וַיִּקַץ כְּיָשֵׁן ה'
>
> Then the Lord awaked as one out of sleep (*Psalm* 78:65).
>
> עוּרָה לָמָּה תִישַׁן ה'
>
> Awake, why sleepest thou, O Lord? (*Psalm* 44:23).

The awakening of the Lord will mean the full disclosing of Divine light [אוֹר אֵין סוֹף] when God opens His essence in all glory, through internal aspects of His will as it is written:

> יָאֵר פָּנָיו אִתָּנוּ

> … and cause his face to shine upon us (*Psalm* 67:1)

Hence, *qelipah* becomes a source of souls at those people who profess social constructivism and are in a hurry to live, plucking fruits of pleasure. On the other hand, holiness [קְדוּשָׁה] is a source of souls at those people who obey eternal social technologies.

Eternal social technologies cannot be thought up, designed. They always already exist, we should just make them out. They are pragmatic limits of our thinking and our behavior. Outside of eternal social technologies there is nothing, only emptiness and madness. For instance, if I was not respectful in relation to my parents, it would lead to my own destruction and would make me worse. Therefore since the very Ancient times it is accepted to say that *the eternal social technology is an outcome of revelation*. In other words, the Lord is a source of eternal technologies, He is above any worlds (*'olamot*, עוֹלָמוֹת), any types of social practice, any social frames. As it is written:

> Let them praise the name of the Lord: for his name alone is excellent; his glory is above the earth and heaven (*Psalm* 148:13).

In Hebrew the eternal social technology is called the Torah [תוֹרָה], at once it is meant that the Pentateuch is a legislative basis of the Torah.

The Pentateuch and its interpretations were given at the Mountain Sinai. Since then the Divine laws and their interpretations have been transmitted orally from generation to generation by the sages of Israel, then by the heads of the schools of learning. The first generations of the teachers (doctors) from the time of Simeon the Just, who lived about 300 B.C.E., have been known as Tanaim. Almost all of them have served either as nasi (the chief of Sanhedrin, i.e. of the highest Judaic court, נָשִׂיא) or ab-beth-din (president of the tribunal, i.e. of rabbinical court, אב בית דין). Rabbi

Judah the Prince (Yehudah ha-Naśi') was one of the last among these Tanaim. He decided to write down the traditional part of the law and made a compilation of all the laws, versions and interpretations. The code of Rabbi Judah the Prince, called *Mishnah* (repetition, מִשְׁנָה), became authoritative and was accepted by all the later generations of the people of Israel.

Then the Mishnah was included as the nucleus into the larger and more comprehensive work, called *Talmud* (learning, תַּלְמוּד). The additional part to the Mishnah in the Talmud is called *Gemara* (complement, גְּמָרָא). The Talmud contains constructions of the law, comments, explanations and glosses upon the Holy Scriptures, ordinances, decrees, disputes and debates of the sages, who lived from 300 B.C.E. to 500 C.E. The Talmud is a Judaic *corpus juris civilis*. The teachers (doctors) of the Gemara are known as Amoraim (debaters).

There are two Gemaras, one of which was probably compiled by Rabbi Yoḥanan bar Nafḥa', in Tiberias, about 380 C.E., and contains the learning of the schools of Palestine and the other one was perhaps compiled by Rav 'Aši and Ravina', in Babylon, in the 5th century, and it contains the learning of the Babylonian schools of Nehardea, Nisibis, Mahoza, Pumbeditha and Sura. As a result, there are two versions of the Talmud, respectively, called Jerushalmi (Jerusalem) and Babli (Babylonian). Notice that after Amoraim, the teachers are called Saburaim (expounders).

Many other treatises known under the names of *Barayta* (external, בָּרַיְתָא), *Tosefta* (addenda, תּוֹסֶפְתָּא), *Mekhilta* (roll, מְכִילְתָּא), *Sifra* (book, סִפְרָא), and *Sifre* (books, סִפְרֵי) were not completely accepted by the doctors of the Talmud, however, these treatises are often cited and commented upon in the Gemara in the course of the debate.

The Mishnah is written in a pure Hebrew. The Jerusalem Gemara is written in a Hebrew-Syriac dialect and the Babylonian Gemara is an Eastern-Aramaic dialect. The Talmud is divided into six Sedarim (orders). Each Seder is subdivided into treatises called

Masekhtoth (texture). Further, each Masekhta is subdivided into Perakim (sections), and each Perek into paragraphs, which refer to the head words of the Mishnah.

The first Seder is called *Zera'im* (seeds, זרעים) and contains eight treatises of the grace to be said, tithes, first fruits, free gifts to the priests, Levites and the poor, prohibited intermixtures of different varieties of seeds, etc. The second Seder, *Mo'ed* (festivals, מועד), is divided into eleven treatises, treating of the Sabbath, the Passover and other days to be observed. The third Seder, *Našim* (women, נשים), has seven treatises and is devoted to the institution of the levirate (i.e. the duty devolving on a person to marry a deceased brother's widow if he died without issue), marriage and divorce and their incidents, adultery, vows of married and unmarried females, and also abstinence. The fourth Seder, *Neziqin* (injuries, נזיקין), contains eight treatises dedicated to the civil and criminal laws, methods of procedure, pleading and evidence, tribunals, modes of punishment, incidents of trial. This Seder also contains the *Pirqe Abot* (sayings of the fathers, פרקי אבות), ethical teachings of Tanaim. The fifth Seder, *Kodoshim* (sacred things, שימקד), has nine treatises, treating of the sacrifices in the temple, of the first-born animals, of forbidden food and the prescribed mode of slaughtering animals. It also contains a detailed description of the Herodian Temple. The sixth Seder, *Taharot* (purifications, טהרות), in nine treatises, is devoted to disqualification for communion through diseases and impurities as well to the modes of cleansing required in appropriate cases.

2. Are either charisma or logic true signs of revelation?

Charisma is a key notion in explicating the nature of leadership. In classical sociology charisma was defined as follows:

> Charisma is a certain quality of an individual personality by virtue of which he is set apart from ordinary men and treated as

endowed with supernatural, superhuman, or at least specifically exceptional powers or qualities. These are such as are not accessible to the ordinary person, but are regarded as of divine origin or as exemplary, and on the basis of them the individual concerned is treated as a leader (*Weber*).

The term *charisma* comes from the Greek word *χαρισμα*, which means 'favor freely given' or 'gift of grace.' Basic connotations of this word Max Weber refers to are Christian. For instance, gifts (charisma) are understood by the Apostle as favor of the Lord, namely:

ܘܐܠܗܐ ܣܝܡ ܒܥܕܬܗ ܩܕܡܝܐ ܫܠܝܚܐ ܬܪܝܢ ܒܬܪ ܗܠܝܢ ܢܒܝܐ ܬܠܬ ܒܬܪ ܗܠܝܢ ܡܠܦܢܐ ܬܠܬ ܒܬܪ ܗܠܝܢ ܥܒܕ ܢܣܝܬܐ ܒܬܪ ܗܠܝܢ ܐܣܝܐ ܘܦܣܝܩܐ ܣܓܝ̈ܐ ܒܬܪ ܗܠܝܢ ܣܓܝ̈ܐ ܠܫܢ̈ܝ̈ܐ

For God hath placed in his church, first, legates; after them, prophets; after them, teachers; after them, workers of miracles; after them, the gifts of healing, and helpers, and leaders, and [various] kinds of tongues (*1 Corinthians* 12:28).

Also, the following passage concerns the same connotations:

ܗܟܝ ܓܒ ܐܢܐ ܕܟܠ ܐܢܫܐ ܐܝܟ ܢܦܫܝ ܢܗܘܝ ܐܠܐ ܟܠ ܚܕ ܐܝܬ ܠܗ ܡܘܗܒܬܗ ܡܢ ܐܠܗܐ ܚܕ ܗܟܢܐ ܘܚܕ ܗܟܢܐ

For I would that all men might be like me in purity. But every man is endowed with his gift of God; one thus, and another so (*1 Corinthians* 7:7).

The highest charisma from the Christian point of view is said to be love, first the love for God and second for other people. The ability to love all others is the most exceptional quality:

ܐܦܠܘ ܕ ܐܢܫ ܠܐ ܐܫܬܘܝ ܐܢܐ ܠܝ ܚܒܐ ܠܐ ܐܫܬܘܝ ܐܢܐ ❖ ܐܦܠܘ ܕ ܐܢܫ ܢܒܝ ܠܗ ܢܒܝܐ ܐܢܐ ❖ ܐܦܠܘ ܕ ܐܢܫ ܢܬܠ ܟܠ ܡܕܥ̈ܐ ❖ ܐܦܠܘ ܕ ܐܢܫ ܐܝܬ ܠܗ ܟܠ ܗܝܡܢܘܬܐ ❖ ܐܦܠܘ ܕ ܐܢܫ ܢܬܠ ܓܫܡ̈ܐ ❖ ܐܦܠܘ ܕ ܐܢܫ ܢܬܠ ܓܘܫܡܗ ❖ ܐܦܠܘ ܕ ܐܢܫ ܢܬܠ ܟܠ ܡܕܥ̈ܐ

If I could speak in every tongue of men, and in that of angels, and there should be no love in me, I should be like brass that resoundeth, or the cymbal that maketh a noise. And if there should be in me [the gift of] prophecy, and I should understand all the mysteries, and every science; and if there should be in me all faith, so that I could move mountains, and love should not be in me, I should be nothing. And if I should feed out to the destitute all I possess; and if I should give my body to be burned; and there should be no love in me, I gain nothing. Love is long-suffering, and is kind; love is not envious; love is not boisterous; and is not inflated; and doth nothing that causeth shame; and seeketh not her own; is not passionate; and thinketh no evil; rejoiceth not in iniquity, but rejoiceth in the truth; beareth all things, believeth all things, hopeth all, and endureth all. Love will never cease. But prophesyings will end; and tongues will be silent; and knowledge will vanish. For we know but partially; and we prophesy but partially. But when completeness shall come, then that which is partial will vanish away. When I was a child, I talked as a child, and I reasoned as a child, and I thought as a child: but when I became a man, I laid aside the things of childhood. And now we see, as by a mirror, in similitude; but then face to face: now I know partially; but then shall I know, just as I am known. For these three things are abiding, faith, and hope, and love; but the greatest of these is love (*1 Corinthians* 13:1—13).

Charisma or God's gift to be an extraordinary man enables the leadership of that man. So, the Qur'an describes leadership as an appointment from the Lord and His gift as follows:

أَهُمْ يَقْسِمُونَ رَحْمَةَ رَبِّكَ نَحْنُ قَسَمْنَا بَيْنَهُم مَّعِيشَتَهُمْ فِي الْحَيَاةِ الدُّنْيَا وَرَفَعْنَا بَعْضَهُمْ فَوْقَ بَعْضٍ دَرَجَاتٍ لِيَتَّخِذَ بَعْضُهُم بَعْضًا سُخْرِيًّا وَرَحْمَتُ رَبِّكَ خَيْرٌ مِّمَّا يَجْمَعُونَ

> Is it they who would portion out the Mercy of thy Lord? It is We Who portion out between them their livelihood in the life of this world: and We raise some of them above others in ranks, so that some may command work from others. But the Mercy of thy Lord is better than the [wealth] which they amass (*Qur'an* 43:32; translated by Yusuf Ali).

Theistic religions consider prophets as true and honest leaders, i.e. as leaders with charisma; just they must be followed by all others. Maimonides describes the essence of prophecy, its social role and its Divine origin as follows:

> מיסודי הדת, לידע שהאל מנבא את בני האדם; ואין הנבואה חלה אלא על חכם גדול בחכמה, גיבור במידותיו, ולא יהא יצרו מתגבר עליו בדבר בעולם אלא הוא מתגבר בדעתו על יצרו תמיד, בעל דעה רחבה נכונה עד מאוד. אדם שהוא ממולא בכל המידות האלו, שלם בגופו, כשייכנס לפרדס ויימשך באותן העניינות הגדולים הרחוקים, ותהיה לו דעת נכונה להבין ולהשיג, והוא מתקדש והולך ופורש מדרכי כלל העם ההולכים במחשכי הזמן, והולך ומזרז עצמו ומלמד נפשו שלא תהיה לו מחשבה כלל באחד מדברים בטילים, ולא מהבלי הזמן ותחבולותיו, אלא דעתו תמיד פנויה למעלה, קשורה תחת הכיסא, להבין באותן הצורות הקדושות הטהורות, ומסתכל בחכמתו של הקדוש ברוך הוא כולה מצורה ראשונה עד טבור הארץ, ויודע מהן גודלו-- מיד רוח הקודש שורה עליו.
>
> It is [one] of the foundations of [our] faith that God conveys prophecy to man.
> Prophecy is bestowed only upon a very wise sage of a strong character, who is never overcome by his natural inclinations in any regard. Instead, with his mind, he overcomes his natural inclinations at all times. He must [also] possess a very broad and accurate mental capacity.
> A person who is full of all these qualities and is physically sound [is fit for prophecy]. When he enters the *Pardes* and is

drawn into these great and sublime concepts, if he possesses a accurate mental capacity to comprehend and grasp [them], he will become holy. He will advance and separate himself from the masses who proceed in the darkness of the time. He must continue and diligently train himself not to have any thoughts whatsoever about fruitless things or the vanities and intrigues of the times.

Instead, his mind should constantly be directed upward, bound beneath [God's] throne [of Glory, striving] to comprehend the holy and pure forms and gazing at the wisdom of the Holy One, blessed be He, in its entirety, [in its manifold manifestations] from the most elevated [spiritual] form until the navel of the earth, appreciating His greatness from them. [After these preparations,] the divine spirit will immediately rest upon him.

When the spirit rests upon him, his soul becomes intermingled with the angels called *ishim,* and he will be transformed into a different person and will understand with a knowledge different from what it was previously. He will rise above the level of other wise men, as [the prophet, Samuel] told Saul [I *Samuel* 10:6]: "[The spirit of God will descend upon you] and you shall prophesy with them. And you will be transformed into a different person." (Maimonides. *Mishneh Torah. Yesodey haTorah* 7:1—2; translated by Eliyahu Touger).

The Hebrew word 'prophet' (*nby'*, Hebrew: נביא, Arabic: **نبي**) with the Judaic meaning is used in Islam with the close meaning as well. It also denotes human beings chosen by God to deliver revelation. However, in Islam we find out another additional designation of prophecy, 'messenger' or 'apostle' (*rśwl*, Arabic: **رَسُول**). The messenger is sent to bring the revelation, religion:

هُوَ الَّذِي أَرْسَلَ رَسُولَهُ بِالْهُدَى وَدِينِ الْحَقِّ لِيُظْهِرَهُ عَلَى الدِّينِ كُلِّهِ وَكَفَى بِاللَّهِ شَهِيدًا

It is He Who has sent His Messenger with Guidance and the Religion of Truth, to proclaim it over all religion: and enough is Allah for a Witness (*Qur'an* 48:28, see also 9:33, 61:9; translated by Yusuf Ali).

In the Qur'an, the word 'messenger' is used only for Moses, Jesus and Muhammad. Thereby, Muhammad is evaluated as the highest apostle. Accordingly, the notion of leadership in Islam is modelled by references to him:

> لَقَدْ كَانَ لَكُمْ فِي رَسُولِ اللَّهِ أُسْوَةٌ حَسَنَةٌ لِّمَن كَانَ يَرْجُو اللَّهَ وَالْيَوْمَ الْآخِرَ وَذَكَرَ اللَّهَ كَثِيرًا
>
> Ye have indeed in the Messenger of Allah a beautiful pattern *(of conduct)* for any one whose hope is in Allah and the Final Day, and who engages much in the Praise of Allah (*Qur'an* 33:21; translated by Yusuf Ali)

To sum up, in theistic religions the leader should possess charisma, strongly feel his dependence upon the Lord and must observe His commands in relation to any work. The leader cannot transgress Divine commandments or his own instructions, he cannot change the religion as well:

> וכן נביא שעבר על דברי עצמו והכובש נבואתו חייב מיתה בידי שמים ובשלשתן נאמר אנכי אדרוש מעמו וכן אם יאמר לנו הנביא שנודע לנו שהוא נביא לעבור על אחת מכל מצות האמורות בתורה או על מצות הרבה בין קלות בין חמורות לפי שעה מצוה לשמוע לו
>
> Similarly, a prophet who violated his own prophetic instructions, and one who refrains from prophesying, are liable for death at the hand of God, since concerning the three of them, it is said, "I will seek [retribution] from him."
> When a prophet—who has already proven himself to be a prophet—instructs us to violate one of the mitzvot [commandments] of the Torah or many mitzvot, whether they be of a severe or light nature, for a limited amount of time, it is a mitzvah to listen to him (Maimonides. *Mishneh Torah. Yesodey haTorah* 9:3; translated by Eliyahu Touger).

Since theistic religions were established and formed, prophet's role has been played by sages. Just they possess charisma

and should be true leaders in our society, as it is claimed in theistic faiths. In the Talmud it is said about as follows:

> אמר רבי אבדימי דמן חיפה מיום שחרב בית המקדש ניטלה נבואה מן הנביאים וניתנה לחכמים אטו חכם לאו נביא הוא הכי קאמר אע"פ שניטלה מן הנביאים מן החכמים לא ניטלה אמר אמימר וחכם עדיף מנביא שנאמר (תהילים צ) ונביא לבב חכמה מי נתלה במי הוי אומר קטן נתלה בגדול אמר אביי תדע דאמר גברא רבה מילת' ומתאמר' משמיה דגברא רבה אחרינא כוותי' אמר רבא ומאי קושיא ודילמא תרוייהו בני חד מזלא נינהו אלא אמר רבא תדע דאמר גברא רבה מילתא ומתאמרא
>
> R. Abdimi from Haifa said: Since the day when the Temple was destroyed, prophecy has been taken from the prophets and given to the wise. Is then a wise man not also a prophet? — What he meant was this: Although it has been taken from the prophets, it has not been taken from the wise. Amemar said: A wise man is even superior to a prophet, as it says, And a prophet has a heart of wisdom. Who is compared with whom? Is not the smaller compared with the greater? Abaye said: The proof [that prophecy has not been taken from the wise] is that a great man makes a statement, and the same is then reported in the name of another great man. Said Raba: What is there strange in this? Perhaps both were born under one star. No, said Raba; the proof is this, that a great man makes a statement and then the same is reported (*Baba Bathra* 12a).

Thus, in theistic religions true leaders are presented just by men, who have charisma, follow God's commands and protect their religion. Charisma is a necessary ability to become a leader. Such understanding of charisma as origin of leadership is almost classical. Max Weber defines charisma in the same way. The only difference is that he eliminates the religious content and proposes the more formal approach applied to any kind of social behavior, including theistic religions as well.

In the modern/postindustrial world, however, the charisma nature has considerably changed. In theistic religions and Weber's texts another meaning was contained. Now charisma is not directly connected with a certain quality of an individual personality,

because it is formed as a media image and it grows out of work of many people on the creation and distribution of this media image. Such charisma satisfies values of open society and it is not possible behind its limits.

In religious communities, the traditional, Weber's understanding of charisma has kept validity. Personal abilities have still their high value in the religious charisma and there is no such dependence on mass media. Also, it is possible to claim that in conservative religious communities, the traditional feature of social reflexion (ways of estimations of social behavior) remains in force. These estimations considerably differ from appropriate estimations of open society. As a result, their conflict takes place sometimes.

Religious leaders of conservative religious communities are negatively perceived in mass media of open society very often. The ground is in a distinction of social reflexions. For this reason, open society shows a steady interest to scandals connected with religious leaders. These scandals usually cause a high public resonance.

There are funny and ridiculous scandals. For example, the story of Abrahim Ghait, the young Libyan influential imam who was invited to the UK for Ramadan's sermons (i.e. dedicated to the sacred month of Moslems). This young imam appeared an exhibitionist, who showed himself before two UK citizens, one 28-year-old and another 12-year-old:

> Abrahim Ghait, who was invited to Britain to celebrate Ramadan, exposed himself to a 28-year-old woman as she made her way home from the gym. Three days later he repeated the offence in front of a 12-year-old girl—telling her his genitalia was named 'Lexie.' Ghait is famous in the Muslim world for memorising the Koran by the age of 17. Yesterday he pleaded guilty to two charges under the Sexual Offences Act of exposing his genitals with the intention that someone would see them and be caused alarm or distress. Magistrates jailed him for six weeks on each charge, to serve side-by-side, and placed him on the Sex Offenders' Register for seven years (*Dailymail*, 23rd September 2010).

Other sex scandals with religious leaders are more everyday and have been implied, for example, by harassment. These stories do not differ from secular scandalous events such as a story of Clinton and Lewinsky. The scandal with Rabbi Leib Tropper became such an event. He has offered giurs (conversion to Jewry) to women for sexual actions:

> NEW YORK (JTA)—The head of an organization that oversees conversions to Judaism has resigned amid accusations of sexual improprieties.
> Rabbi Leib Tropper, head of the Eternal Jewish Family based in New York, is leaving the independent organization he founded five years ago.
> Tropper, in a statement released this week by the organization, said his desire to "pursue a variety of other interests" led him to resign.
> Stories have surfaced recently of his sexual misconduct while overseeing the conversion process. A woman, whose name has not been released, told FailedMessiah.com that Tropper allegedly asked for sexual favors in return for completing her conversion process.
> Tropper, the woman claims, also arranged for sexual encounters between herself and Tropper's wife that the rabbi allegedly watched. Tropper also is accused of allegedly pressuring woman to engage in sexual encounters with other men.
> According to FailedMessiah.com, the alleged female victim made unauthenticated video and audio tapes of sexual encounters with Tropper intended for leading rabbis involved with Eternal Jewish Family, yet the tapes were leaked through the Internet to various bloggers over the past few days.
> "These tapes appear to support her claims," FailedMessiah.com wrote.
> Tropper did not address the accusations in the statement announcing his resignation (*JTA*, December 17, 2009).

Conservative religious communities present minorities in our secular world. Therefore very often religious people are close-

mouthed and prefer to communicate to coreligionists, i.e. within a communion. As a result, the mutual support carried out by coreligionists may disembogue into different forms of public corruption. Many mediated public scandals are connected with disclosers of corruptions caused by this closeness of conservative religious communities, e.g.:

> WASHINGTON (JTA)—Several rabbis were arrested as part of a public corruption and international money-laundering investigation in New Jersey.
> According to reports, among the 44 people arrested Thursday morning by the FBI along with the rabbis were the mayors of three New Jersey towns, a deputy mayor and a state assemblyman. They were to appear in federal court in Newark later in the day.
> The money-laundering suspects were accused of moving "at least tens of millions of dollars through charitable, nonprofit entities controlled by rabbis in New York and New Jersey," according to a release by acting U.S. Attorney Ralph Marra.
> The rabbis arrested were religious leaders from Syrian Jewish neighborhoods in Deal and Elberon, N.J., as well as Brooklyn, N.Y. The Asbury Park Press reported that the investigation involved the Deal Yeshiva, a Sephardic institution that has a boys' division in Ocean Township and a girls' school in West Long Branch.
> The charged rabbis include Rabbi Saul Kassin, 87, of the Sharee Zion synagogue in Brooklyn, N.Y.; Eliahu Ben Haim, 58, the principal rabbi of Congregation Ohel Yaacob in Deal; Edmond Nahum, 56, of the Deal Synagogue; Mordchai Fish, 56, of Congregation Sheves Achim in Brooklyn; and Lavel Schwartz, 57, Fish's brother. They were charged with money laundering (*JTA*, July 23, 2009).

The human culture has considerably changed for the past hundred years. One of the appreciable changes is in an expansion of the horizon what should be considered as social. It attracts the gap between religious outlook and religious behavior. On the one hand, a person can follow religious outlook, be ethic, know

religious dogmatics and it is possible that, at the same time, he does not keep the obligatory religious sacramental. On the other hand, there can be people who thoroughly follow the religious sacramental, but their outlook precisely is not ethic. Sex scandals with religious leaders illustrate just the second category of people.

Thus, thanks to occurring mass media as the major component of open society, the charisma nature is alienated (*Entfremdung*) from the person. Today's mass media has brought in a verdict of who has charisma indeed and who not. Nowadays, possessing charisma is a competence to behave as celebrity, not more. To exemplify our words, let us consider two cases of theistic religious leaders as true mass media celebrities.

Our first case of religious celebrity is presented by Mother Teresa (26 August 1910—5 September 1997), a Roman Catholic nun with the Albanian origin and Indian citizenship. She founded the Missionaries of Charity in Calcutta in 1950 (notice that in 1965 this society became an International Religious Family by a decree of Pope Paul VI). Then, two years after with the support of Indian officials she converted an abandoned Hindu temple of goddess Kali into the 'Kalighat Home for the Dying' (later 'Kalighat, the Home of the Pure Heart'), a free hospice made available by the city of Calcutta. About patients of this hospice she said:

> A beautiful death is for people who lived like animals to die like angels—loved and wanted.

The first documentary film on Mother Teresa was made in 1969 by BBC, one of the world's leading English-speaking public service broadcasters. Then the first devoted to her book 'Something Beautiful for God' (Harper Collins Publishers, 1972) written by the well-known media personality, Malcolm Muggeridge, appeared at one of the world's leading English-language publishers. Since then she became a public icon, mass media symbol—a nun who ministered to the poor, sick, orphaned, and dying. Since then Mother Teresa has been given some of the most prestigious awards:

the Pope John XXIII Peace Prize (1971), the Kennedy Prize (1971), the Nehru Prize (1972), the Albert Schweitzer International Prize (1975), the Nobel Peace Prize (1979), States Presidential Medal of Freedom (1985), Congressional Gold Medal (1994), Honorary citizenship of the United States (November 16, 1996).

Mother Teresa never looked for popularity, but she knew how to be a true celebrity: she was attractive in her openness, sincerity, mercy. She did not hesitate to tell in public what she in fact thought of, e.g. once she expressed doubts about God's existence:

> Where is my faith? Even deep down ... there is nothing but emptiness and darkness ... If there be God—please forgive me. When I try to raise my thoughts to Heaven, there is such convicting emptiness that those very thoughts return like sharp knives and hurt my very soul ... How painful is this unknown pain—I have no Faith. Repulsed, empty, no faith, no love, no zeal, ... What do I labor for? If there be no God, there can be no soul. If there be no soul then, Jesus, You also are not true.

The Catholic Church recognized Mother Teresa's efforts a bit later than she became a celebrity and followed mass media attitudes towards her. In fine, she was beatified by Pope John Paul II and given the title Blessed Teresa of Calcutta (19 October 2003). In turn, in the first encyclical Deus Caritas Est (25 January 2006), Pope Benedict XVI mentioned Blessed Teresa of Calcutta three times and referred to her life as to an example of holiness:

> In the example of Blessed Teresa of Calcutta we have a clear illustration of the fact that time devoted to God in prayer not only does not detract from effective and loving service to our neighbour but is in fact the inexhaustible source of that service.

The role of mass media in public perceptions of Mother Teresa is too evident. Let us consider another case of religious celebrity, present-day's Judaic authority, Menachem Mendel Schneerson (5 Apri 1902—12 June 1994), known in many Hasidic Judaic communities as the Lubavitcher Rebbe or just the Rebbe.

Perhaps, he was the most prominent Hasidic rabbi of the 20th century and became the last (seventh) Hasidic leader of the Chabad-Lubavitch movement.

For the Hasidic leadership, it is an important thing to have a special origin and to belong to a Hasidic dynasty. Within a dynasty family's conflicts implying ideological oppositions happened time and again. For example, the third Rebbe of Chabad (a Hasidic movement established by Rabbi Shneur Zalman of Liadi, ר' שניאור זלמן מליאדי), Rabbi Menachem Mendel Schneersohn, Tzemach Tzedek (September 9, 1789—March 17, 1866) did not leave a clear successor. As a result, his son, Rabbi Shmuel Schneersohn became the Rebbe in the city of Lubavitchi (today's Russia, near Belarus) and founded the Lubavitcher Hasidic dynasty supporting the doctrine Chabad-Lubavitch. Another son, Rabbi Yehuda Leib Schneersohn became the Rebbe in the city of Kopust or Kopys in today's Belarus and founded the Kopuster Hasidic dynasty supporting the doctrine Chabad-Kopust. Between two dynasties the relationships were quite stressful. There were also other Chabad dynasties.

The father of Rabbi Menachem Mendel Schneerson we will talk about was Levi Yitzchak Schneerson, an authority on Kabbalah and Jewish law, rabbi of Yekaterinoslav from 1907 to 1939. His great-great grandfather was the third Chabad Rebbe, Tzemach Tzedek. In 1939 Levi Yitzchak was arrested by the Communist regime and after more than a year in Stalin's prisons, he was sentenced to exile to a remote village Chiali in Kazakhstan. He died in Almaty in 1944.

Rabbi Menachem Mendel Schneerson was the eldest of three sons of Levi Yitzchak and hence he belonged to a Hasidic dynasty. His brother Dov Ber was mentally ill from childhood and spent his years in an institution for the mentally disabled near Nikolaiev. He was killed in 1944 by Nazis. The youngest brother, Yisrael Aryeh Leib, became a Communist, later a Trotskyite. After he left the Soviet Union he changed his name to Mark Gourary and became a

Picture 3 Rebbe's grave in Lubavitchi

businessman in Israel, but later moved to England where he began doctoral studies at Liverpool University but died in 1951 before he completed them.

In 1923 Menachem Mendel Schneerson visited Rabbi Yosef Yitzchok Schneersohn, the sixth Rebbe of Chabad-Lubavitch, for the first time and five years later he became his son-in-law by getting married to Schneersohn's middle daughter, Chaya Mushka. After a lot of travels, Menachem Mendel joined his father-in-law in the Crown Heights section of Brooklyn, New York in 1941. Soon after, in 1942, Rabbi Yosef Yitzchok Schneersohn appointed him

director of the Chabad-Lubavitch newly-founded central organizations across the world. Before joining his father-in-law, Menachem Mendel has never served as rabbi.

The sixth Rebbe of Chabad-Lubavitch was first who began to teach publicly, gather and send out a small amount of his newly trained rabbis to other cities, use also mass media technologies for distributing Chabad-Lubavitch ideas. He died in 1950 and one year later Rabbi Menachem Mendel Schneerson was crowned as the seventh Rebbe of Chabad-Lubavitch. He emulated and amplified the trend initiated by his predecessor to teach publicly.

Menachem Mendel Schneerson had abilities to behave publicly as true celebrity, he knew how to be a leader in any situation. For example, once the Rebbe stated that the Gematria of the word kugel (it means a baked Jewish pudding or casserole) and Shabbat are equal. However, when someone calculated the numeric values of both words one found that the word Shabbat equals a much higher number than kugel, then to this the Rebbe responded that you must eat more kugel for them to become even.

With the lapse of time, the Lubavitcher Rebbe became a pop-star in many Hasidic communities. For instance, Schneerson's portraits were placed almost in all Lubavitch homes, shops, and synagogues. His empire has grown with a personality cult around him. Many rabbis have proclaimed him as the Messiah. One of the earliest proponents of this statement was Rabbi Avraham Parizh who printed a poster proclaiming him as having near God-like essence in 1952.

Mass media technologies in popularizing and distributing Chabad-Lubavitch ideas allowed Rabbi Menachem Mendel Schneerson to become a great celebrity not only in the Jewish world. On March 25, 1983, on the occasion of the 80th birthday of the Rebbe, the United States Congress proclaimed Rabbi Schneerson's birthday as 'Education Day, USA,' and awarded him the National Scroll of Honor.

In the modern world, charisma is an ability to behave publicly as true celebrity, i.e. an ability and opportunity to evoke a wide positive resonance in mass media. It cannot be already understood as God's gift, since such charisma has no connections to religion and its content nowadays.

If we assume an event of Revelation, then its dissemination cannot be a result of making images in mass media, because for mass media there are no differences what content is. Charisma became secular and religion-outsider. If we are interested indeed in the revelation and its nature still, then we should avoid public (charismatic) presentations of religious ideas and should consider the Revelation as mental event and investigate it by pure logical means.

3. Logical interpretation of the Pentateuch in Judaism

In Judaism the referential structure of any statement is called the Torah, at the same time the Pentateuch is the legislative foundation of the Torah describing what each statement may mean and how each act may be evaluated. The Torah as Pentateuch has the following two dimensions: *haggadah* and *halakhah*. The first dimension, *haggadah* [הַגָּדָה], reflects the historic facts connected to Israel, the second dimension, *halakhah* [הֲלָכָה], expresses the compiled laws which have to be accepted by any Jew. These laws are scattered all over the books of the Pentateuch and explicated only on the basis of the grammatical form: 'And the Lord spoke/said do/do not.' All these laws are mentioned in a context of historical events. For example, the first commandment (in Hebrew *miẓwah*) given man is as follows: "be fruitful and multiply" (*Gen.* 1:28). This commandment was given to Adam and Eve to obey. Thus, *haggadah* (in this case it is a history about the creation of Adam and Eve) is to show the historical context of the commandments, therefore *haggadah* plays the subordinated role in relation to *halakhah*.

Haggadah is no more than an addition to *halakhah*. For example, in the opinion of the Talmudic authority, Rabbi Isaac, the Torah should begin with the verse

> This month shall be unto you the beginning of months: it shall be the first month of the year to you (*Exodus* 12:2),

as this verse contains the first commandment given to Jews.

In Christianity, the dimension of *halakhah* is completely eliminated, and the Bible starts to be considered especially as a historical narration (even about sacred events). All the Lord's commandments start to have only the form of a recommendation. As a result, a contemplation appears in Christianity. Commandments are not taken literally and are interpreted allegorically with the application of analogies. For example, the Christian commandment "To him that smiteth thee on thy cheek, offer the other: and from him that taketh away thy cloak, keep not back thy tunic" (*Luke* 6:29) in the most sense, cannot be taken literally.

The Christian contemplation caused the further development of secular culture. Such a contemplation and allegorization is, perhaps, a distinctive feature of Christianity. On the other hand, in Judaism we find the detailed consideration of how the commandments should be obeyed depending on concrete everyday situations. For instance, in the case of the absence of water it is ordered to wash hands by fine sand which can be found in huge quantities in the climatic conditions of the Near East.

In Muslim faith, as well as in Judaism, there are commandments which should necessarily be obeyed. For example, in the same measure it is ordered to keep to one's own hygiene and in the case of the absence of water it is necessary to carry out the lavabo with fine sand.

يَا أَيُّهَا الَّذِينَ آمَنُواْ لاَ تَقْرَبُواْ الصَّلاَةَ وَأَنتُمْ سُكَارَى حَتَّىَ تَعْلَمُواْ مَا تَقُولُونَ وَلاَ جُنُبًا
إِلاَّ عَابِرِي سَبِيلٍ حَتَّىَ تَغْتَسِلُواْ وَإِن كُنتُم مَّرْضَى أَوْ عَلَى سَفَرٍ أَوْ جَاء أَحَدٌ مِّنكُم

مِّن الْغَآئِطِ أَوْ لاَمَسْتُمُ النِّسَاء فَلَمْ تَجِدُواْ مَاء فَتَيَمَّمُواْ صَعِيدًا طَيِّبًا فَامْسَحُواْ بِوُجُوهِكُمْ وَأَيْدِيكُمْ إِنَّ اللّهَ كَانَ عَفُوًّا غَفُورًا

> O ye who believe! Approach not prayers with a mind befogged, until ye can understand all that ye say,—nor in a state of ceremonial impurity (Except when travelling on the road), until after washing your whole body. If ye are ill, or on a journey, or one of you cometh from offices of nature, or ye have been in contact with women, and ye find no water, then take for yourselves clean sand or earth, and rub therewith your faces and hands. For Allah doth blot out sins and forgive again and again (*Qur'an* 4:43; translated by Yusuf Ali).

The fact that Arabs in the ceremonial purposes are washed sometimes by fine sand is mentioned in a commentary by Rashi (Rabbi Solomon Isaac, the best known commentator of the Torah and the Talmud) in the following verse:

> Let a little water, I pray you, be fetched, and wash your feet, and rest yourselves under the tree (*Gen.* 18:4).

In this verse Abraham invites three people to enter his tent, and before this he suggests they wash their feet. Rashi explains that Abraham assumed these people were Arabs who worship fine sand on their feet. Abraham very strictly obeys the Judaic laws, e.g., he does not admit an object of idolatry in his house. In Rashi's opinion, Lot is connected to this commentary. He, who was not such a perfect man as Abraham, does not obey that graven images do not penetrate into his house, therefore in the beginning Lot suggested strangers to enter the house and only later wash their feet: "tarry all night and wash your feet" (*Gen.* 19:2). Foreign religious sacraments are an idolatry for Judaism (in Hebrew: *avodah zarah*, the literal translation: "foreign worship," therefore the Arabian ceremonial washing with fine sand is a version of idolatry in the Judaic viewpoint.

The example of Rashi's commentary is a bit astonishing. At first sight, the explanation seems to be completely inappropriate: Arabs as a nation come from Abraham, and more precisely they are Ishmael's descendants, although by the time of the described events, Ishmael was 13 years old! Besides, the ceremonial washing with fine sand was accepted by Arabs only after Muhammad's sermon. It is natural that Rashi, living in France in the 11th century and being familiar with the Arabian culture, knew all these historical details. Why has he afforded an obvious historical inaccuracy? The point is that *haggadah*, i.e. the historical narration of the Bible, does not play a significant role and the Torah is not a historical story. In this way, the Judaic understanding of the Biblical books is considerably different from the Christian one. In Judaism, the Torah is a compendium of laws for which different events occurring at different times are mapped onto a general plane: *dura lex sed lex*.

For Rashi, therefore, it is very important to have a *logical coordination* of the Torah verses, instead of its historical one. For example, Abraham suggests strangers to wash their feet in the beginning and only later to enter his tent. On the other hand, Lot suggests the same strangers to enter his house in the beginning and only after to wash their feet. The different sequence of actions is obvious. It is our *first premise*. Further, it is known that Lot, though he was Abraham's nephew, was not a perfect man. On the other hand, Abraham was such a man. It is our *second premise*. Notice that due to the given premise we know that Abraham's actions were faultless from the standpoint of *halakhah* (namely, of the 613 Jewish commandments). Now we need to draw a conclusion from both premisses and that conclusion becomes an explanation of the Biblical verse. For this purpose I should correlate Abraham and Lot's acts with *halakhah*. The following explanation suits it best here: *fine sand on the feet of strangers could be ritually unclean*, therefore it could profane Abraham's dwelling, so Abraham decided to suggest them to wash their feet. The most widespread example in Rashi's

days of ritually unclean fine sand on feet is a result of the ceremonial washing of feet by Arabs (in fact, every sacrament of alien religion is a ceremonial defiling for Judaism). For this reason, Rashi refers to this example.

It is necessary to notice that all of Rashi's commentaries are rather the result of a logical processing of the Biblical text, and in most cases his commentaries are legal conclusions, connecting concrete acts of Biblical characters to *halakhah*, the laws. The logical portrayal of all the smallest details in the works of Judaic authorities contrasts very sharply with the general contemplation of Christian thinkers.

In Judaism, any details are completely coordinated among themselves from the standpoint of logic. Logic is a unique science which for the whole history of its official existence (more than two thousand years) has not changed in its foundations. Even mathematics has been changed essentially, in the beginning it was based on geometry (Ancient Greek mathematics), then on algebra (classical mathematics), and now it is based on mathematical logic. Every scientific knowledge is corruptible and changeable, except for logic. Only this science sets the border for thinking, outside of which there is nothing, only silence. Logic is an eternal science.

One of the well-known Rabbis of present days, Rabbi Adin Even Israel Steinsaltz (*Steinsaltz*), compares logic of the Talmud (i.e. the logic used by Talmudists for inferring Judaic laws from the Pentateuch) to mathematical logic, affirming that *the Talmudic logic, different from mathematical logic, gives the description and explanation for all in view of case studies*, i.e. it is constructed on the basis of the analysis of all equiprobable cases whereas mathematical logic eliminates any concrete context.

Many laws described in the Talmud are taken directly from the written Torah (Pentateuch). But there are also laws which were logically inferred. These laws are called the laws of wise men. "It would be possible to include laws of wise men in the Torah, but the Lord has decided that they should go from us," as Rabbi Moses

Chaim Luzzatto said. Thus, while the written Torah (Pentateuch), in the Judaic view, is a result of the Revelation, the oral Torah (the laws of wise men) is a logical analysis of any details from the standpoint of the Pentateuch, i.e. it represents the logic which takes into account all equiprobable cases. Logic as the basis of the oral Torah goes from us, Pentateuch, from the Lord.

The basic motto of the oral Torah is expressed in the verse: "do a fencing for the doctrine (Torah);" וַעֲשׂוּ סְיָג לְתוֹרָה (*Pirqe Abot 1, 1*). This appeal means that it is necessary to complete Judaic laws logically. In other words, 'fencing for the doctrine' is presented by a logical coordination of all details as well as an elimination of any possible contradiction.

Let us consider an example how the unsoluble contradiction in the statement of claim may be eliminated:

> Two persons, who hold a garment, and each of them claims that he has found it, or that the whole belongs to him, (in such a case) each of them shall take an oath that no less than a half belongs to him, and then its value shall be divided. If, however, one claims the whole and the other half of it, then the oath for the first must be for no less than three quarters, and for the second no less than a quarter, and it is to be divided accordingly. The same is the case with an animal, if both are riding; or, if one is riding and one leading, each of them must take an oath that no less than a half belongs to him, if both claim for the whole, and so they divide. If, however, there are witnesses, or they admit the fact, then it is to be divided without any oath (*Baba Metzia, Mishnah* 1:1).

This example says that if there is no possibility to resolve the contradiction in the statement of claim, the contradiction should be eliminated by means of a mutual compromise.

In the Pentateuch there are no logical connectives in the sense of conventional (European) logic.

Definition 3.1 *One of the logical connectives in the Pentateuch (that is used very often) has the form of the conjunction "and" (in Hebrew "we" or "u"), and it is called the Judaic conjunction and denoted by* $*$. *This connective is not idempotent and commutative, but it is associative:*

$$A * A \neq A,$$
$$A * B \neq B * A,$$
$$A * (B * C) = (A * B) * C,$$

where A, B, C *are either names/concepts or propositions/sentences.*

The Judaic conjunction differs from the conventional one, because the latter is idempotent, commutative, and associative. For example, in the Judaic view, Lot's event '*enter my dwelling and wash your feet*' $(A * B)$ is not equivalent to Abraham's event '*wash your feet and enter my dwelling*' $(B * A)$.

In conventional semantics, referred to as Russellian, facts are referents of sentences: "sentences are used to express propositions, claims about the world, and these claims are true just in case the world is as it is claimed to be" (*Barwise & Etchemendy*). Sentences and propositions held in Russellian semantics are called *Russellian sentences* and *Russellian propositions*, respectively.

However, the given Russellian semantics is limited, because self-referential sentences have no meaning in it. For example, take the Liar sentence: '*This sentence is not true.*' It has no meaning according to the Russellian account. Indeed, assume that it is a fact that the Liar proposition is true, then the same fact shows that the Liar proposition is untrue. Since there exists no fact with such a property, the Liar proposition has no truth valuation. Thus, the Liar sentence concludes that Russellian propositions cannot make claims about the whole world.

Another approach to semantics was developed by Austin. According to him,

> a legitimate statement A provides two things: a historical (or actual) situation S_A, and a type of situation T_A. The former is just some limited portion of the real world; the speaker refers to it with what Austin [in his paper 'Truth'] calls "demonstrative conventions." The latter is, roughly speaking, a property of situations determined from the statement by means of "descriptive conventions" associated with the language. The statement A is true if S_A is of type T_A; otherwise it is false. While Austin did not use the term "proposition," it seems in the spirit of his account to identify what we will call the Austinian proposition expressed by A with the claim that S_A is of type T_A, and to individuate such a proposition by its two components, the situation referred to and the type of situation it is claimed to be (*Barwise & Etchemendy*).

Sentences and propositions held in Austian semantics are called *Austian sentences* and *Austian propositions*, respectively. Also, an Austinian statement is a reference to a situation and an utterance of a sentence. The proposition expressed by that statement is a claim that the situation is of a type, and that proposition is true if and only if the claim holds.

For instance, let us consider the following statement:

> ארי עלה מבבל
>
> The lion came from Babylon (*Baba Qama* 177a).

According to the Russelian semantic approach, there should exist the fact (a direct meaning) that the lion exists and it came from Babylon. If there is an appropriate fact indeed, then this statement is true, otherwise it is false. However, this statement was aimed to express something other than an animal coming from somewhere. In other words, its meaning cannot be direct; it does not refer to facts. However, it refers to contexts of utterances and means Rav Kahana' [רב כהנא] who came from Babylon. This meaning can be understood only within the Austian approach.

In Austinian semantics, the Liar sentence occurs in statements that refer to different situations, and so it expresses a family of propositions indexed by the situations. The Liar shows that each of these propositions is untrue. Therefore, it cannot be a fact in the world that a Liar proposition is true (the Liar sentence refers to the *greatest set of situations* and shows that for each situation of that set, the claim does not hold). Thus, we do not obtain a paradox here like that, which was in Russellian semantics: the Liar proposition has a negative truth valuation.

Judaic semantics are similar to Austian ones, they can express self-reference too. Sentences and propositions held in Judaic semantics will be called *Judaic sentences* and *Judaic propositions*, respectively. In Judaism, a legitimate statement A also provides two things: a historical, i.e. hagaddic, (or actual) situation S_A, and a type of situation T_A (halakhic, or legislative, situation). The Judaic proposition in the Pentateuch expressed by A is the claim that S_A is of type T_A, i.e. S_A corresponds to the halakhic situation T_A. The statement A is true if S_A is of type T_A; otherwise it is false.

Definition 3.2 *In Judaic semantics, a model of a Judaic proposition A is a pair comprising the situation S_A and a type T_A; $\langle S_A, T_A \rangle$ models the proposition A that situation S_A is of type T_A ($S_A \in T_A$). The Judaic conjunction $A * B$ models the type of situation B such that the type of B includes the type of A. So, if T_{A*B} is a Judaic conjunctive type, then proposition $A * B$ is true (we have $\langle S_{A*B}, T_{A*B} \rangle$) if and only if $\langle S_{A*B}, T_B \rangle$ takes place and $T_A \subseteq T_B$.*

For example, Lot's sentence '*enter my dwelling and wash your feet*' ($A * B$) is false, because $\langle S_{A*B}, T_{A*B} \rangle$ is not true for T_B and $T_A \not\subset T_B$, and Abraham's sentence '*wash your feet and enter my dwelling*' ($B * A$) is true, because $\langle S_{B*A}, T_{B*A} \rangle$ is true for T_A and $T_B \subseteq T_A$.

As we see, the Judaic exegesis is a kind of collective solipsism: there are no facts in the Russellian meaning (no facts outside the *halakhah*, the Judaic laws), there exist only contexts of

utterance that have reference to halakhic types of situations. This situation semantics for Judaic reasoning may allow us to explicate formally the Talmudic case study and situation analysis.

Different versions of Austian-style semantics are applied now in different fields of modern logic including logical analysis of self-referent statements, deductions in natural language and so on. Thus, one may affirm that one of the promising aims of modern logic dealing with Austian-style semantics also is to analyze, from the viewpoint of modern logic, all the logical rules scattered through the Talmudic and Midrashic literature, rules that are used for making halakhic (legal) deductions from the Pentateuch in Judaism up until today. Many of them were collected by Malbim (1809—1879), born in Volhynia (today's Ukraine), in the *Ayyelet ha-Shachar* (the introduction to his commentary on the *Sifra*). As a result, one may expect to find formal explications of the Judaic rules and methods for the investigation and exact determination of the meaning of the Scriptures, including the logical deduction of the *halakhah* from the Scriptural text or from another law.

4. Inference rules of Judaic logic

The Judaic reasoning has been established by Tannaim (the first Judaic commentators of the Bible) for inferring deductions from the Pentateuch and entailing the Judaic laws. For the effective entailing of these deductions, Tannaim have developed special logical inference rules (called *middot*). The Talmudic tradition, continued by Amoraim (the second generation of Judaic commentators, from about 200 to 500 C.E.), has systematized these rules and now *middot* set up the logical foundations of Rabbinic traditions.

At the first glance, the Talmudic and Rabbinic reasoning used in the deductions of Jewish laws seems to be very paradoxical and cloudy because of multi-agent parallel inferring, but after more attentive studying it can be viewed as a harmonious logical system

of a special kind. Unfortunately, up until now, the Talmudic reasoning has not been formalized from the point of view of modern logic. However, many experts in Talmudic studies have very strong feelings that Judaic logic can be regarded as a formal system ordered by a set of inference rules.

The point is that Judaic logic cannot be formalized in the standard way that was successfully used, as an example, for formalizing Ancient Greek logics (namely, Aristotle's syllogistics and Stoic propositional logic, see *Corcoran 1972, 1973, Łukasiewicz 1957*).

By reason of its general sense, the modern logic (including classical and some non-classical logics) has its roots in Ancient Greek logical systems assuming that each proposition is either true or false and its truth values are verified thanks to the facts (on an appropriate model). These both assumptions are obvious within Russellian semantics underlying mathematical logic (more precisely, first-order predicate logic and some of its natural extensions), but they are not applicable for the Judaic reasoning. The problem is that for the latter there is no classical understanding of truth value; because for the Judaic reasoning there are no facts in Russell's sense, this reasoning cannot be verified on an appropriate model. Such a feature is initially caused by the Jewish language practice different from the Greek one. For example, in Old Hebrew there is no distinction between words and things and as a result between language and physical world (this feature of the Old Hebrew language has been very accentuated by the Judaic religious doctrine). The Hebrew word *davar* (as well as the Aramaic *milah*) means 'thing' and 'word' simultaneously.

The correctness of Judaic reasoning is checked over not on the facts, but in the context of its use. Therefore Russellian semantics (as well as standard logical methods) are absolutely inapplicable for formalizing Judaic logic. In the Talmudic reasoning we cannot find out the notion of proposition and its truth value in the sense of modern logic. Judaic propositions are self-referential as

I could say now from the standpoint of modern logic. In some logical Hebrew books of the 18th century (e.g. see *Derekh Tvunot* by Rabbi Moses Chaim Luzzatto) one can find an opinion that some (but not all) propositions are defined thanks to the facts, but these propositions in general do not concern Judaic logic and Judaic way of entailing legal propositions concerning the Judaic laws from the Pentateuch. Rabbi Moses Chaim Luzzatto distinguishes two modes of meaning: (i) "each proposition [*hama'amarim*] is either true or false" (*Derekh Tvunot*, *Perek 6*), (ii) "each proposition [*ma'amar*] refers to words with general or particular meaning" (*Derekh Tvunot*, *Perek 5*). The second mode assumes self-reference.

The best known Judaic inference rules are said to be the thirteen rules compiled by Rabbi Ishmael. They form an extension of the seven rules of Hillel which Rabbi Ishmael elaborated and strengthened by illustrating them with examples taken from the Scriptures. Those thirteen rules are collected in the *Barayta* of Rabbi Ishmael, forming the introduction to the *Sifra*. His rules were universally adopted by his successors, many Tannaim, as well as Amoraim. Notice that the oldest Hebrew commentary tradition (i.e. Tannaitic tradition) recognizes mainly three collections of logical inference rules:

- the seven rules of Hillel;
- the thirteen rules of Rabbi Ishmael;
- the thirty-two rules of Rabbi Eliezer ben Jose ha-Gelili.

The seven rules of Hillel are included into the thirteen rules of Rabbi Ishmael. In turn, the latter are applied in Judaism just for analyzing legislative (halakhic) aspects of the Scripture and the thirty-two rules are described as those which are applied in narrative (haggadic) interpretations, i.e. for historical exegesis. According to the orthodox rabbinic tradition, all those rules were not invented; they were received within the Revelation at Mount Sinai. So, Maimonides claims that any Judaic law reasoning (*svara'*) is closed under the thirteen rules of Rabbi Ishmael (see Maimonides' *Introduction* to his *Seder Zera'im*). In other words, according to him,

the system of those rules is 'complete' (as we would say from the standpoint of modern logic) for inferring law sentences from the Pentateuch.

Let us consider basic Judaic inference rules. The majority of them concern the Judaic conjunction, *.

The seven rules of Hillel

H1. *Qal wa-ḥomer* [קל וחומר] *('parallel concurrent deductions')*, this rule partly corresponds to the scholastic proof *a fortiori* ('*a minori ad majus*' or '*a majori ad minus*'), according to the latter what applies in less important cases will apply in more important ones too, i.e. this rule allows to entail from the simple to the complex or vice versa. However, there are important distinctions from the scholastic proof *a fortiori*. The process of deduction in the *qal wa-ḥomer* is proceeded under the assumption that the inferred statement conclusion may contain nothing more than is found in the premise. This limitation is called the *dayo* [דיו] principle. A syllogism implicitly drawn from a minor case upon a more important one: "If *X* is true of *Y* and *Z* is of greater weight than *Y*, then how much more *X* must be true of *Z* (but not more than of *Z*)." Example: using the following passage as premise "If thou meet thy enemy's ox or his ass going astray, thou shalt surely bring it back to him again" (*Ex*. 23:4), we can conclude that if that be one's conduct toward an enemy, how much more should one be considerate toward a friend. In *qal wa-ḥomer* two or more parallel deductions concur under the following conditions: (i) they have joint premises (ii) one deduction of the set of concurrent deductions is much more certain. Thus, a certainty of that deduction is expanded to cases of other concurrent deductions. Notice that *qal wa-ḥomer* does not hold in Judaic criminal procedure, i.e. by using this rule nobody can be sentenced to an execution. Its deduction might be presented in the following symbolic form:

$$\frac{\begin{array}{c} A_1, A_2, ..., A_n \subset A \text{ (i.e. } A_1, A_2, ..., A_n \text{ are part., } A \text{ is gen.)}, \\ B_1, B_2, ..., B_{n-k} \subset B \text{ (i.e. } B_1, B_2, ..., B_{n-k} \text{ are part., } B \text{ is gen.)}, \\ A > B \text{ (i.e. } A \text{ is of greater weight than } B), \end{array}}{B_{n-k+1}, ..., B_n \subset B, \quad [B_{n-k+1} \text{ is similar to } A_{n-k+1}, ..., B_n \text{ is similar to } A_n]}.$$

We have some particulars, A_1, A_2, ..., A_n, of the one general, A, and some particulars, B_1, B_2, ..., B_{n-k}, of the other general, B. Further, A is of greater weight than B. Thus, we can predict that there are other particulars B_{n-k+1}, ..., B_n, of the same general, B, such that B_{n-k+1} is similar to A_{n-k+1}, ..., B_n is similar to A_n.

H2. *Gezerah šawah* [גזירה שווה] *('analogy')*, this rule can be described as proof by analogy, which infers from the similarity of two cases that the legal decision given for the one holds for the other also. This rule may be valid if we observe the use of a similar phrase, word, or root of word in Hebrew in different contexts. This means that the same considerations may apply to each context if the words of the text which form the basis of the deduction from analogy must be free. This rule is to be regarded as implying that every *gezerah šawah* must have been handed down from Sinai. No one may draw a conclusion from analogy upon his own authority.

$$\frac{A * X \text{ or } X * A, \quad B * Y \text{ or } Y * B, \quad [a \text{ word}] \in A, B, \quad A \subset C \text{ (} A \text{ is part. of } C\text{)}}{B \subset C \text{ (} B \text{ is part. of } C\text{)}}.$$

H3. *Binyan 'av mi-katub 'eḥad* [בנין אב מכתוב אחד] *('a genus* [in Hebrew: 'father'] *from a passage of the Scripture')*, according to that, a certain passage serves as a basis for the interpretation of all similar texts or cases.

$$\frac{A * X \text{ or } X * A, \quad A \subset C \text{ (} A \text{ is part. of } C\text{)}}{\exists D.\, D \subset C \text{ (there exists } D \text{ such that } D \text{ is part. of } C\text{)}}.$$

H4. *Binyan 'av mi-šene ketubim* [בנין אב משני כתובים] *('a genus from two passages of the Scripture')*, by this rule, a principle established by relating two texts to each other and having a characteristic in common is applied to many other laws which have the same characteristic. Example:

> And if a man smite the eye of his servant, or the eye of his maid, that it perish; he shall let him go free for his eye's sake. And if he smite out his manservant's tooth, or his maidservant's tooth; he shall let him go free for his tooth's sake (*Ex.* 21:26—27).

By applying this rule we can entail, an appropriate predicate ("let him go free for the sake of...") may be applied for other body parts too.

$$\frac{A * X \text{ or } X * A, \quad B * Y \text{ or } Y * B, \quad A, B \subset C}{\exists D.\, D \subset C}.$$

H5. *Kelal u-fraṭ and fraṭ u-kelal* [כלל ופרט] *(species of a genus*: '*general and particular*' and '*particular and general')*, according to this rule, if the particular follows the general, then the whole passage is a definition of the general by the particular, and if the general follows the particular, then the whole passage is a definition of the particular by the general. In other words, if the particular follows the general or the general the particular, then the particular is a specification which is explained by a general rule, and vice versa. Example: A Nazarite

> shall separate himself from wine and strong drink [a *specification*], and shall drink no vinegar of wine, or vinegar of strong drink [a *general rule*], neither shall he drink any liquor of grapes, nor eat moist grapes, or dried [a *specification again*] (*Num.* 6:3).

For the formal meaning see rules I4, I5, I6 mentioned below.

H6. *Ka-yoẓe' bo be-maqom aḥer* [כיוצא בו במקום אחר] *(induction: 'like that in another place')*, the explanation of a Biblical passage according to another of similar content. For instance, two passages that seem to contradict each other should be resolved by a third passage that solves the apparent conflict.

$$\frac{A * X \text{ or } X * A, \quad [A \text{ is unclear}], \quad A \subset B}{\exists C.\, C \subset B, \quad [C \text{ is clear}]}.$$

H7. *Dabar lamed me-'inyano* [דבר למד מענינו] *(contextual reasoning: 'something proved by the context')*, here a meaning is established by the context, i.e. a verse cannot be isolated from its general context.

$$\frac{A * X \text{ or } X * A, \quad [X \text{ is context of } A]}{[\text{some properties of } X \text{ are properties of } A]}.$$

The thirteen rules of Rabbi Ishmael

I1. *Qal wa-ḥomer* [קל וחומר] *(parallel concurrent deductions)*, this rule is the same as the first rule of Hillel.

$$\frac{\begin{array}{c} A_1, A_2, \ldots, A_n \subset A \ (\textit{i.e. } A_1, A_2, \ldots, A_n \textit{ are part., } A \textit{ is gen.}), \\ B_1, B_2, \ldots, B_{n-k} \subset B \ (\textit{i.e. } B_1, B_2, \ldots, B_{n-k} \textit{ are part., } B \textit{ is gen.}), \\ A > B \ (\textit{i.e. } A \textit{ is of greater weight than } B), \end{array}}{B_{n-k+1}, \ldots, B_n \subset B, \quad [B_{n-k+1} \textit{ is similar to } A_{n-k+1}, \ldots, B_n \textit{ is similar to } A_n]}.$$

I2. *Gezerah šawah* [גזרה שווה] *('analogy')*, this rule is the same as the second rule of Hillel.

$$\frac{A * X \text{ or } X * A, \quad B * Y \text{ or } Y * B, \quad [a \text{ word}] \in A, B, \quad A \subset C \ (A \textit{ is part. of } C)}{B \subset C \ (B \textit{ is part. of } C)}.$$

I3. *Binyan 'av* [בנין אב] *('genus'),* it unites the third and the fourth rules of Hillel. The example of a genus: "He shall even pour out the blood thereof, and cover it with dust" (*Lev.* 17:13). This means that all aspects of two actions should be considered in common, e.g. just as the pouring out of the blood is performed with the hand, so must the covering be done with the hand, not with the foot.

$$\frac{A * X \text{ or } X * A, \quad B * Y \text{ or } Y * B, \quad A, B \subset C}{\exists D.\, D \subset C}.$$

I4. *Kelal u-fraṭ* [כלל ופרט] *('general and particular' or 'particular after general')*. If in the Bible a specification follows a general rule, then this specification not just exemplifies, but also restricts the contents of the general rule. Example: "Ye shall bring your offering of the cattle, even of the herd, and of the flock" (*Lev.* 1:2). The latter specification is to exclude all undomesticated animals. The name 'animals' is general, the name 'cattle' or 'flock' is particular. This rule is formalized as follows:

$$\frac{A * B, B \subset A \text{ (}i.e.\ A \text{ is general, } B \text{ is particular)}}{B}.$$

I5. *Fraṭ u-kelal* [פרט וכלל] *('particular and general' or 'general after particular')*. If a general rule follows a particular one, then we accept just the general rule. Example: "If a man deliver unto his neighbour an ass, or an ox, or a sheep [*specifications*], or any beast [*a general*]" (*Ex.* 22:10). This implies that all lost things are included. Another example: "do not sacrifice … any bullock and sheep, wherein is blemish, any defect" (*Deut.* 17:1). A 'blemished one' is particular, 'one with any defect' is general. Hence, it is forbidden to sacrifice the bullock and sheep with any defect.

$$\frac{A * B,\ A \subset B\ (\textit{i.e. A is particular, B is general})}{B}.$$

I6. *Kelal u-fraṭ u-kelal* [כלל ופרט וכלל] *('general and particular and general' or 'general after particular after general')*. If a general follows a particular and the latter follows another general, then we accept the last general exemplified by a previous specification. Example:

> Thou shalt bestow the money for whatsoever thy soul lusteth after [*a general*] for oxen, or for sheep, or for wine, or for strong drink [*a specification*] or for whatsoever thy soul desireth [*a general*] (*Deut.* 14:26).

Other things than those specifications may be purchased, but only if they are food or drink like those, specified in the previous particular. Another example: we are told that an embezzler shall pay double to his neighbour

> for all manner of trespass [*a general*], whether it be for ox, for ass, for sheep, for raiment [*a particular*], or for any manner of lost thing [*a general*] (*Ex.* 22:9).

Taking into account that the particular includes only moveable property, the fine of double payment does not apply to embezzled real estate, notes or bills. In the mathematical form:

$$\frac{(A * B) * C,\ B \subset A,\ B \subset C\ (\textit{i.e. A is gener., B is part., C is gener.})}{C}.$$

I7. *Kelal ha-ẓarikh li-fraṭ u-fraṭ ha-ẓarikh li-kelal* [כלל הצריך לפרט ופרט הצריך לכלל] *('the general which requires elucidation by the particular, and the particular which requires elucidation by the general')*. This is a case of cyclic relation between particular and general. Example: "All the firstling males that come of thy herd and of thy flock thou shalt sanctify unto the Lord thy God" (*Deut.* 15:19), "Sanctify unto Me all the

first-born," "whatsoever openeth the womb" (*Ex.* 13:2). We reconstruct here the following cycle: 'first-born' ⇒ 'males' ⇒ 'openeth the womb' ⇒ 'first-born.' This means that a first-born male is a particular and as that he is included in the term "all the first-born" even if a female had previously been born to that mother. On the other hand, the particular limiting expression "whatsoever openeth the womb" is stated. But this term does not exclude one born after a previous Caesarean birth, because the general term is "all the first-born."

$$\frac{A \subset B,\ B \subset A\ (i.e.\ A\ is\ part.\ of\ B,\ B\ is\ part.\ of\ A)}{A * B\ and\ B * A}.$$

I8. *Davar še-hayah bi-khelal we-yaẓa' min ha-kelal lelammed lo' le-lammed 'al 'aẓmo yaẓa' 'ella' le-lammed 'al ha-kelal kullo yaẓa'* [דבר שהיה בכלל ויצא מן הכלל ללמד לא ללמד על עצמו יצא אלא ללמד על הכלל כולו יצא] *('the particular implied in the general and excepted from it for pedagogic purposes elucidates the general as well as the particular')*. If a specification of a general rule is singled out for special treatment, whatever is postulated of this specification is to be applied to all the instances embraced by the general rule. Example:

> A man also or woman that hath a familiar spirit, or that is a wizard, shall surely be put to death: they shall stone them with stones: their blood shall be upon them (*Lev.* 20:27).

Divination by a ghost or familiar spirit is included in the general rule against witchcraft (*Deut.* 18:10). From this it follows that the penalty of stoning is applied to other instances of witchcraft too.

$$\frac{A \subset B,\quad X * A\ or\ A * X,\quad X * B\ or\ B * X}{C \subset A\ for\ all\ C\ such\ that\ C \subset B}.$$

I9. *Davar še-hayah bi-ḵhelal we-yaẓa' liṭ'on ṭo'an 'aḥer še-hu' ḵhe'inyano yaẓa' lehaqel we-lo' lehaḥmir* [דבר שהיה בכלל ויצא לטעון טען אחר שהוא כענייננו יצא להקל ולא להחמיר] *('a particular that was implied in a general principle and was later singled out to discuss another point similar to the general principle was singled out in order to decrease rather than to increase the rigidity of its application'),* when particular instances of a general rule are treated specifically, in details similar to those included in the general rule, then the general rule in those instances is to be more lenient, but not to be more stringent. Example: the laws of the boil (*Lev.* 13:18—21) and the burn (*Lev.* 13:24—28) are treated specifically even though these are particular instances of the general rule regarding plague-spots (*Lev.* 13:1—17). Therefore the general restrictions regarding the law of the second week (*Lev.* 13:5) and the quick raw flesh (*Lev.* 13:10) are not be applied to them.

$$\frac{A \subset B, \quad Y * A \text{ or } A * Y, \quad X * B \text{ or } B * X, \quad Y \subset X}{[B \text{ is restricted } (\text{more lenient}) \text{ for } Y \text{ and } A]}.$$

I10. *Davar še-hayah bi-ḵhelal we-yaẓa' liṭ'on ṭo'an 'aḥer še-lo' ḵhe'inyano yaẓa' lehaqel we-lehaḥmir* [דבר שהיה בכלל ויצא לטעון טען אחר שלא כענייננו יצא להקל ולהחמיר] *('a particular that was part of a general principle and was later singled out to discuss another point not similar to the general principle was singled out in order to be more lenient as well as to be more stringent'),* when particular instances of a general rule are treated specifically in details dissimilar from those included in the general rule, then both lenience and stringency are to be applied in those instances. Example: the details of the laws of plague in the hair or beard (*Lev.* 13:29—37) are dissimilar from those in the general rule of plague spots. Thus, both the lenience regarding the white hair mentioned in the general rule (*Lev.*, 13:4) and the stringency of the yellow hair mentioned in the particular instance (*Lev.* 13:30) are to be applied.

$$\frac{A \subset B, \quad Y * A \text{ or } A * Y, \quad X * B \text{ or } B * X, \quad Y \not\subset X, \quad X \not\subset Y}{[B \text{ can be more lenient and more stringent for } Y \text{ and } A]}.$$

I11. *Davar še-hayah bi-khelal we-yaẓa' lidon ba-davar ḥadaš 'i 'attah yakhol lehaḥaziro li-khelalo 'ad še-yaḥazirennu ha-katuv be-piruš* [דבר שהיה בכלל ויצא לדון בדבר חדש אי אתה יכול להחזירו לכללו עד שיחזירנו הכתוב בפירוש] *('a particular that was part of a general principle and was singled out to be considered in a new matter, you cannot return it to its general principle unless the Scripture does so explicitly'),* when a particular instance of a general rule is singled out for completely new treatment, the details of the general rule must not be applied to this instance unless the Scripture makes an explicit reference to it. Example: the trespass offering of the leper requires the placing of the blood on the ear, thumb, and toe (*Lev.* 14:14). Consequently, the laws of the general rule trespass offering, such as the sprinkling of the blood on the altar (*Lev.* 7:2) would not have applied, were it not for Scripture's stating: "For as the sin offering is the priest's, so is the trespass offering: it is most holy" (*Lev.* 14:13), so as we see this is like other guilt offerings.

$$\frac{A \subset B, \quad X * A \text{ or } A * X, \quad Y * B \text{ or } B * Y, \quad [X \neq Y]}{C \not\subset A \text{ for all } C \text{ such that } C \subset B}.$$

I12. *Davar ha-lamed me-'inyano we-davar ha-lamed misofo* [דבר הלמד מענייני ודבר הלמד מסופו] *(deduction from the context: 'a matter derived from its context, or a matter derived from its end'),* the meaning of a passage may be deduced: (i) from its context, (ii) from a later reference in the same passage. Example of (i): "Thou shalt not steal" in the Decalogue (*Ex.* 20:15) must refer to the capital offence of kidnapping, because the two other offenses mentioned in the same passage, "Thou shalt not murder" and "Thou shalt not commit adultery," are both capital offences and offences of 9 other Decalogue's commandments are to be sentenced to execution. Example of (ii): "I put the plague of leprosy in a house of the land of your possession" (*Lev.* 14:34), refers only to a house built with

stones, timber, and mortar, since these materials are mentioned later in passage 14:45.

$$\frac{A * X \text{ or } X * A, \quad [X \text{ is context of } A]}{[\text{some properties of } X \text{ are properties of } A]}.$$

I13. *Šenei khetuvin ha-makhḥišin zeh 'et zeh 'ad šeyavo' ha-katuv ha-šeliši we-yakhri'a beneyhem* [שני כתובין המכחישין זה את זה עד שיבוא הכתוב השלישי ויכריע ביניהם] *("two verses contradict one another until a third verse reconciles them"),* when two Biblical passages contradict each other the contradiction in question must be solved by reference to a third passage. Example: one verse states that God came down to the top of the mountain (*Ex.* 19:20), another that His voice was heard from heaven (*Deut.* 4:36). A third verse (*Ex.* 20:18) solves this contradiction. He brought the heavens down to the mount and spoke.

$$\frac{A, \quad B, \quad [A \text{ contradicts } B]}{[\text{there exists } C \text{ such that the contradiction is solved by } C]}.$$

The thirty-two rules of Rabbi Eliezer

E1. *Ribuy* [ריבוי] *('extension')*, the particle *'et, gam*, *'af* or *kol* found out in a Scripture passage indicates that something stated must be regarded in a wider meaning than it may be read in literal sense. Example: the duty of recalling the Exodus "all [*kol*] the days of thy life" (*Deut.* 16:3) devolves upon one at night as well as by day. Another example:

> Thy servant slew both the [*gam 'et*] lion and [*'et*] the bear: and this uncircumcised Philistine shall be as one of them, seeing he hath defied the armies of the living God (1 *Sam.* 17:36).

According to this rule we should deduce three extensions in the statement, it means that the lion was with its two young lions [*gam, 'et*] and the bear with its bear cup [*'et*]. This rule symbolically:

$$\frac{B * X \text{ or } X * B, \quad [\textit{one of the particles } \text{'et, gam, 'af, kol}] \in B}{B \text{ is extended}}.$$

E2. *Mi'uṭ* [מיעוט] *('restriction')*, the particle *'ak*, *raq* or *min* indicates that something in the statement read in literal sense must be excluded in a specific case. As an example consider the verse

> and they said, Hath the Lord *however* [haraq] spoken *only* ['akh] by Moses? (*Numbers* 12:2).

As we see, two restrictions occur in the verse. Therefore it is inferred that not only Moses, but Aaron and Miriam (the two restrictions) had prophecy too.

$$\frac{B * X \text{ or } X * B, \quad [\textit{one of the particles } \text{'ak, raq, min}] \in B}{B \text{ is restricted}}.$$

E3. *Ribuy 'aḥar ribuy* [אין רבוי אחר רבוי אלא למעט :or ריבוי אחר רבוי] *('extension after extension')*, when one extension follows another and both are the same, it indicates that something restricted must be regarded as implied, i.e. if the same extended concept/sentence occurs just after the extended concept/sentence, then we introduce the restricted concept/sentence. Hence a double extension teaches not to extend the point of view, but rather to interpret it in a limited sense

$$\frac{A * A, \quad A \text{ is an extension}}{A, \quad A \text{ is restricted}}.$$

E4. *Mi'uṭ 'aḥar mi'uṭ* [מיעוט אחר מיעוט or: אין רבוי אחר רבוי אלא למעט] *('restriction after restriction')*, when one restriction follows another it indicates that we may introduce the extended. For instance, "the pit was empty, there was no water in it" (*Gen.* 37:24). We see that the Torah repeats the same thing twice. Therefore we have to extend the statement: 'there is no water, but there are snakes and scorpions.' The rule in the mathematical form is:

$$\frac{A * A, \quad A \text{ is a restriction}}{A, \quad A \text{ is extended}}.$$

E5. *Qal wa-ḥomer meforaš* [קל וחומר מפורש] *('an explicit case of qal wa-ḥomer')*, see the first rule of Hillel. Example:

> If thou hast run with the footmen, and they have wearied thee, then how canst [*we'ekh*, i.e. *qal wa-ḥomer*] thou contend with horses? (*Jer.* 12:5).

$$\frac{\begin{array}{c} A_1, A_2, \ldots, A_n \subset A \ (\textit{i.e. } A_1, A_2, \ldots, A_n \textit{ are part., } A \textit{ is gen.}), \\ B_1, B_2, \ldots, B_{n-k} \subset B \ (\textit{i.e. } B_1, B_2, \ldots, B_{n-k} \textit{ are part., } B \textit{ is gen.}), \\ A > B \ (\textit{i.e. } A \textit{ is of greater weight than } B), \end{array}}{B_{n-k+1}, \ldots, B_n \subset B, \quad [B_{n-k+1} \textit{ is similar to } A_{n-k+1}, \ldots, B_n \textit{ is similar to } A_n]}.$$

E6. *Qal wa-ḥomer satum* [קל וחומר סתום] *('an implicit case of qal wa-ḥomer')*, a fortiori argument, but only implied, not explicitly declared to be one in the text. Example: "He that sweareth to hurt" (*Ps.* 15:4) By *qal wa-ḥomer*, we infer that he that sweareth to good as well. This rule has the same formal meaning as E5.

E7. *Gezerah šawah* [גזרה שווה] *('analogy')*, the same as the second rule of Hillel, but applied to historical exegesis. Example: Samuel was Nazarite, since there is a *gezerah šawah* between three verses: the passage "and there shall no *razor* come upon his head" (1 *Sam.* 1:11)

towards Samuel, the passage "no *razor* shall come on his head" (*Judg.* 13:5) towards Samson, and the general rule in the statement:

> All the days of the vow of his separation there shall no *razor* come upon his head: until the days be fulfilled, in the which he separateth himself unto the Lord, he shall be holy, and shall let the locks of the hair of his head grow (*Num.* 6:5).

$$\frac{A * X \text{ or } X * A, \quad B * Y \text{ or } Y * B, \quad [a \text{ word}] \in A, B, \quad A \subset C}{B \subset C}.$$

E8. *Binyan 'av* [בנין אב] *('genus')*, it is like the third rule of Rabbi Ishmael. Example: All God's words unto Moses began with calling and Moses' reply according to the verse: "God called unto him out of the midst of the bush, and said, Moses, Moses. And he said, Here am I" (*Ex.* 3:4). It is a genus. In the mathematical form:

$$\frac{A * X \text{ or } X * A, \quad B * Y \text{ or } Y * B, \quad A, B \subset C}{\exists D.\, D \subset C}.$$

E9. *Derek qeẓarah* [דרך קצרה] *('abbreviation')*, this is used in the text when the subject of discussion is self-explanatory.

$$\frac{A * X \text{ or } X * A}{B, \quad [B \text{ is an abbreviation of } A]}.$$

E10. *Dabar šehu' šanuy* [דבר שהוא שנוי] *('repeated expression')*, repetition always implies a special meaning. Example:

> Trust ye not in lying words, saying, The temple of the Lord, The temple of the Lord, The temple of the Lord, are these (*Jer.* 7:4).

This passage hints at two things: three temples of the Lord were destroyed and the temple was visited by Israelites three times a year.

$$\frac{\overbrace{A * A * \ldots * A}^{n}}{B_{[n]}, A \subset B_{[n]}\ (B_{[n]}\ \textit{is a property of}\ A\ \textit{connected to}\ n)}.$$

E11. *Siddur še-neḥlaq* [סדור שנחלק] *('omitting sequence/order'),* where in the text a clause or sentence is divided not logically by the punctuation, the proper sequence and the division of the verses must be restored according to the logical connection.

$$\frac{[\textit{the phrase contains}\ A, X]}{A * X\ \textit{or}\ X * A}.$$

E12. [דבר שבא ללמד ונמצא למד] Anything introduced as a comparison to illustrate and explain something else, itself receives in this way a better explanation and elucidation. Example: "The voice [of Egypt] thereof shall go like a serpent" (*Jer.* 46:22). On the one hand, it states that Egypt will be destroyed and devastated and, on the other hand, the voice of serpent hints at the event when angels have torn off arms and legs of the same serpent who talked to Eva.

$$\frac{A,\quad B,\quad [A\ \textit{is made clearer by}\ B]}{[B\ \textit{is made clearer by}\ A]}.$$

E13. [דבר שאחריו מעשה והוא פרטו של ראשון] When the general action is followed by the particular, the latter restricts the former and merely defines it more exactly (comp. the fifth rules of Hillel). Example: the general action is "So God created man in his own image, in the image of God created he him; male and female created he them" (*Gen.* 1:27), its two specifications

> And the Lord God formed man of the dust of the ground, and breathed into his nostrils the breath of life; and man became a living soul (*Gen.* 2:7),

> And the Lord God caused a deep sleep to fall upon Adam, and he slept: and he took one of his ribs, and closed up the flesh instead thereof. And the rib, which the Lord God had taken from man, made he a woman, and brought her unto the man (*Gen*. 2:21—22).

$$\frac{A * B, B \subset A \text{ (}i.e.\ A \text{ is general, } B \text{ is particular)}}{B}.$$

E14. [דבר גדול שנתלה בקטן הימנו להשמיע האזן בדרך שהיא שומעת] Something important is compared with something unimportant to elucidate it and render it more readily intelligible. Example: "My doctrine shall drop as the rain [*important*], my speech shall distil as the dew [*less important*]" (*Deut*. 32:2).

$$\frac{[A \text{ is compared with an unimport. } B]}{[A \text{ is to elucidate } B]}.$$

E15. [שני כתובים המכחישים זה את זה עד שיבוא הכתוב השלישי ויכריע ביניהם] When two Scripture verses contradict each other, the contradiction must be solved by reference to a third verse (see the 13rd rule of Rabbi Ishmael). Example: we observe a contradiction in numbers between the following passages:

> And Joab gave the sum of the number of the people unto David. And all they of Israel were *a thousand thousand and an hundred thousand men* that drew sword: and Judah was four hundred threescore and ten thousand men that drew sword (1 *Chron*. 21:5);

> And Joab gave up the sum of the number of the people unto the king: and there were in Israel *eight hundred thousand valiant men* that drew the sword; and the men of Judah were five hundred thousand men (2 *Sam*. 24:9).

We find out a solution due to the verse:

> Now the children of Israel after their number, to wit, the chief fathers and captains of thousands and hundreds, and their officers that served the king in any matter of the courses, which came in and went out month by month throughout all the months of the year, of every course *were twenty and four thousand* (1 *Chron.* 27:1).

This means that we should add 12 × 24,000 = 288,000 to the number 800,000. We obtain 1,088,000. Further, we should add 12,000 in respect to the number of chief fathers and captains for 288,000 Israelites who have been served the king.

$$\frac{A,\quad B,\quad [A \textit{ contradicts } B]}{[\textit{there exists } C \textit{ such that the contradiction is solved by } C]}.$$

E16. *Dabar šehu' meyuḥad bi-meqomo* [דבר שהוא מיוחד במקומו] *('rare expressions take own places, they are not causal')*, an expression which occurs in only one passage have an additional meaning.

$$\frac{A * X \textit{ or } X * A,\quad [\textit{a rare word}] \in A}{[\textit{an additional meaning of } A]}.$$

E17. [דבר שאינו מתפרש במקומו ומתפרש במקום אחר] A point which is not clearly explained in the main passage (at the beginning) may be better elucidated in another passage (at the end).

$$\frac{A * X,\quad [A \textit{ is unclear}]}{\exists Y.\, Y * A,\quad [A \textit{ is much clearer}]}.$$

E18. [דבר שנאמר במקצת והוא נוהג בכל] A statement with regard to a part may imply the whole. Example: "neither shall ye eat any flesh that is torn of beasts in the field" (*Ex.* 22:31). This flesh could be torn of beasts in another place (e.g. in the forest) to be forbidden for eating.

$$\frac{A * X \text{ or } X * A, \quad [A \text{ is particular}]}{[\text{perhaps } A \subset B \text{ and } (B * X \text{ or } X * B)]}.$$

E19. [דבר שנאמר בזה והוא הדין בחברו] A statement concerning one thing may hold with regard to another as well. A predicate is mentioned in connection with a subject, but refers also to other subjects. Example: "Light is sown for the righteous, and gladness for the upright in heart" (*Ps.* 97:11). Both of these predicates "light for…" and "gladness for…" refer to either of these subjects.

$$\frac{[A \text{ concerns } B]}{[\text{perhaps there exists } C \text{ and } A \text{ concerns } C \text{ as well}]}.$$

E20. [דבר שנאמר בזה ואינו עניין לו אבל הוא עניין לחברו] A statement concerning one thing may apply only to something else. A predicate which only nominally refers to the subject, but in reality it alludes to a subject which is connected with the first one. Example: "And this is the blessing of Judah: and he said, Hear, Lord, the voice of Judah" (*Deut.* 33:7). The first part of this blessing ("this is the blessing of Judah") refers to Simon who didn't receive a blessing yet.

$$\frac{[A \text{ tells about } B]}{[\text{perhaps there exists } C \text{ and } A \text{ concerns } C]}.$$

E21. [דבר שהוקש בשתי מידות ואתה נותן לו כח היפה שבינותיהן] If one object is compared to two properties (predicates), the best part of both the latter holds for this object. A subject compared with two predicates has to be taken in the light of all their advantages and merits. Example: "The righteous shall flourish like the palm tree: he shall grow like a cedar in Lebanon" (*Ps.* 92:12). A palm tree bears fruit, but does not give umbrage and a cedar does not bear fruit, but gives umbrage. This verse means that the righteous bear fruit like a palm tree and give umbrage like a cedar.

$$\frac{[A, B \text{ are predicates of } C]}{[\text{the best parts of } A, B \text{ are properties of } C]}.$$

E22. [דבר שחברו מוכיח עליו] A verse may be supplemented and explained by a parallel verse. A subject is defined by another subject. Example: "O Lord, rebuke me *not in* thy wrath and chasten me in thy hot displeasure" (*Ps.* 38:1). "Not in" of the first passage refers also to the second one.

$$\frac{A * B, \quad [a \text{ word } C] \in A}{[\text{perhaps a word } C \text{ concerns } B \text{ as well}]}.$$

E23. [דבר שהוא מוכיח על חברו] A passage serves to elucidate and supplement its parallel passage. A subject explains another one. Example: "A wise son his father's instruction: but a scorner heareth not rebuke" (*Prov.* 13:1). The word heareth refers to the first part, and it ought to read: A wise son heareth the instruction.

$$\frac{A * B, \quad [a \text{ word } C] \in B}{[\text{perhaps a word } C \text{ concerns } A]}.$$

E24. [דבר שהיה בכלל ויצא מן הכלל ללמד על עצמו] When the particular implied in the general statement is especially excepted from the general, it serves to emphasize some property characterizing this particular. Example: "Go view the land, even Jericho" (*Josh.* 2:1). Jericho is especially excepted, because it was capital. Another example: "there lacked of David's servants nineteen men and Asahel" (2 *Sam.* 2:30). Hence, Asahel belonging to nineteen servants was equal to all other in respect of his abilities.

$$\frac{A * B, \quad B \subset A,}{[\text{there exists a special property of } B]}.$$

E25. [דבר שהיה בכלל ויצא מן הכלל ללמד על חברו] The particular implied in the general statement is frequently excepted from the general to elucidate some other specific property.

$$\frac{A * X \text{ or } X * A, \quad B * Y \text{ or } Y * B, \quad A \subset B}{[\text{there exists a special property of } A]}.$$

E26. *Mašal* [משל] *('parable')*, a metaphorical meaning of passages. Example: "If he rise again, and walk abroad upon his staff" (*Ex.* 21:19). The expression "upon his staff" means that he is healthy. It is out of a logical interpretation.

E27. *Mi-neged* [מנגד] *('interpretation by symmetry')*. Example:

> After the number of the days in which ye searched the land, even *forty days*, each day for a year, shall ye bear your iniquities, even *forty years*, and ye shall know my breach of promise (*Num.* 14:34).

$$\frac{A * X \text{ or } X * A, \quad B * Y \text{ or } Y * B, \quad [a \text{ word } C] \in A, B}{[\text{there exists a common property of } A, B]}.$$

E28. *Mi-ma'al* [ממעל] *('interpretation by a preceding action')*. Example: Jeremiah's prophecy "Therefore thus saith the Lord God of Israel against the pastors that feed my people" (*Jer.* 23:2) was later, than Isaiah's words:

> But ye said, No; for we will flee upon horses; therefore shall ye flee: and, We will ride upon the swift; therefore shall they that pursue you be swift (*Is.* 30:16).

The former is saying about punishment and the latter about sin.

$$\frac{A * X, \quad X * B}{[\text{perhaps } A \text{ is about } B]}.$$

E29. *Gematria'* [גימטריא] *(Gematria: 'numerical meaning of words')*, interpretation according to the numerical value of the letters. *Gematria* refers to the numerical equivalent of a word, e.g., the name Eliezer, Abraham's servant, has the same numerical value as the number of soldiers (318) Abraham takes out to battle (*Gen.* 14:14). By *Gematria*, one may suppose that Abraham sent only Eliezer into the battle.

$$\frac{A * X \text{ or } X * A, \quad [\textit{the numeric value of } A] = [\textit{the numeric value of } B]}{A = B}.$$

E30. *Noṭariqon* [נוטריקון] *('shorthand'),* the letters of a word represent the initial letters of other words. Some examples are: *nimreẓet* ('grievous;' 1 *Kin.* 2:8) alludes to *no'ef* ('adulterer'), *mo'avi* ('Moabite', i.e. non-Jew), *roẓeaḥ* ('murderer'), *ẓorer* ('enemy'), *to'evah* ('abomination').

$$\frac{A * X \text{ or } X * A, \quad [A \textit{ is a shorthand of } B, C, ...]}{A = B, C, ...}.$$

E31. [מוקדם שהוא מאוחר בעניין] Postposition of the precedent. Many phrases which follow must be regarded as properly preceding. Example:

> And ere the lamp of God went out in the temple of the Lord, where the ark of God was, and Samuel was laid down to sleep (1 *Sam.* 3:3).

The event that Samuel was laid down to sleep should be understood as a preceding action.

$$\frac{A * B}{[\textit{perhaps } B \textit{ then } A]}.$$

E32. [מוקדם שהוא מאוחר בפרשיות] Many portions of the Bible refer to an earlier period than do the sections which precede them, and vice versa. Example: in reality the event "In the same day the Lord made a covenant with Abram" (*Gen.* 15:18) was earlier than the event of war

> And there went out the king of Sodom, and the king of Gomorrah, and the king of Admah, and the king of Zeboiim, and the king of Bela (the same is Zoar) and they joined battle with them in the vale of Siddim (*Gen.* 14:8).

$$\frac{A * X, \quad X * B}{[perhaps\ B\ then\ A]}.$$

Hillel, Ishmael, and Eliezer ben Jose ha-Gelili decided to omit from their collections many rules, because they restricted themselves to a compilation of the principal methods of logical deduction, which they called *middot* (measures, rules), although the many other rules were known by that term too.

Among middot there exists an original analogue of classification of knowledge into two groups: a priori analytic and a priori synthetic. In Kant's view, a reasoning in which all concepts have a common genus is called an analytic reasoning, i.e. between them it is easy to find a semantic connection of including. For example, the reasoning '*every man is an animal*' is analytic, because the concept '*animal*' includes the concept '*man*' ('*man*' is a specie of '*animal*'). On the other hand, a priori synthetic reasoning is regarded as a reasoning in which there exists no semantic connection of including between concepts, but it is possible to find an empirical connection of ordering (e.g., relationship of cause and effect). A priori synthetic reasoning can be exemplified as '*if there is fire, then there is smoke.*'

Let us remember what Kant said about such a classification.

> In all judgments in which the relation of a subject to the predicate is thought (I take into consideration affirmative

judgments only, the subsequent application to negative judgments being easily made), this relation is possible in two different ways. Either the predicate to the subject A, as something which is (covertly) contained in this concept A; or outside the concept A, although it does indeed stand in connection with it. In the one case I entitle the judgment analytic, in the other synthetic. Analytic judgments (affirmative) are therefore those in which the connection of the predicate with the subject is thought through identity; those in which this connection is thought without identity should be entitled synthetic … If I say, for instance, '*All bodies are extended,*' this is an analytic judgment. For I do not require to go beyond the concept which I connect with '*body*' in order to find extension as bound up with it. To meet with this predicate, I have merely to analyze the concept, that is, to become conscious to myself of the manifold which I always think in that concept. The judgment is therefore analytic. But when I say, '*All bodies are heavy,*' the predicate is something quite different from anything that I think in the mere concept of body in general; and the addition of such a predicate therefore yields a synthetic judgment.

Judgments of experience, as such, are one and all synthetic. For it would be absurd to found an analytic judgment on experience. Since, in framing the judgment, I must not go outside my concept, there is no need to appeal to the testimony of experience in its support. That a body is extended is a proposition that holds a priori and is not empirical. For, before appealing to experience, I have already in the concept of body all the conditions required for my judgment. I have only to extract from it, in accordance with the principle of contradiction, the required predicate, and in so doing can at the same time become conscious of the necessity of the judgment—and that is what experience could never have taught me. On the other hand, though I do not include in the concept of a body in general the predicate '*weight,*' none the less this concept indicates an object of experience through one of its parts, and I can add to that part other parts of this same experience, as in this way belonging together with the concept.

Thus it is evident: 1. that through analytic judgments our knowledge is not in any way extended, and that the concept

> which I already have is merely set forth and made intelligible to me; 2. that in synthetic judgments I must have besides the concept of the subject something else (X), upon which the understanding may rely, if it is to know that a predicate, not contained in this concept, nevertheless belongs to it. In the case of empirical judgments, judgments of experience, there is no difficulty whatsoever in meeting this demand. This X is the complete experience of the object which I think through the concept A—a concept which forms only one part of this experience (*Kant*).

For the two kinds of reasoning (a priori analytic and a priori synthetic) the following two groups of inference rules are applied in the Talmud:

- *basic Judaic inference rules for a priori analytic reasoning*—H5, I4, I5, I6;
- *basic Judaic inference rules for a priori synthetic* reasoning—E1, E2, E3, E4.

In these rules we can find four semantic sorts: particularity, generality, extension, restriction. *Particularity* (*frat*) and *generality* (*kelal*) are understood in the standard (Aristotle's) way. *Extension* (*ribuy*) is applied in case the following words occur in reasoning: 'together,' 'also' or 'however'. For example, the word 'together' (*'et*, אֵת) occurs twice in the following verse: "in the beginning God created the heaven [in Hebrew: אֵת הַשָּׁמַיִם] and the earth [in Hebrew: וְאֵת הָאָרֶץ]" (*Gen.* 1:1). This means that for the understanding of this verse it is necessary to extend the concept/statement that was used; as a result, this verse is understood that the heaven and the earth had been created immediately with everything that is contained in them (stars, trees, grass, etc.). *Restriction* (*mi'ut*) is used if there are the following words in reasoning: 'but,' 'only' or 'of.' For example, the occurrence of the particle 'of' (*min*) in the verse "any man bring your offering *of* the cattle" (*Lev.* 1:2) says that not any cattle may be sacrificed, the exceptions are animals that are idols for somebody.

Two of the derivations of H5, I4, I5, I6 are as follows:

I'1. The rule '*general after general*:' if the general name/proposition occurs just after the general name/proposition, then we introduce the particular name/proposition between them and use rule I6.

$$\frac{A * B,\ A \text{ is general},\ B \text{ is general}}{(A * C) * B,\ A \text{ is gener.},\ C \text{ is part.},\ B \text{ is gener.}}.$$

I'2. The rule *fraṭ u-kelal u- fraṭ* [פרט וכלל ופרט] *('particular and general and particular' or 'particular after general after particular')*: if the particular property occurs just after the general property that occurs just after the particular, then we accept the latest particular.

$$\frac{(A * B) * C, A \subset B, C \subset B (\text{i.e. } A \text{ is part.},\ B \text{ is gener.},\ C \text{ is part.})}{C}.$$

Some derivations of E1, E2, E3, E4 are as follows:

E'1. The rule '*restriction after extension'* (*ribuy u-mi'uṭ*): if the restricted concept/sentence occurs just after the extended concept/sentence, then we accept the restricted concept/sentence. Formally:

$$\frac{A * B,\ A \text{ is extended},\ B \text{ is restricted}}{B}.$$

E'2. The rule '*extension after restriction*' (*mi'uṭ u-ribuy*): if the extended concept/sentence occurs just after the restricted concept/sentence, then we accept the extended concept/sentence taking into account the previous restriction. Symbolically:

$$\frac{A * B,\ A \text{ is restricted},\ B \text{ is extended}}{[B, \text{but } A \text{ restricts } B]}.$$

E'3. The rule '*extension after restriction after extension*' (*ribuy mi'uṭ ribuy*): if the extended concept/sentence occurs just after the restricted concept/sentence, then we accept the latest extended concept/sentence. For instance, take the reasoning

> thou shalt bestow that money for whatsoever thy soul lusteth after, for oxen, for sheep, for wine, for strong drink, for whatsoever thy soul desireth (*Deut.* 14:26).

The concept 'whatsoever thy soul lusteth after' is extended, the concept 'oxen, sheep, wine, strong drink' is restricted, and the concept 'whatsoever thy soul desireth' is extended again. Then we accept the latest concept taking into account the examples 'oxen, sheep, wine, strong drink.'

$$\frac{(A * B) * C,\ A \text{ is ext.}, B \text{ is rest.}, C \text{ is ext.}}{[C, \text{but } B \text{ restricts } C]}.$$

Notice that there is no rule '*restriction after extension after restriction*' (*mi'uṭ ribuy mi'uṭ*). All Tanaim agree with this claim, namely they teach that when the verse contains a limiting clause, a broader clause, and then another limiting clause, it should be interpreted by the rule I'2:

> ורבי אלעזר דדריש מיעט וריבה פרט וכלל ופרט מנא ליה א"ר אבהו נפקא ליה מהאי קרא (שמות כב) וכי יתן איש אל רעהו חמור או שור או שה פרט וכל בהמה כלל לשמור חזר ופרט פרט וכלל ופרט אי אתה דן אלא כעין הפרט רבא אמר נפקא ליה מהאי קרא (ויקרא א) ואם מן פרט הצאן כלל כבשים ועזים חזר ופרט פרט וכלל ופרט אי אתה דן אלא כעין הפרט אמר ליה רב יהודה מדיסקרתא לרבא ולילף מן הדין קרא (ויקרא א) מן פרט הבהמה כלל בקר וצאן חזר ופרט פרט וכלל ופרט אי אתה דן אלא כעין הפרט א"ל מן האי ליכא למשמע מינה דאי מהתם הוה אמינא הבהמה **דף לה,ב גמרא** חיה בכלל בהמה א"ל חיה בכלל בהמה הא כתיב בקר וצאן והוה ליה פרט וכלל ואי אתה דן אלא כעין הפרט ומנלן דהכי הוא דתניא (דברים יד) ונתת הכסף בכל אשר תאוה נפשך כלל בבקר ובצאן וביין ובשכר פרט ובכל אשר תשאלך נפשך חזר וכלל כלל ופרט וכלל אי אתה דן אלא כעין הפרט מה הפרט מפורש פרי מפרי וגידולי קרקע אף כל פרי מפרי

וגידולי קרקע מכדי כלל ופרט וכלל כעין פרטא דיינינן כללא בתרא מאי אהני אהני לאוסופי כל דדמי ליה ותו פרט וכלל ופרט כעין הפרט דיינינן פרטא בתראה מאי אהני אי לאו פרטא בתראה ה"א נעשה כלל מוסף על הפרט ומכדי תרין כללי ופרטא ותרין פרטי וכללא <כללא> כעין פרטא דיינינן מאי איכא ביני וביני איכא דאילו תרתין כללי ופרטא אי איכא פרטא דדמי ליה אפילו בחד צד מרבינן תרי פרטי וכללא אי איכא פרטא דדמי משני צדדין מרבינן בחד צד לא מרבינן מכדי פרט וכלל נעשה כלל מוסף על הפרט ואיתרבי כל מילי ומיעט וריבה נמי ריבה הכל ואיתרבי כל מילי מאי איכא בין מיעט וריבה לפרט וכלל איכא דאילו פרט וכלל מרבינן אפילו עלין ולולבין ומיעט וריבה לולבין אין עלין לא:

Now R. Eleazar interprets [the text as consisting of] a clause that amplifies and a clause that limits. Where then does he find [in the Scripture the typical example of] specification, general statement and second specification?—R. Abbahu said that he finds it in the following verse. If a man deliver unto his neighbour an ass, or an ox, or a sheep, is a specification; or any beast is a generalisation; to keep is a further specification and so we may infer only what is similar to the specification.

Raba said that [R. Eleazar] could find one in the following verse. And if [his offering] be of [the flock] is a specifications the flock a general statement, and [whether of] the sheep, [or of] the goats a further specification, and so we may infer only what is similar to the specification.

Rab Judah of Diskarta asked Raba: Why should not [R. Eleazar] find it in the following verse? [Ye shall bring your offering] of is a specification the cattle [beasts] a general statement, and [of] the herd [or of] the flock a further specification, and so only what is similar to the specification can be inferred?—He replied: This is not a clear case, for if [he inferred it] from there it could be argued that [in the expression] 'the cattle,' cattle includes beasts of chase.—[Rab Judah] retorted: Could beasts of chase be included In 'cattle' [in this instance]? For 'the herd and the flock' are mentioned, making in fact a specifications a general statement, and a specifications and only what is similar to the specification can be inferred!

How do we know that [the rule] is correct?—It has been taught: And thou shalt bestow the money for whatsoever thy soul desireth is a general statement, for oxen or for sheep or for

wine or for strong drink a specification, and or for whatsoever thy soul asketh of thee a further general statement, making a general statement, a specification and a second general statement. Only what is similar to the specification may be inferred, and so because the specification particularises the product of that which is itself a product, whose sustenance is drawn from the earth, whatever is a product of a product-bearing species that draws its sustenance from the earth [may be purchased].

Seeing that when there is a general statement, a specifications and a general statement, we infer whatever is similar to the specification, what is then the function of the second general statement? It is to add whatever resembles the things specified.

Again, seeing that when there is a specifications a general statement, and a specifications what is similar to the specification is inferred, what is the purpose of the second specification?—But for its presence it would be said that it is a case of general statement being added to the [first] specification. Further, seeing that both when there are two general statements [separated by] a specification and when there are two specifications [separated by] a general statement, what is similar to the specification is inferred, what then is the difference between the two cases?—It is that whereas in the former case we include even things that resemble the specification In one respect only, in the latter case we include only what resembles [the specification] in two respects, but not what resembles it in one respect.

Seeing that when a specification is followed by a general statement, the general statement supplements the specification, all things being included, and again when a limitation is followed by an amplifying clause, this amplifies to the fullest extent, all things being included, what then is the difference between [the two cases]?—The difference is that whereas in the case of a specification followed by a general statement, both shoots and leaves [say], would be included, in the case of a limitation followed by an amplifying clause, Only the shoots, but not the leaves [would be included] (*Nazir* 35a—35b).

As we see, the Gemara cited above states that R. Eleazar applies the rule E'2 to the text. Why not the rule I5? The point is

that I5 includes every object in the general without exception, and it means in this context of the debate that I5 would prohibit a Nazir from eating all parts of the vine, even the branches. However, in R. Eleazar's words, a Nazir is liable only for eating edible parts of the grapevine. The rule E'2 has been used more often, than I'2 that has been regarded as a rare and unusual method. The mistake of R. Eleazar consists in that a broader clause is followed by another limiting clause. Therefore it is just the case of I'2, not of E'2.

Notice that rules H5, I4, I5, I6, I'1, I'2 assume the fixed genera inside a reasoning, while rules E1, E2, E3, E4, E'1, E'2, E'3 have no fixed genus inside a reasoning. The difference of the rule 'restriction after extension' ('extension after restriction') from the rule 'particular after general' ('general after particular') is that in the first case there is no genus relation inside the statement. For example, in the reasoning "thou shalt bestow that money for whatsoever thy soul lusteth after, for oxen, for sheep" (*Deut.* 14:26) the concepts 'whatsoever thy soul lusteth after' and 'oxen' have no common genus.

In Judaic logic, the inference rules H5, I4, I5, I6, I'1, I'2, E3, E4, E'1, E'2, E'3 are used for simplifying the Torah statements containing the Judaic conjunction $*$. Thus, the statements of the Pentateuch closed under the Judaic conjunction $*$ are axioms of *Judaic conjunctive logic* with inference rules H5, I4, I5, I6, I'1, I'2, E3, E4, E'1, E'2, E'3.

Definition 4.1 *The Judaic simple conjunctive logic is a system closed by using inference rules* H5, I4, I5, I6, I'1, I'2, E3, E4, E'1, E'2, E'3.

Definition 4.2 *The Judaic complete conjunctive logic is a system closed by using inference rules* H3—H7, I2—I13, I'1, I'2, E1—E4, E7, E8, E10, E11, E13, E16—E18, E22—E25, E27, E28, E31, E32, E'1, E'2, E'3.

In order to present a modern logical analysis of basic *middot* used by Talmudists (first of all *qal wa-ḥomer*, see: H1, I1, E5, E6) we should remark that these rules have nothing in common with

Ancient Greek logics. For instance, truth and falsity cannot be regarded as meaning of Scriptural passages. Evidently, the latter are viewed as absolutely true. By Judaic inference rules H3—H7, I2—I13, I'1, I'2, E1—E4, E7, E8, E10, E11, E13, E16—E18, E22—E25, E27, E28, E31, E32, E'1, E'2, E'3, other meanings of Biblical statements are introduced, namely the phrases may be either 'general' or 'particular.' As a result, logical connectives are defined in a unique way. For example, the connective "…and…" (it is said to be Judaic conjunction) is defined by the following matrix:

Definition 4.3

A	***B***	***A* and *B***
Particular	Particular	Particular
Particular	General	General
General	Particular	Particular
General	General	General

According to this meaning, Judaic conjunction is not commutative (the equality '*A* and *B*' = '*B* and *A*' is not valid, see rules H5, I4, I5), but it is idempotent (the equality '*A* and *A*' = '*A*' holds) and associative (the equality '*A* and (*B* and *C*)' = '(*A* and *B*) and *C*' holds). This understanding is implied by inference rules H5, I4, I5, I6.

The connective "…or…" (it is called Judaic disjunction) is defined as follows:

Definition 4.4

A	***B***	***A* or *B***
Particular	Particular	General
Particular	General	General
General	Particular	Particular
General	General	General

It is not commutative also, because of rules H5, I5, I6, I7. The first and fourth table lines may be exemplified by the verse:

> Behold, the money, which we found in our sacks' mouths, we brought again unto thee out of the land of Canaan: how then should we steal out of thy lord's house silver *or* gold? (*Gen.* 44:8).

They have been just convicted of stealing a silver thing and they are saying that they cannot steal not only silver [*a particular*], but also other things including gold [*a general*]. Another example:

> And the Lord said unto him, Who hath made man's mouth? *or* who maketh the dumb, *or* deaf, *or* the seeing, *or* the blind? have not I the Lord? (*Ex.* 4:11).

Judaic disjunction introduces here a general by combining either just general or particular notions. As Rashi comments on:

> **מי שם פה וגו':** מי למדך לדבר כשהיית נדון לפני פרעה על המצרי:
> **או מי ישום אלם:** מי עשה פרעה אלם שלא נתאמץ במצות הריגתך, ואת משרתיו חרשים, שלא שמעו בצוותו עליך, ולאספקלטורין ההורגים מי עשאם עורים, שלא ראו כשברחת מן הבימה ונמלטת:
> **הלא אנכי:** ששמי ה' עשיתי כל זאת:
>
> *Who hath made man's mouth:* Who taught you to speak when you were being judged before Pharaoh concerning the Egyptian [you killed]?
> *or who maketh the dumb:* Who made Pharaoh dumb, that he did not exert any effort [to issue his] command to kill you? And [who made] his servants deaf, so that they did not hear his commandment concerning you? And who made the executioners blind, that they did not see when you fled from the [executioner's] platform and escaped?
> *have not I:* Whose name is the Lord [י-ה-ו-ה], [Who] has done all this.

Another example:

If thy brother, the son of thy mother, *or* thy son, *or* thy daughter, *or* the wife of thy bosom, *or* thy friend, which is as thine own soul, entice thee secretly, saying, Let us go and serve other gods, which thou hast not known, thou, nor thy fathers (*Deut.* 13:6).

And Rashi's comment:

כי יסיתך: אין הסתה אלא גרוי, שנאמר (שמואל א' כו יט) אם ה' הסיתך בי, אמיטר"א בלע"ז [יסית], שמשיאו לעשות כן:
אחיך: מאב:
בן אמך: מאם:
חיקך: השוכבת בחיקך ומחוקה בך אפיקייד"א בלע"ז [דבוקה], וכן (יחזקאל מג, יד) ומחיק הארץ, מיסוד התקוע בארץ:
אשר כנפשך: זה אביך. פירש לך הכתוב את החביבין לך, קל וחומר לאחרים:
בסתר: דבר הכתוב בהווה, שאין דברי מסית אלא בסתר. וכן שלמה הוא אומר (משלי ז, ט) בנשף בערב יום באישון לילה ואפלה:
אשר לא ידעת אתה ואבותיך: דבר זה גנאי גדול הוא לך שאף האומות אין מניחין מה שמסרו להם אבותיהם, וזהו אומר לך עזוב מה שמסרו לך אבותיך:

If [your brother…] entice thee: The term הֲסָתָה always denotes inciting as it is said, "If the Lord have stirred [incited] thee up against me incited you against me [אם ה' הסיתך]" (I *Sam.* 26:19); *ametra* in Old French, will incite, that he tempts him to do so.
thy brother: from the father['s side].
the son of thy mother: from the mother['s side].
of thy bosom: The one that lies in your bosom and is attached [מָחֳקָה] to you; *afichiede* in Old French, bound, attached, *afichier* in Old French, to attach. This is similar to וּמֵחִיק הָאָרֶץ (*Ezek.* 43:14), which refers to the base [of the altar] that is affixed firmly in the ground.
[Or thy friend,] which is as thine own soul: This refers to your father. Scripture enumerates those [persons] who are dear to you; how much more so [must you apply this] to others [and not spare them]!
[If thy brother… entice thee] secretly: Scripture speaks of what common, for usually the one who entices speaks clandestinely.

> Similarly, Solomon says: "In the twilight, in the evening, in the black and dark night" (*Prov.* 7:9).
> *[Let us go and serve other gods,] which thou hast not known, thou, nor thy fathers:* This would be a great disgrace for you, for not even the heathen nations forsake what their forefathers have handed down to them. Yet this [man] says to you, "Abandon what your forefathers have handed down to you!"

Judaic disjunction is idempotent and associative, if we use only general notions, otherwise it is not.

The meaning-matrix of the connective "if... , then..." (it is said to be Judaic implication) is as follows:

Definition 4.5

A	***B***	**If *A* then *B***
Particular	Particular	Particular
Particular	General	General
General	Particular	General
General	General	General

Judaic implication is commutative, idempotent and associative. An instance of idempotency could be provided with the following passage: "If I be bereaved of my children, I am bereaved" (*Gen.* 43:14), an illustration of commutativity: "*If* Cain shall be avenged sevenfold [*a particular*], truly Lamech seventy and sevenfold [*a particular*]" (*Gen.* 4:24) we could add that if Lamech shall be avenged seventy and sevenfold, Cain sevenfold. An example of the case 'if/when a particular, then a particular:'

> *If* thou wilt take the left hand [*a particular*], *then* I will go to the right [*a particular*]; or *if* thou depart to the right hand [*a particular*], *then* I will go to the left [*a particular*] (*Gen.* 13:9).

An example of the case 'if/when a particular, then a general:'

> And *when* a stranger shall sojourn with thee, and will keep the passover to the Lord [*a particular*], let all his males be circumcised, and then let him come near and keep it; and he shall be as one that is born in the land [*a general*]: for no uncircumcised person shall eat thereof (*Ex*. 12:48).

An example of the case 'if/when a general, then a particular:'

> And said, *If* thou wilt diligently hearken to the voice of the Lord thy God, and wilt do that which is right in his sight, and wilt give ear to his commandments, and keep all his statutes [*a general*], I will put none of these diseases upon thee, which I have brought upon the Egyptians: for I am the Lord that healeth thee [*a particular*] (*Ex*. 15:26).

An example of the case 'if/when a general, then a general:'

> Now therefore, *if* ye will obey my voice indeed, and keep my covenant [*a general*], *then* ye shall be a peculiar treasure unto me above all people: for all the earth is mine [*a general*] (*Ex*. 19:5).

Judaic logic for inferring legislative statements from the Pentateuch is closed under connectives defined above (definitions 4.3—4.5). The simplest case of deductions in such logic is built up by using *qal wa-ḥomer* (see H1, I1, E5, E6). This inference rule differs from all conventional inference rules involved in a deduction procedure of classical and non-classical logics. It is connected with a parallelism and non-recursiveness assumed in its using. Let us consider a simple traditional example of *qal wa-ḥomer*. In the *Baba Qama*, different kinds of damages (*nezeqin*) are analyzed, among which three genera are examined: foot action (*regel*), tooth action (*šen*) and horn action (*qeren*). These three are damages that could be caused by an ox (he can trample (foot), eat (tooth) and gore (horn)). Due to the Torah it is known that tooth damage (as well as foot damage) by an ox at a public place needs to pay zero compensation. Horn damage at a public place pays 50% the cost of damage as

compensation. In a private area foot/tooth damage must be paid in full. What can we say now about payment for horn action at a private place?

Damages (nezeqin)	**Public place**	**Private place**
Foot action (*regel*)	0	100%
Tooth action (*šen*)	0	100%
Horn action (*qeren*)	50%	?

In order to draw up a conclusion by *qal wa-ḥomer*, we should define a two-dimensional ordering relation on the set of data: (i) on the one hand, we know that payment for horn action in a private area cannot be smaller than the same in a public area, (ii) on the other hand, payment for horn action at a private place cannot be greater than foot/tooth action at the same place, (iii) according to the *dayo* principle, we choose the minimal value (it is equal to 50%). Hence, we infer that payment of compensation for horn action at a private place is equal to 50% cost of damage. This conclusion in a formal notation:

$$\begin{array}{lccc} & \text{Tooth action} & & \text{Horn action} \\ \textit{public place}: & \begin{pmatrix} \textit{pay } 0\% \\ \textit{pay } 100\% \end{pmatrix} & \leq & \begin{pmatrix} \textit{pay } 50\% \\ \textit{pay } x\% \end{pmatrix} \\ \textit{private place}: & & & \end{array}$$
$$x = 50\%$$

A modification of Aristotelian syllogism may be presented as the simplest case of *qal wa-ḥomer*. Let us examine the following famous syllogism: "All men are mortal. Socrates is a man. Therefore Socrates is mortal." Its *qal wa-ḥomer* analogue:

$$\begin{array}{lccc} \textit{general notion (predicate)}: & \begin{pmatrix} \textit{mortal} \\ \textit{man} \end{pmatrix} & \leq & \begin{pmatrix} x \\ \textit{Socrates} \end{pmatrix} \\ \textit{particular notion (subject)}: & & & \end{array}$$
$$x = \textit{Socrates' mortality}$$

Continuing our reasoning in the same way, we should define a two-dimensional ordering relation: (i) we know that the notion 'Socrates' is more general than the notion x whose value we are looking for, (ii) the notion 'mortal' is more general than this x as well, (iii) by the *dayo* principle, we find the minimal value of x, it is 'Socrates' mortality' exactly.

The Biblical example of *qal wa-ḥomer* occurs in the following passages:

> And the Lord spake unto Moses, saying, go in, speak unto Pharaoh king of Egypt, that he let the children of Israel go out of his land. And Moses spake before the Lord, saying, behold, the children of Israel have not hearkened unto me; how then shall Pharaoh hear me, who am of uncircumcised lips? (*Ex.* 6:10—12).

$$\frac{\begin{pmatrix} \textit{Jews do not hear Moses} \\ \textit{does Pharaoh hear Moses}? \end{pmatrix} \geq \begin{pmatrix} \textit{because} \\ \textit{because} \end{pmatrix} \begin{pmatrix} \textit{Moses is of uncircumcised lips} \\ \textit{Moses is of uncircumcised lips} \end{pmatrix}}{\textit{No, he does not}}$$

All examples of *qal wa-ḥomer* regarded above show that this inference rule cannot be presented in a linear form and assumes a multi-dimensional ordering relation (the simplest case of two-dimensional order was considered in instances above). As opposed to this, usual inference rules in modern logic suppose linearity, therefore by combining these rules we obtain conventional proof trees. *Qal wa-ḥomer* provides us with an algorithm for massively-parallel proofs. Hence, a deductive system of Judaic logic may be presented as a hybrid cellular automaton. We know that all Judaic statements have just two meanings, they are either 'general' or 'particular.' However, the ordering relation over them is partial, i.e. some statements are incompatible. Obviously, *qal wa-ḥomer* can be applied just for compatible statements.

Definition 4.6 *A hybrid cellular automaton for Judaic deduction is defined as follows: it is a 4-tuple* $\langle\ \mathbf{Z}^d, S, N, q\ \rangle$, *where*

- $\mathbf{Z}$ *is the set of all integers,* $d \in N$ *is a number of dimensions and the members of* $\mathbf{Z}^d$ *are referred as cells,*
- S *is a finite or infinite set of elements called the states of an automaton, the members of* $\mathbf{Z}^d$ *take their values in* S, *the set* S *is collected from statements of the Pentateuch.*
- $N \subset \mathbf{Z}^d \setminus \{0\}^d$ *is a finite set of n elements,* N *is said to be a radius of q, this set consists of statements that are compatible in accordance with d-dimensional ordering relation,*
- $q: S^{n+1} \rightarrow S$ *that is q is one of the Judaic inference rules, e.g. qal wa-ḥomer.*

As we see, Judaic reasoning may be formalized by using the non-well-founded mathematics and process semantics, both for the first time began to be used in computer science. In particular, we can assume that the Judaic semantics and Judaic formal logic can be developed within the framework of *interactive-computing/concurrency paradigm*. One of its means is the coinduction principle. For more details see *Abramsky, Jacobs & Rutten, Pavlovic & Escardo, Rutten*. However, the concepts of coalgebra and coinduction have not yet had much impact in the pure logical investigations as well as in the logical-historical studies.

Chapter 2. Non-Aristotelian roots of Talmudic syllogistics

1. Elementary logical operations

The reality around us has been collected from the facts. These facts are described in the language by means of propositions. If a proposition represents an appropriate fact rightly, it is true and if not rightly, it is false. Thus, due to the propositions we are fixing the facts and describing reality. Chrysippus of Soli, the Stoic philosopher, spoke about this reference relation between propositions and facts as follows:

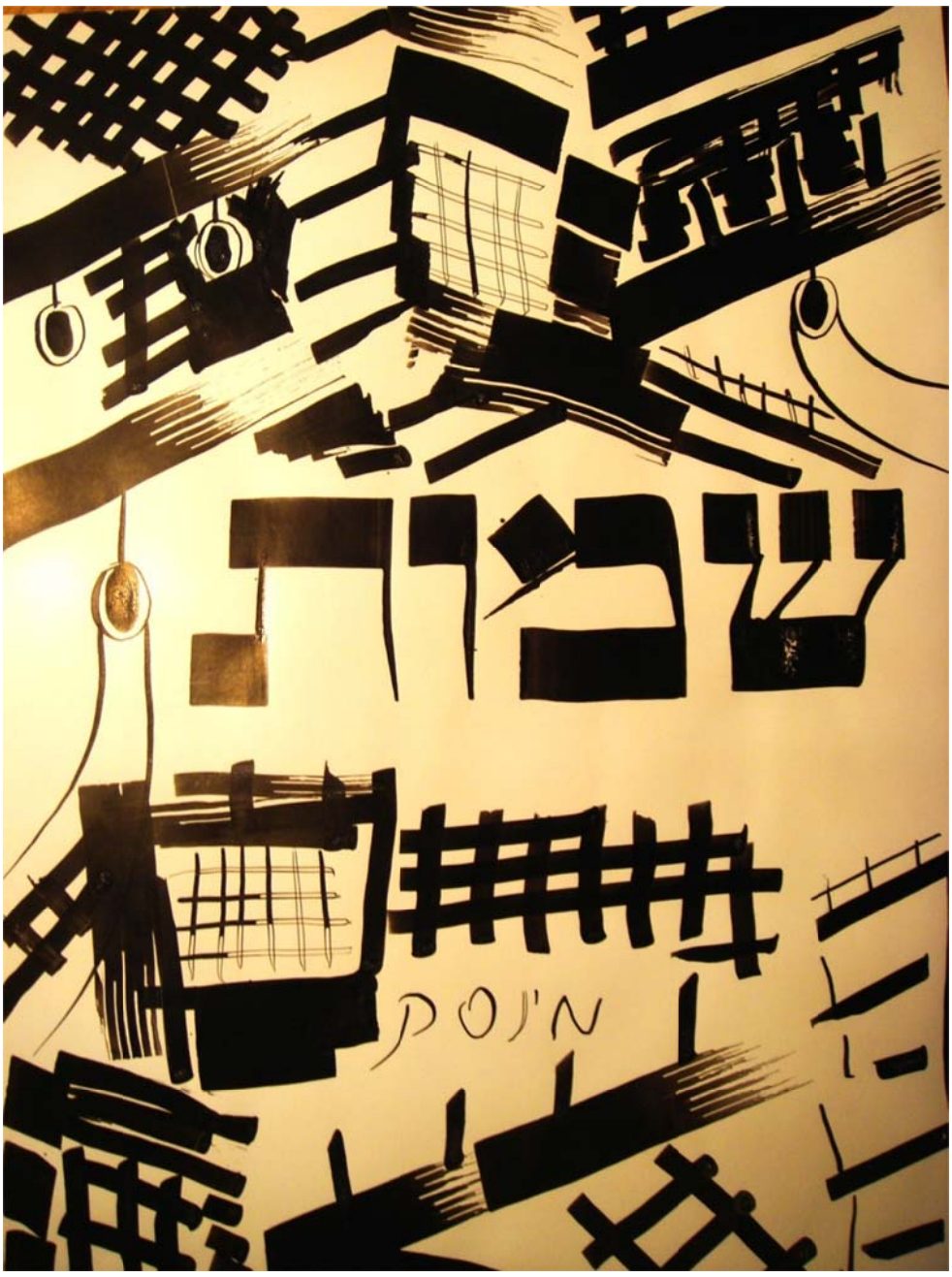
> Proposition is that we express by means of words and that happens either true, or false (Diocles Magnes apud *Diog. Laert.* VII 66).

In order to build up propositions which correctly describe reality, there is a special science, logic. For the first time, it was developed by philosophers of the Early Stoa, first of all, by Chrysippus. He was one of the first who began to distinguish between simple propositions and composite ones:

> Composite propositions consist of implicative, subimplicative and [explanatory propositions] which express smaller or bigger, but also, of disjunctive [propositions] and of all similar (*Philo de agricultura* § 139. Vol. II p. 122, 18).

Composite propositions are derivable from simple ones by means of connective words: "not …," "if …, then …," "either …, or …," "… or …," "… and …" A truth valuation of a simple proposition is that it is true if it corresponds to the fact which it describes, and it is false, if it is inconsistent with that fact. A truth valuation of a composite proposition depends on a connective word which was used for its derivation.

Let A and B are atomic propositions. Then molecular propositions have the following meanings:



Picture 4 The title page of the Book of Exodus

Definition 1.1 *The negation 'not A' (symbolically: ¬A) is true if and only if 'A' is false, and 'not A' is false if and only if 'A' is true.*

Definition 1.2 *The implication 'if A, then B' (symbolically: A⇒B) is false if and only if 'A' is true and 'B' is false, and it is true in all other cases*:

> It is a question of propositions, such as: 'If Dion is alive, Dion will be alive [and further].' Even if this proposition starts with the true premise 'Dion is alive,' has the true conclusion 'Dion will be alive' and consequently [as a whole] is true [now], in the future there will occur the moment when the second premise 'but Dion is alive' is true and for all that the whole implication is false—therefore, that in the future there will occur the moment when the premise 'but Dion is alive' is true still, but the conclusion 'Dion will be alive' is untrue. And if this last proposition is untrue, then the implication as a whole is false. After all not always, if the proposition 'Dion is alive' is true, then the proposition 'Dion will be alive' is true also—wherefore otherwise Dion should be immortal. And, of course, it is impossible to specify with accuracy, during what moment of Dion's life the proposition 'Dion will be alive' will not be true already. Therefore, they speak, these propositions change their values in indefinite unstated time. They are so-called propositions "indefinitely" changing values (Alexander Aphrod. apud *Siraplicium in Aristot. phys.* p. 1299, 36).

Definition 1.3 *The conjunction 'A and B' (symbolically: A&B or A∧B) is true if and only if simultaneously both 'A' and 'B' are true, and it is false in all other cases*:

> If it is a question that something is neither in relations of conclusion, nor in relation of incompatibility, we name such propositions conjunctive. For example: 'Dion walks and Dion ratiocinates.' These things are not concerning relations of conclusion and relations of incompatibility and consequently are expressed by means of conjunction (Galenus *introductio dialectica* 4 p. 10,13).

Definition 1.4 *The disjunction 'either A or B' (symbolically: A⊕B) is true if and only if both 'A' and 'B' are false simultaneously or true simultaneously,*

and it is true in all other cases.

Definition 1.5 *The subdisjunction 'A or B' (symbolically: A∨B) is false if and only if both 'A' and 'B' are false simultaneously, and it is true in all other cases (notice that in modern logic subdisjunction is called disjunction or inclusive disjunction and disjunction is called exclusive disjunction)*:

> Est item aliud, quod Graeci διεζευγμένον ἀ̓ξίωμα, nos "disiunctum" dicimus. Id huiuscemodi est: "aut malum est voluptas aut bonum aut neque bonum neque malum est." Omnia autem, quae disiunguntur, pugnantia esse inter sese oportet, eorumque opposita, quae ἀ̓ντικείμενα Graeci dicunt, ea quoque ipsa inter se adverse esse. Ex omnibus, quae disiunguntur, unum esse verum debet, falsa cetera. Quod si aut nihil omnium verum, aut omnia plurave quam unum vera erunt, aut quae disiuncta sunt non pugnabunt, aut quae opposita eorum sunt contraria inter sese non erunt, tunc id disiunctum mendacium est et appellatur παραδιεζευγμένον; sicuti hoc est, in quo quae opposita non sunt contraria: "aut curris aut ambulas aut stas."— — possis enim simul eodemque tempore neque ambulare neque stare neque currere (Gellius *Noct. Att.* XVI 8, 12).

> There is another sort of propositions which is called by Greeks διεζευγμένον ἀ̓ξίωμα, and by us disjunctive, for example: "Enjoying is either evil or good, or neither evil nor good." Thus, all constituents of the proposition which stand in the relation of disjunction should be exclusive; besides, and [contradictory] oppositions (opposita) (which by Greeks are called ἀ̓ντικείμενα) should be exclusive with members of disjunction. In disjunction one member should be true necessarily, and others false. If any of members is not true, all members or more than one member are true, or these members are not exclusive as well as their [contradictory] oppositions are not exclusive, in this case the disjunction is incorrect and it is called παραδιεζευγμένον as [subdisjunction]. It concerns, for example, those cases when oppositions to members of disjunction are not exclusive: "You either run or walk or stand,"—members of disjunction are exclusive here, whereas

> their [contradictory] oppositions [i.e. you do not run, you do not walk, you do not stand] are not exclusive... At the time it is quite possible not to run, not to walk and not to stand.

In modern logic the value to be 'true' is usually designated by 1, the value to be 'false' by 0. Ascribing a value to a proposition is called interpretation. If the set of propositions is finite, then interpretation can be defined in the form of a finite table. This table is called the *table of truth values* (or simply *truth table*). It allows us to see easier, how truth values of composite propositions are derivable on the basis of the value of simple ones:

A	***not A***
0	1
1	0

A	***B***	***if A then B***
0	0	1
0	1	1
1	0	0
1	1	1

A	***B***	***A and B***
0	0	0
0	1	0
1	0	0
1	1	1

A	***B***	***either A or B***
0	0	0
0	1	1
1	0	1
1	1	0

A	***B***	***A or B***
0	0	0
0	1	1
1	0	1
1	1	1

In definitions 1.1—1.5, the basic logical operations are set up. As we see, first they were defined by Ancient Greek philosophers (first of all by Stoics). Hence, modern symbolic logic has its roots in the Ancient Greek philosophy (e.g. in Aristotelian and Stoic ideas). This propinquity allows to formalize the Ancient Greek logics (namely, Aristotle's syllogistics and Stoic propositional logic, *Corcoran 1972, 1973, Łukasiewicz 1957*) from the standpoint of modern logic. However, the Judaic reasoning has not been formalized up until now, although this reasoning seems to be very logical and system-oriented. The problem is that taking into account the feature of some native algorithms in Judaic conclusions, the goal to formalize the Talmudic reasoning and to obtain the Judaic logic as formal language may be possible to accomplish only by using the most modern methods of computational logic, namely by using tools of unconventional computing (*Adamatzky* et al. *2005*) and non-well-founded mathematics (*Barwise & Etchemendy 1987, Barwise & Moss 1996*) which allow us to describe the Judaic metaphysics of massive-parallelism and concurrency. The point is that Judaic reasoning may be presented as a behavioural logic formalizing massive-parallel and concurrent behaviour in Judaic deductions (*Schumann 2008a*).

Since the role of unconventional computing, massive-parallel computing and non-well-founded mathematics has been increasing recently, we could claim that formalizing the Judaic reasoning will provide modern logic with novel horizons, similarly to how formalizing the Ancient Greek logical systems provided logic of the 20th century with new horizons allowing non-classical logics to come into being and to develop.

On the one hand, the Ancient Greek logic laid the foundation of classical logic. The Aristotle's *Prior Analytics* and *Posterior Analytics* (*Barnes 1993, Smith 1989*) contained the first proof theory for deductive sciences (in Aristotle's words, the theory of demonstration). Notice that as in much of the early history of modern symbolic logic, the Ancient logical theory arose within the philosophy of mathematics too. Aristotle's syllogistics has had an enormous influence in Western thought. For instance, its two basic assumptions were as follows: (1) every premise must be true, i.e. applicable to a number of cases (otherwise it will not be true in every instance); (2) true premises always give a true conclusion if we use correct inference rules. These assumptions are basic for deductions in modern symbolic logic as well. For the first time, Łukasiewicz began exploring the logical theories of Aristotle and the Stoics and formalizing them as modern logical systems (*1957*). The possibility of formalizing Aristotle's syllogistic and Stoic reasoning from the viewpoint of modern logic shows that these systems satisfy all the basic assumptions of conventional symbolic logic, in particular all assumptions of the Russellian semantics.

On the other hand, modern analysis of Aristotelian works caused a development of non-classical logics. Let us recall that the logical studies of Aristotle's *De Interpretatione* (ch. IX) (*Ackrill 1963*) implied one of the strongest polemics in the symbolic logic of the 1930—1950s years. Using some philosophical ideas of Aristotle (namely, his asserting that the application of the law of excluded middle to future propositions like "There will be a sea-battle tomorrow" should be categorically restricted), Łukasiewicz proposed the first version of many-valued logic (*1920*). His approach was espoused by many logicians (*Butler 1955, Prior 1953*). In the issue, this approach has had the hugest and most important influence on the development of the following very significant branches of non-classical logic: many-valued logic, modal logic, temporal logic, and dynamic logic. However, in the beginning, there were a lot of critical estimations of Łukasiewicz's non-classical

understanding of the logical sense of *De Interpretatione* (ch. IX) (*Baylis 1936, Ducase 1941, Quine 1953, Williams 1951*). But as it turned out, Łukasiewicz was right. As a result, his book *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic* had a very appreciable success—it had many editions in different languages (three editions only in Russian). This history gives evidence that the logical formalisation of reasoning belonging to Ancient logicians (including Greek, Indian, Hebrew) may be very surprising. Thus, we could assume that the formalisation of logical reasoning belonging to Talmudists may be a bit surprising in the future too.

2. Porphyrian tree and genus-species relations in the Talmud

In Talmudic reasoning we cannot face classical logic at all, e.g. deductions on the basis of definitions 1.1—1.5 (in lieu of them there are postulated definitions 4.3—4.5 of the first chapter). However, the notion of Judaic (simple or complete) conjunctive logic (see definitions 4.1—4.2 of the previous chapter) alludes that there exist almost classical relations between genus (the general, *kelal*) and species (the particular, *prat*) in the Judaic Universe as well as in Aristotelian syllogistics. Such relations are said to be *syllogistic*. Nevertheless, we will show that there are too deep differences between the Judaic treatment of syllogistic relations and the classical approach inspired by Ancient Greek philosophers.

The first significant difference consists in that syllogistic relations in Old Greek philosophy have an ontological dimension presented by the *Porphyrian tree*, a hierarchical structure that analytically describes the Universe. According to one of the classical versions, the Porphyrian tree grows from the top (the first node of the tree) designating the highest genus, e.g. essence. The latter puts the series of genera and species (other nodes connected by branches of the tree), e.g. the two first species of essence stood out in the tree (the first nodes after the top) are as follows: substance

and non-substance. Then the following branches shown up connect those nodes with material and non-material. The next pair of branches moves to the nodes of living and non-living. Further, the pair of sentient and non-sentient grows up. Hereafter the pair of rational and non-rational is emphasized. At last, real things (Plato, Socrates, etc.) stand out in the tree. In the *Porphyrii Isagoge* we can read about:

> Sit autem in uno praedicamento manifestum quod dicitur. Substantia est quidem et ipsa genus, sub hac autem est corpus, sub corpore vero animatum corpus sub quo animal, sub animali vero rationale animal sub quo homo, sub homine vero Socrates et Plato et qui sunt particulares homines. Sed horum, substantia quidem generalissimum est et quod genus sit solum, homo vero specialissimum et quod species solum sit; corpus vero species quidem est substantiae, genus vero corporis animati; et animatum corpus species quidem est corporis, genus vero animalis; animal autem species quidem est corporis animati, genus vero rationalis; sed rationale animal species quidem est animalis, genus autem hominis; homo vero species quidem est rationalis animalis, non autem etiam genus particularium hominum, sed solum species; et omne quod ante individua proximum est, species erit solum, non etiam genus. Quemadmodum igitur substantia quae cum suprema sit, eo quod nihil sit supra eam, genus est generalissimum, sic et homo cum sit species post quam non sit alia inferior species neque aliquid eorum quae possunt dividi sed solum individuorum (individuum enim est Socrates et Plato), species erit sola et ultima species et, ut dictum est, specialissima. Quae vero sunt in medio, eorum quidem quae supra ipsa sunt, erunt species, eorum vero quae post ipsa sunt, genera. Quare haec quidem duas habent habitudines: eam quae est ad superiora secundum quam species ipsorum esse dicuntur, et eam quae est ad posteriora, secundum quam genera ipsorum esse dicuntur. Extrema vero unam habent habitudinem; nam et generalissimum ad ea quidem quae posteriora sunt habet habitudinem, cum genus sit omnium id quod est supremum, eam vero quae est ad superiora non habet, cum sit supremum et primum principium; specialissimum autem unam habet

habitudinem, eam quae est ad superiora quorum est species, eam vero quae est ad posteriora non diversam habet, sed etiam individuorum species dicitur (sed species quidem individuorum velut ea continens, species autem superiorum velut quae ab eis continetur) (Anicius Manlius Severinus Boethius. *Porphyrii Isagoge translation*).

Between the most generic and the most special, there are others which are alike both genera and species, referred, nevertheless, to different things, but what is stated may become clear in one category. Substance indeed, is itself genus, under this is body, under body animated body, under which is animal, under animal rational animal, under which is man, under man Socrates, Plato, and men particularly. Still, of these, substance is the most generic, and that which alone is genus; but man is most specific, and that which alone is species; yet body is a species of substance, but a genus of animated body, also animated body is a species of body, but a genus of animal; again, animal is a species of animated body, but a genus of rational animal, and rational animal is a species of animal, but a genus of man, and man is a species of rational animal, but is no longer the genus of particular men, but is species only, and every thing prior to individuals being proximately predicated of them, will be species only, and no longer genus also. As then, substance being in the highest place, is most generic, from there being no genus prior to it, so also man being a species, after which there is no other species, nor any thing capable of division into species, but individuals, (for Socrates, Plato, Alcibiades, and this white thing, I call individual,) will be species alone, and the last species, and as we say the most specific. Yet the media will be the species of such as are before them, but the genera of things after them, so that these have two conditions, one as to things prior to them, according to which they are said to be their species, the other to things after them, according to which they are said to be their genera. The extremes on the other hand, have one condition, for the most generic has indeed a condition as to the things under it, since it is the highest genus of all, but has no longer one as to those before it, being supreme, and the first principle, and, as we have said, that above which there cannot be another higher genus. Also, the

> most specific has one condition, as to the things prior to it, of which it is the species, yet it has not a different one, as to things posterior to it, but is called the species of individuals, so termed as comprehending them, and again, the species of things prior to it, as comprehended by them, wherefore the most generic genus is thus defined to be that which being genus is not species, and again, above which there cannot be another higher genus; but the most specific species, that, which being species is not genus, and which being species we can no longer divide into species; moreover, which is predicated of many things differing in number, in reply to what a thing is (Porphyry, *Introduction (or Isagoge) to the logical Categories of Aristotle*).

In genus-species relations presented in the Porphyrian tree we can consider a branch (a relation between a genus and species) as implication, where the top of branch (genus) is regarded as consequent of implication and the bottom of branch (species) as antecedent of implication. Then for each node of the tree, we may define an intension as all reachable genera (all higher nodes) and an extent as all reachable species (all lower nodes):

Definition 2.1 *Let us take the set of all consequents of true implications, where the name/proposition A is an antecedent. Then this set is an intension (logical content) of A. For instance, let A be a human being. Then its intension is built up by the set* {'rational,' 'animate,' 'social,' etc.}.

Definition 2.2 *Let us take the set of all antecedents of true implications, where the name/proposition A is a consequent. Then this set is an extent (logical scope) of A. For instance, let A be a human being. Then its extent is built up by the set* {'worker,' 'woman,' 'scientist,' etc.}.

Due to both definitions, we can precisely say what position in the Porphyrian tree each node has:

Proposition 2.1 *There exists the law of the inverse relation between intension and extent: the greater intension is, the smaller extent is and, on the contrary, the smaller intension is, the greater extent is.*

The main feature of Judaic syllogistic reasoning is that it is prohibited to build up own deductions on relations between genera and species, i.e. there is no ontologization of syllogistics, there is no Porphyrian tree in Judaism. Human beings cannot hammer out relations between genera and species (i.e. deductive syllogisms) at all. It is permitted only to find out syllogistic relations in the Torah, because just the Lord made up all syllogistic relations for us. We can claim that the principal aim of Tannaim and Amoraim was to reconstruct the Universe as Divine creation by looking for all deductive syllogisms implicitly or explicitly contained in the Torah. From the Judaic point of view, the Porphyrian tree is just for men who do not believe in the Divine Revelation, who believe in themselves and the own intelligence.

While for the West philosophers initiated by the Old Greek ones all the syllogistic relations (e.g. in the Porphyrian tree) should be considered as analytic a priori, i.e. they are found out in my thought (it means they are analytic) before any experience (it means they are a priori), in Judaism all the syllogistic relations should be regarded as analytic a posteriori, because, on the one hand, they are constructed purely logically (they are analytic), but on the other hand, they occur as result of studying the Torah (they are a posteriori). Thus, the intellectual status of Judaic syllogistics and Aristotelian one has a devastating contrast. If the first is about analytic a posteriori knowledge, then the second is about analytic a priori knowledge.

Each Talmudic book is a narration on its own syllogistic tree reconstructed by studying the Torah. All such trees assembled in the Talmud as a whole are aimed to present a Judaic view on the Universe. As an example, let us consider the *Baba Qama* as one of the Talmudic books with its own syllogistic tree.

3. The *Baba Qama* as example of genus-species relations in the Talmud

ספר המדות

מאמר ראשון

סירוש

פרק א׳ כל מלאכה

סירוש

פרק א׳ כל מלאכה וכל לימוד גם כל יראה מעשה וכל בחיר׳

כי הוא להאות דבר טוב . על כן המיבו אשר דברו כי טוב הוא אשר הכל יתאוו . נראה שיש הפרש בין התכליות . כי מהם פעולות ואחרים זולתם הם הפעולים . אמנם מהדברי׳ אשר להם התכליות בלתי הפעולות יותר טובים הם הפעולים מהפעולות . תכליות רבים לרבים מהפעולות והנמצאות והמלאכות והלמודים . הרפואה תכליתה הבריאות . עשית הספינה הליכתה בים . תכלית הנהגת צבא המלחמה המלחמה הנצחון׳ תכלית הנהגת הבית העושר׳ רבים מאלה תחת מעלה אחת כמו עשית הרסנים שהיא תחת רכיבת הסוסים . וכל מלאכות שהם הכנות הכלים לרכיבת הסוסים . וכל אלה המעשים המכונים לדבר מלחמה הם נכנסים תחת הנהגת הצבא ובזה יסודרו אלה תחת אלה . בכל הדברים תכליות המלאכות העליונות הם נכספים מאשר תחתיהם כי הם דרושים בעבור העליונים . אמנם אין הפרש בין שיהיו הפעולות תכלי׳ המעשים או זולתם דבר אחר . ר״ל הפעולי׳ כמו בלמודים הנזכרים . הנה אם יש שם תכלית אחד לנפעלים אשר בעבורו אנו רוצים אותו

כי הקודמים לו ומשמשים לו שימום העבד לרב : נכספים . הפעולים ומחויים וכוספי׳ להסיגם לכן הם נכספי׳ : ומזה לין הפרש . לין הפרש בין התכלי׳ הראשי׳ הנכסף אחר תכליות הקודמים לו והוא המכוסף בשם תכלי׳ הפעול . לבין התכלי׳ אשר מחתיו שהוא מהתכליות הקודמים והוא המכוסה בזה בשם תכלי׳ פעולה : הנה אם ים שם תכלי׳ לאחר . לאחר שניאר נקודם שכל הדברים הם בעלי תכליות לם לאחד לו רבים מתחומים ועליונים . דאל לומר חולי יאמר מי שאין ניכלתנו להסיג חוסר הדבר והכליתו העליון כי עליון לעליונים וכל מה שנדמהו לזה העליון האחרון לינו אומר כי עוד עליונים לחדיו ואומנו לא נדע . על זה הוא כוונ׳ לומר שנשתדל להסיג הטוב הנקדם הוא יאות על יכולת השנתנו אותה כ״א יהיו התכלי׳ היה נאות על יכולת השנתנו אותה כ״א יהיו התכלי׳ בעלי בלי תכלי׳ תהי׳ התשוקה לריק כי אין ניכלתנו להסיג הנע״ת . והטבע לא יפעל דבר לנטלה . כאן הבן שומל . הן ידענו שאת שהסי״ת הוא עלם נע״ת

Figure 2.1 Aristotle. *Sefer ha-Middot* (Nichomachean Ethics, in the translation from the Latin by Meir Alguadez [c. 1400]).

The Talmudic book *Baba Qama* analyzes the problem of damages and injuries, explicitly or implicitly caused to a person or his property. The first passages of the Mishnah of this book show the highest genera of Baba Qama's syllogistic tree:

> ארבעה אבות נזיקין השור והבור והמבעה וההבער לא הרי השור כהרי המבעה ולא הרי המבעה כהרי השור ולא זה וזה שיש בהן רוח חיים כהרי האש שאין בו רוח חיים ולא זה וזה שדרכן לילך ולהזיק כהרי הבור שאין דרכו לילך ולהזיק הצד השוה שבהן שדרכן להזיק ושמירתן עליך וכשהזיק חב המזיק לשלם תשלומי נזק במיטב הארץ
>
> The principal categories of damage are four: the ox, the pit, the 'spoliator' [*mab'eh*] and the fire. The aspects of the ox are [in some respects] not [of such low order of gravity] as those of the 'spoliator;' nor are [in other respects] those of the 'spoliator' [of such low order of gravity] as those of the ox; nor are the aspects of either of them, in which there is life, [of such low order of gravity] as those of the fire which is not endowed with life; nor are the aspects of any of these, the habit of which is to be mobile and do damage, [of such low orders of gravity] as those of the pit of which it is not the habit to move about and do damage. The feature common to them all is that they are in the habit of doing damage; and that they have to be under your control so that whenever any one [of them] does damage the offender is liable to indemnify with the best of his estate (*Baba Qama* 2a).

In total this book regards the four highest categories of damages: *šor* (the ox, שור), *bor* (the pit, בור), *mav'eh* (the spoliator, מבעה) and *heb'er* (the fire, הבער). Now we are trying to formalize the basic reasoning in respect to damages within the limits of the special version of formal syllogistics. This system will be called *Baba Qama's syllogistics* (or BK-syllogistics). As we will see, it explicitly differs from the Aristotelian one. I called genera-species relations syllogistic ones, because they were first considered from the logical point of view in Aristotle's syllogistics and in traditional logic they are called syllogistic. In the Talmud genera-species relations are supposed in *klalim u-praṭim*.

BK-syllogistics, as well as other syllogistical systems, is based on propositional logic.

Definition 3.1 *The alphabet of propositional logic is an ordered system* $\mathsf{A} = \langle V, L_1, L_2, K \rangle$*, where*

1. V is a set of propositional variables p, q, r, …;

2. L_1 is a set of unary propositional connectives consisting of the only element $\neg$ called the symbol of negation;

3. L_2 is a set of binary propositional connectives consisting of three elements: $\wedge$, $\vee$, $\Rightarrow$ called the symbols of conjunction, disjunction, and implication respectively;

4. K is the set of auxiliary symbols containing brackets: (,).

Definition 3.2 *The language of propositional logic is an ordered system* $\mathsf{L} = \langle \mathsf{A}, \mathsf{F} \rangle$*, where*

1. A is the alphabet of propositional logic;

2. F is a set of all formulas that are formed by means of symbols in A.

Notice that elements of F are defined by induction:

(a) every propositional variable p, q, r, ... is a formula of propositional logic;

(b) if φ, ψ are formulas, then expressions $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, $\varphi \Rightarrow \psi$ are formulas of propositional logic;

(c) a finite sequence of symbols is called a formula of propositional logic if that sequence satisfies two above mentioned conditions.

Definition 3.3 *The propositional logic (or propositional calculus) is the ordered system* $\mathsf{S} = \langle \mathsf{A}, \mathsf{F}, \mathsf{C} \rangle$*, where*

1. A is the alphabet of propositional logic;

2. F is the set of all formulas formed by means of symbols in A;

3. C is an inference operation that is the map of formulas in $\mathsf{F}_0 \subseteq \mathsf{F}$ to formulas in $\mathsf{C}(\mathsf{F}_0)$, i.e., to the set of all corollaries from F_0.

The inference rules of propositional logic are as follows:

1. *the substitution rule*, according to that we replace a propositional variable p_j of a formula $\alpha(p_1,\ldots,p_n)$, containing propositional variables p_1, ..., p_n, by a formula $\beta(q_1,\ldots,q_k)$, containing propositional variables $q_1,\ldots,q_k$, and this entails a new formula $\alpha'(p_1,\ldots,p_{j-1}, \beta(q_1,\ldots, q_k), p_{j+1},\ldots,p_n)$:

$$\frac{\alpha(p_1,\ldots,p_j,\ldots,p_n)}{\alpha'(p_1,\ldots,p_{j-1},\beta(q_1,\ldots,q_k),p_{j+1},\ldots,p_n)};$$

2. *modus ponens*, according to that if two formulas α and $\alpha \Rightarrow \beta$ hold, then we deduce a formula β:

$$\frac{\alpha, \alpha \Rightarrow \beta}{\beta}.$$

We use the set of axioms of *Łukasiewicz's propositional calculus* S as the input set of $\mathsf{C}(\varnothing)$ (see *Łukasiewicz 1920*):

$$(p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow r)), \tag{1}$$
$$(\neg p \Rightarrow p) \Rightarrow p, \tag{2}$$
$$p \Rightarrow (\neg p \Rightarrow q). \tag{3}$$

The implication and negation are given here as basic operations. Other operations are derivable, e.g. the conjunction, disjunction and equivalence are defined as follows:

$$p \wedge q ::= \neg(p \Rightarrow \neg q), \tag{4}$$
$$p \vee q ::= \neg p \Rightarrow q, \tag{5}$$
$$p \Leftrightarrow q ::= (p \Rightarrow q) \wedge (q \Rightarrow p). \tag{6}$$

Combining axioms (1)—(3) and using inference rules, we obtain all other tautologies of the set $\mathsf{C}(\varnothing)$ for S.

BK-syllogistics is an extension of propositional logic.

Definition 3.4 *The alphabet of BK-syllogistics is an ordered system* $\mathsf{A}_{BK} = \langle V, Q_1, Q_2, Q, L_1, L_2, L_3, K\rangle$, *where*

1. V is the set of propositional variables p, q, r, ...;

2. Q_1 is a set of first-order BK-syllogistic constants k, sh, r, b, m_1, m_2, h_1, h_2;

3. Q_2 is a set of second-order BK-syllogistic constants Sh, Sh_1, Sh_2, B, M, H;

4. Q is a set of BK-syllogistic variables P, Q, R;

5. $L_1 = \{\neg\}$;

6. $L_2 = \{\wedge, \vee, \Rightarrow\}$;

7. L_3 is a set of binary BK-syllogistic connectives containing three elements $=_p$, $=_d$, $=_c$ called the functors "… is because of …," "… is a damage of the kind …," "… belongs to the category of …"

8. K is the set of auxiliary symbols containing two brackets: (,).

Let us consider the informal meaning of BK-syllogistic constants:

- Sh—*šor* (the ox) designates any domestic animal. It is known that the domestic animal can butt, bite, lie down on a grass and crush a vessel standing at this place, can be scratched by a fence and break it down or brush of a think by the tail. The owner of such an animal bears liability for a damage to a property of another person.

- Sh_1—The *tam*-ox designates any domestic animal that has never butted earlier, it has pushed nobody, has not bitten, has not kicked. Such an ox did not cause damage. Nevertheless if it spoilt something, the owner pays half of cost of the spoilt thing. For example, if such an ox kicked and broke something, its owner first compensates only half of damage. In practice it is fulfilled as follows: an ox is sold; if the money obtained covers half of cost or more, the owner of ox pays the appointed sum; if for the ox its owner could obtain less than half of cost of damage, the owner of ox is not obliged to add from his property. This rule is learned from the phrase: "And if one

man's ox hurt another's, that he die; then they shall sell the live ox, and divide the money of it" (*Ex.* 21:35). For example, the ox that costs 20 coins, butted the second ox and the latter died/suffered. Let us assume that the dead/suffered ox costs 100 coins. Hence, the owner of the first ox is obliged to pay 50 coins for the claimant. Taking into account that his ox costs just 20 coins, in law the claimant will not receive more than 20 coins.

- Sh_2—The *mu'ad*-ox designates any domestic animal that caused a damage not less than two times. Its owner pays full cost of damage. The *mu'ad* status of the ox is established on the basis of six eyewitnesses (three pairs)—each pair of eyewitnesses should confirm an appropriate case of the given damage. So, if the first two witnesses told about how the ox had hurt for the first time, the others two about another similar case, and the third two about the third one, and it is found out that one of the three pairs of eyewitnesses had given the false testimony, all testimonies are considered to void. Whereby in law all six witnesses are regarded as one group. As we have already said, for the *mu'ad*-ox its owner has to pay the complete cost of damage. This payment should be made from the best part of respondent's property (from his best stead). In the Torah we can read about the respondent for the *mu'ad*-ox: "he shall surely pay ox for ox" (*Ex.* 21:36). Also there are no here instructions that the damage is compensated at the rate of costs of the ox that harmed. About money it is not written, too. Such distinction between the *tam*-ox and the *mu'ad*-ox is based on the following verse of the Torah: "And if one man's ox hurt another's, that he die; then they shall sell the live ox, and divide the money of it; and the dead ox also they shall divide. Or if it be known that the ox hath used to push in time past, and his owner hath not kept him in; he shall

surely pay ox for ox; and the dead shall be his own" (*Ex.* 21:35—36).

- k—*qeren* (the horn) designates a damage caused by an aggressive behaviour of domestic animal (for example, a damage caused by ox's push).
- sh—*šen* (the tooth) designates a damage caused by animal's action when it did something for the sake of own pleasure (i.e. it is a damage caused when a domestic animal eats something or does another harm, satisfying needs, but without aggression).
- r—*regel* (the leg) designates a casual action of domestic animal (a damage caused to somebody's property without any ox's intention, without its aggression and without ox's desire to satisfy own needs). For example, while the ox is going along the road, it touches someone's bucket with milk. The bucket falls, milk spreads.
- B, b—*bor* (the pit) designates any obstacle or barrier at a public place (in public possession), i.e. a think which caused a damage (for example, a person left in the middle of road any cargo at that place where it is not accepted to do it). If a person lays down a jug at the place where one usually lays down vessels, and the passersby stumbles and breaks a jug, then the passersby has to compensate the vessel cost. If he was injured thereby, the owner of the jug is free from liability. However, if it happened to him at dark or vessels partitioned off the road, then the passersby is free from responsibility, but if he was wounded, the owner of jug compensates the whole damage. Actually a pit, capable to cause a damage, is defined as a lacuna that is not less than 10 *ṭfaḥim* (*ṭefaḥ* is a linear measure approximately corresponding to 8cm). A person is liable for a pit dug by him not only at a public place, but also at the own (private) one in case it is possible to get this from the public territory. If a person had dug a pit in 9 *ṭfaḥim*,

and another person then came and deepened this pit up to one *ṭefaḥ*, for a damage if the animal falls into this pit and goes under or it is hurt, the second person bears an appropriate responsibility. If a person found out a pit in public possession and covered it, and then again opened it, the damage is paid by someone who had dug a pit, and the second man is free from responsibility, because he returned a pit to its initial state. But if a person had backed fill the pit found out in public possession by earth, and again dug, he compensates the complete damage because of the pit. It is explained as follows. The pit dug by another man was backed fill by him, that is, liquidated. Later he dug (created) another pit for which, actually, he responds.

- M—*mav'eh* (the spoliator) designates any damage which is directly caused by a person. It is affirmed in the Talmud: *'adam mu'ad le'olam* (the person is always warned). That is, the person is always *mu'ad* and hence he bears responsibility for all his acts. Therefore it is unimportant, how he acted wrongdoing either by mistake or intentionally, being awake or in sleep, being sober-minded or drunk wines. The person always is responsible for his act. One of the major morally-ethical Torah's principles states: *'ein šaliaḥ ledvar 'averah* (nobody can be an envoy for the fulfilment of malicious act). On the other hand, the gender difference also does not belittle fault. We could cite the Torah: "And the Lord spake unto Moses, saying, Speak unto the children of Israel, When a man or woman shall commit any sin that men commit, to do a trespass against the Lord, and that person be guilty" (*Num.* 5:5—6). It is a question of laws of returning thinks that were stolen. From here we learn that the Lord equalizes the man and the woman in legal liability questions. We may

notice that payments of guilty person is not only the punishment, but also *kaparah* (the redemption).

- m_1—An assignment of somebody's property without his agreement. The thief pays a basis of indemnification (200% of the stolen) according to cost of the stolen think at the moment of theft. However, if the thief stole an animal, and then he slaughtered and sold it, he pays the extra penalty: in total the quadruple payment for a sheep (200%+200%) and the fivefold payment for an ox (200%+300%). Those extra penalties (200% for a sheep and 300% for an ox) are defined on the basis of costs of the stolen property at the moment of trial court. If the thief admitted that he had stolen, then killed or sold an animal, he is free from the payment of extra penalty. He has to pay the basic cost (200%) of the stolen animal. Even if then there will be eyewitnesses of slaughtering. If the thief does not admit his larceny, his fault is established on the basis of eyewitnesses. Let us suppose, the two witnesses bore testimony that someone had stolen an ox or a sheep. The other two bore witness of that the thief had killed or sold an animal. And it is soon found out that testimonies of those and others are false. The first pair pays double indemnification (the same what the thief pays for any theft). The second pair pays threefold for an ox or double for a sheep. If it is known that only the second pair of witnesses committed perjury, the thief pays a basis of indemnification for an animal (200%), and perjurers threefold for an ox and double for a sheep. If one witness of the second pair told lies, testimonies of both have to be ignored. The thief in that case pays just the double indemnification for the stolen animal. But if one witness of the first pair slandered, all testimonies are liquidated. The theft moment is called *qinyan* (the thing acquisition). It is the moment, when a person is receiving the status of the

thief. It takes place in case the thief at least raised an animal. Even if it occurred within a possession of the owner of that animal. Therefore the thief pays for it the double size as for any theft. In the same way, *qinyan* is observed when the thief has time to lead an animal on a leash out of the gate. The thief, slaughtering an animal on Sabbath, is free from quadruple or fivefold payment. It is connected to that for the infringement of Sabbath's laws he needs stricter penalty. Therefore he pays only for theft, i.e. the double cost of a domestic animal. If a man stole an ox or a sheep and killed an animal to use it for idolatry, he pays only for theft in the same measure. For idolatry he responds under the harder laws. If a person stole an ox or a sheep of his father, and then killed or sold an animal, he pays only for theft (doubly) and does not pay the penalty, because he is one of the inheritors. If a man stole an ox or a sheep and after the owner of an animal had despaired to return the lost, he declared that an animal is *heqdeš* (it is devoted to the Temple), and then killed or sold it, he pays only the double price and is free from penalty payment.

- m_2—A violence of a person against another. If two persons simultaneously did traumas to each other and caused mutual injuries, the one, who did a more significant harm to health, completely pays a difference between the sums what their damages cost. In case someone first had begun fight, the instigator pays, and the second is free from responsibility. The second was entitled to self-defence. The one, who hit somebody, is obliged to pay the indemnification whose sum is constituted by payments according to five aspects: for damage, pain, healing, loss of time and moral loss (degradation). There are the laws causing calculations on each of these aspects. If, let us assume, one's eye was beaten out or a foot was broken, then his efficiency before incident and after incident is

estimated. The difference between the sums is to be payed as indemnification. The sum for a pain is calculated under special tables. The respondent has to pay all expenses of the victim for medicines and visits of the latter to doctors. If as a result of wound there was a sequelae, the respondent continues to pay expenses on healing. For all period when the victim cannot work, the respondent pays him the sum equal to the usual salary. For degradation the court calculates an appropriate sum, estimating respects and merits of guilty person and sufferer. If a person aggrieved somebody's slave (non-Jew), he has to pay all five kinds of indemnification. If a person aggrieved his slave (non-Jew), he pays only for his healing, because the slave property belongs to his owner. However, in case the slave is released, the owner pays him all kinds of indemnification.

- H—*heb'er* (the fire) designates a damage caused by a man who is guilty of the fire. If a person kindled fire on his field and the flame spread out onto neighbour's field, the first person is obliged to compensate a damage, to pay, for example, an all-round price for the burnt down haycock, but he does not have to pay for a thing that was hidden in it (even if this thing is usually covered in a haycock). However, if a person kindled fire on neighbour's field, or the fencing separating one field from another was fallen because of the arisen fire, the originator of fire pays cost not only for haycocks, but also for things which were there in case those things are usually covered in haycocks. If a house was burnt down, the originator of fire pays for each thing which was burnt down in the house.
- h_1—'The flame as arrow' designates such fire which is caused by kindling without necessary precaution. In this case the visible and invisible property of the neighbour is paid. For example, the originator of fire pays both for a

haycock, and for all things covered in it. If as a result of fire another person was hurt, it is also the case of h_1. Here the originator compensates costs of everything that was burnt down, and also pays indemnifications for the caused pain and damage, healing, loss of time and moral loss (degradation).

- h_2—'The flame as property' designates such fire which is caused by kindling with fulfilling all rules of precaution. Here a person compensates only "visible" loss, i.e. in this case he pays only for a haycock.

Definition 3.5 *The language of BK-syllogistics is an ordered system* $\mathsf{L}_{BK} = \langle \mathsf{A}_{BK}, \mathsf{F}_{BK} \rangle$, *where*

1. A_{BK} is the alphabet of BK-syllogistics;
2. F_{BK} is a set of all formulas formed by means of symbols in A_{BK}; this set F_{BK} contains all formulas defined by the rules (a), (b), and (c) of definition 3.2 (i.e. $\mathsf{F} \subset \mathsf{F}_{BK}$) and additionally by the following rules:

(d) if φ, ψ are propositional formulas (i.e. φ, $\psi \in \mathsf{F}$), then the expression $\varphi =_p \psi$ is a formula of BK-syllogistics and if Q is a BK-syllogistic variable and φ is a propositional formula, then the expression $\varphi =_p Q$ is a formula of BK-syllogistics (these formulas called zero-order BK-formulas, the set of such formulas is denoted by T_0);

(e) if φ is a propositional formula and $\mathrm{o} \in \{\mathrm{k}, \mathrm{sh}, \mathrm{r}, \mathrm{b}, \mathrm{m}_1, \mathrm{m}_2, \mathrm{h}_1, \mathrm{h}_2\}$, then the expression $\varphi =_d \mathrm{o}$ is a formula of BK-syllogistics (it is called a first-order BK-formula, the set of such formulas is denoted by T_1);

(f) if φ is a first-order BK-formula (i.e. $\varphi \in \mathsf{T}_1$) and $\mathrm{O} \in \{\mathrm{Sh}, \mathrm{Sh}_1, \mathrm{Sh}_2, \mathrm{B}, \mathrm{M}, \mathrm{H}\}$, then the expression $\varphi =_c \mathrm{O}$ is a formula of BK-syllogistics (it is called a second-order BK-formula, the set of such formulas is denoted by T_2).

Formulas that are defined by rules (d), (e) and (f) of definition 3.5 are called *formulas of BK-syllogistics in the strict sense*. They are denoted by T, i.e. $\mathsf{T} = \mathsf{T}_0 \cup \mathsf{T}_1 \cup \mathsf{T}_2$.

Definition 3.6 *BK-syllogistics is an ordered system* $\mathsf{S}_{BK} = \langle \mathsf{A}_{BK}, \mathsf{F}_{BK}, \mathsf{C} \rangle$, *where*

1. A_{BK} is the alphabet of BK-syllogistics;
2. $\mathsf{F}_{BK} = \mathsf{F} \cup \mathsf{T}$;
3. C is the inference operation in F_{BK}.

The inference rules of BK-syllogistics are as follows:

1. *the substitution rule*, we replace a propositional variable p_j of formula $\alpha(p_1,\ldots,p_n)$, containing propositional variables p_1, ..., p_n, by a formula $\beta(q_1,\ldots,q_k)$, containing propositional variables $q_1,\ldots,q_k$ (as well as by a zero/first/second-order BK-syllogistic formula $\beta(\sigma,\tau)$, containing BK-syllogistic variables or BK-syllogistic constants), and we obtain a new propositional formula $\alpha'(p_1,\ldots,p_{j-1},\ \beta(q_1,\ldots,q_k),p_{j+1},\ldots,\ p_n)$ (as well as a new BK-syllogistic formula $\alpha'(p_1,\ \ldots,p_{j-1},\beta(\sigma,\tau),p_{j+1},\ \ldots,p_n)$):

$$\frac{\alpha(p_1,\ldots,p_j,\ldots,p_n)}{\alpha'(p_1,\ldots,p_{j-1},\beta(q_1,\ldots,q_k),p_{j+1},\ldots,p_n)}$$

or

$$\frac{\alpha(p_1,\ldots,p_j,\ldots,p_n)}{\alpha'(p_1,\ldots,p_{j-1},\beta(\sigma,\tau),p_{j+1},\ldots,p_n)},$$

In the same way, from any zero-order BK-syllogistic formula $\alpha(p_j,P_i)$ a new formula $\alpha'(\beta(q_1,\ldots,q_k),P_i)$ or $\alpha'(p_j,P_l)$ follows if we replace a propositional variable p_j by a formula $\beta(q_1,\ldots,q_k)$, containing propositional variables $q_1,\ldots,q_k$, or a BK-syllogistic variable P_i by P_l:

$$\frac{\alpha(p_j,P_i)}{\alpha'(\beta(q_1,\ldots,q_k),P_i)} \qquad \frac{\alpha(p_j,P_i)}{\alpha'(p_j,P_l)}$$

Further, from any first/second-order BK-syllogistic formula $\alpha(p_j,\tau)$ a new formula $\alpha'(\beta(q_1,\ldots,q_k),\tau)$ follows if we replace a propositional variable p_j by a formula $\beta(q_1,\ldots,q_k)$, containing propositional variables $q_1,\ldots,q_k$:

$$\frac{\alpha(p_j,\tau)}{\alpha'(\beta(q_1,\ldots,q_k),\tau)}$$

2. *modus ponens*, according to that if two formulas of BK-syllogistics α and $\alpha \Rightarrow \beta$ hold, then we deduce a formula β:

$$\frac{\alpha,\alpha \Rightarrow \beta}{\beta}.$$

The axioms of BK-syllogistics consist of axioms of propositional logic (e.g., axioms (1), (2), (3) of the propositional system S), and of the following expressions:

$$p =_d \mathrm{k} \Rightarrow (p =_d \mathrm{k}) =_c \mathrm{Sh}, \tag{7}$$
$$p =_d \mathrm{sh} \Rightarrow (p =_d \mathrm{sh}) =_c \mathrm{Sh}_2, \tag{8}$$
$$p =_d \mathrm{r} \Rightarrow (p =_d \mathrm{r}) =_c \mathrm{Sh}_2, \tag{9}$$
$$(p =_d \mathrm{o}) =_c \mathrm{Sh}_2 \Rightarrow (p =_d \mathrm{o}) =_c \mathrm{Sh}, \tag{10}$$

where $\mathrm{o} \in \{\mathrm{sh},\mathrm{r}\}$,

$$(p =_d \mathrm{k}) =_c \mathrm{Sh} \Leftrightarrow ((p =_d \mathrm{k}) =_c \mathrm{Sh}_1 \vee (p =_d \mathrm{k}) =_c \mathrm{Sh}_2), \tag{11}$$
$$p =_d \mathrm{b} \Rightarrow (p =_d \mathrm{b}) =_c \mathrm{B}, \tag{12}$$
$$p =_d \mathrm{m}_1 \Rightarrow (p =_d \mathrm{m}_1) =_c \mathrm{M}, \tag{13}$$
$$p =_d \mathrm{m}_2 \Rightarrow (p =_d \mathrm{m}_2) =_c \mathrm{M}, \tag{14}$$

$$p =_d \mathrm{h}_1 \Rightarrow (p =_d \mathrm{h}_1) =_c \mathrm{H}, \tag{15}$$

$$p =_d \mathrm{h}_2 \Rightarrow (p =_d \mathrm{h}_2) =_c \mathrm{H}, \tag{16}$$

$$p =_d \mathrm{k} \vee p =_d \mathrm{sh} \vee p =_d \mathrm{r} \vee p =_d \mathrm{b} \vee$$
$$\vee p =_d \mathrm{m}_1 \vee p =_d \mathrm{m}_2 \vee p =_d \mathrm{h}_1 \vee p =_d \mathrm{h}_2, \tag{17}$$

$$q =_p q, \tag{18}$$

$$(q =_p r \wedge r =_p q) \Rightarrow q \Leftrightarrow r, \tag{19}$$

$$(q =_p r \wedge r =_p p) \Rightarrow q =_p p, \tag{20}$$

$$q =_p Q, \tag{21}$$

BK-syllogistic formulas in the strict sense are interpreted in the following models.

Definition 3.7 *A structure*

$\mathsf{M} = \langle O = \{\dot{\mathrm{O}} : \mathrm{O} \in \{\mathrm{Sh}, \mathrm{Sh}_1, \mathrm{Sh}_2, \mathrm{B}, \mathrm{M}, \mathrm{H}\}\}, o = \{\dot{\mathrm{o}} : \mathrm{o} \in \{\mathrm{k}, \mathrm{sh}, \mathrm{r}, \mathrm{b}, \mathrm{m}_1, \mathrm{m}_2, \mathrm{h}_1, \mathrm{h}_2\}\}, I, \doteq_p, \doteq_d, \doteq_c \rangle$ *is a BK-syllogistic model iff:*

1. O and o are sets of objects.

2. I such that $I : \mathsf{T}_1 \to 2^{\mathsf{F} \times o}$ and $I : \mathsf{T}_2 \to 2^{\mathsf{T}_1 \times O}$ associates a set of objects with each first-order and second-order formula of T, so that $I(\alpha =_d \mathrm{o}) = \{x \in \mathsf{F} \times o :$ there is the class of equivalences $[\alpha] \in \mathsf{F}/\Leftrightarrow$, there is the object $\dot{\mathrm{o}}$ such that $[\alpha] \doteq_d \dot{\mathrm{o}}\}$, where $\alpha \in \mathsf{F}$, and $I((\alpha =_d \mathrm{o}) =_c \mathrm{O}) = \{x \in \mathsf{T}_1 \times O :$ there are $I(\alpha =_d \mathrm{o})$ and $\dot{\mathrm{O}}$ such that $I(\alpha =_d \mathrm{o}) \doteq_c \dot{\mathrm{O}}\}$.

3. $\doteq_p$ is a partial ordering relation on F such that $[\alpha] \doteq_p [\beta]$ is read as "$[\alpha]$ is less than $[\beta]$," where $[\alpha]$, $[\beta] \in \mathsf{F}/\Leftrightarrow$, and each member of $\mathsf{F}/\Leftrightarrow$ is less than each member of T_0.

4. $\doteq_d$ is a relation on $(\mathsf{F}/\Leftrightarrow) \times o$ satisfying the condition (17).

5. $\doteq_c$ is a relation on $(\mathsf{T}_1/\Leftrightarrow) \times O$ satisfying the conditions (7)—(16).

We now give the truth conditions of Boolean combinations of BK-syllogistic formulas in a BK-syllogistic model:

Definition 3.8

$\mathsf{M} \models \neg\phi$ iff $\mathsf{M} \not\models \phi$

$\mathsf{M} \models \phi \wedge \psi$ iff $\mathsf{M} \models \phi$ and $\mathsf{M} \models \psi$

$\mathsf{M} \models \phi \vee \psi$ iff $\mathsf{M} \models \phi$ or $\mathsf{M} \models \psi$

$\mathsf{M} \models \phi \Rightarrow \psi$ iff $\mathsf{M} \models \neg\phi$ or $\mathsf{M} \models \psi$

An extension of BK-syllogisctics is used for legislative sentences. It is obtained by adding the following inference rules:

$$\frac{\varphi =_p Q \text{ and } (\varphi =_d \text{k}) =_c \text{Sh}_1 \text{ at any place}}{Q \text{ should pay } 50\%} \quad (22)$$

$$\frac{\varphi =_p Q \text{ and } (\varphi =_d \text{k}) =_c \text{Sh}_2 \text{ at any place}}{Q \text{ should pay } 100\%} \quad (23)$$

Formulas (22), (23) mean that for a *tam*-ox it is necessary to pay 50% if it made a damage *qeren* at any place, and for a *mu'ad*-ox 100% if it acts the same damage at any place.

$$\frac{\varphi =_p Q \text{ and } (\varphi =_d \text{sh}) =_c \text{Sh}_2 \text{ at a private place}}{Q \text{ should pay } 100\%} \quad (24)$$

$$\frac{\varphi =_p Q \text{ and } (\varphi =_d \text{r}) =_c \text{Sh}_2 \text{ at a private place}}{Q \text{ should pay } 100\%} \quad (25)$$

Formulas (24), (25) mean that for a damage *šen* and *regel* it is necessary to pay 100% if it was at a private place. Nothing is paid for the same damage in public possession.

$$\frac{\varphi =_p \psi \text{ and } \neg(\psi =_p \varphi) \text{ and } \psi =_p Q \text{ and } (\varphi =_d \text{r}) =_c \text{Sh}_2 \text{ at a private place}}{Q \text{ should pay } 50\%} \quad (26)$$

Let us comment on the last rule more in detail. According to that if a phenomenon (ψ), whose occurrence had been caused by somebody's animal, caused another phenomenon (φ), but the animal is not responsible for φ directly, the owner of this animal pays only half. For example, if on the ground area of the neighbour the animal crushed a vessel, and this vessel fell to the second that as a result was broken too, the owner of animal pays for the first full cost, and for the second half. For example, cocks and other birds are capable to push off something or to peck up. If the cock has been roped and in movement by paws or by the same cord it touched a vessel and broke it, the owner of cock compensates half of damage. If the cock damaged a vessel by cord, when it was freely walking, and the cord simply was dangling on its paw, the owner of cock pays full cost of damage. The cord is considered here as a part of bird's body.

$$\frac{\varphi =_p Q \text{ and } (\varphi =_d \text{ b}) =_c \text{ B at a public place (on a road)}}{Q \text{ should pay } 100\%} \qquad (27)$$

For a damage *bor* it is necessary to pay 100% if the obstacle was left at an unexpected place about which the passersby cannot know.

$$\frac{\varphi =_p Q \text{ and } (\varphi =_d \text{ m}_1) =_c \text{ M at any place}}{\begin{array}{c} Q \text{ should pay } 200\% \text{ (or } 400\% \text{ for the sheep and} \\ 500\% \text{ for the ox if they were slaughtered)} \end{array}} \qquad (28)$$

For larceny it is necessary to pay 200% from costs of the stolen. In case the sheep was stolen, and then was killed, 400% from its cost is paid and if the ox was stolen and then killed, 500% from its cost is paid.

$$\frac{\varphi =_p Q \text{ and } (\varphi =_d \text{ m}_2) =_c \text{ M at any place}}{\begin{array}{c} Q \text{ should pay } 100\% \text{ of damage plus the penalty} \\ \text{for pain, healing, loss of time and degradation} \end{array}} \qquad (29)$$

If a person caused somebody's mutilation, not only the mutilation is paid, but also pain, healing, loss of time and degradation.

$$\frac{\varphi =_p Q \text{ and } (\varphi =_d h_1) =_c \text{H at a private place}}{Q \text{ should pay 100\% for any damages}} \tag{30}$$

$$\frac{\varphi =_p Q \text{ and } (\varphi =_d h_2) =_c \text{H at a private place}}{Q \text{ should pay 100\% for visible damages}} \tag{31}$$

Formulas (30), (31) mean that for a damage caused by 'flame as arrow,' 100% is paid and for a damage caused by 'flame as property,' we should pay 100% only of visible damages (for example, the haycock is paid except for things it covered).

As we see, reasoning in the *Baba Qama* should be evaluated as absolutely logical and consistent that allows us to formalize it as non-Aristotelian syllogistics, where instead of the law of the excluded middle there is the law of the excluded eighth. This new syllogistics sets up a specific syllogistic tree that became a result of studying the Torah in respect to all questions connected to damages. This syllogistic tree cannot be born in our thought. It went down as a part of the Revelation. Hence, it contains analytic a posteriori knowledge, not analytic a priori.

The reader might focus on the circumstance that only assumptions (data) are in the Torah and all further reasoning are human and then he might wonder if there is a difference between Talmudic logic and general logic. I agree with this opinion that only initial data of Talmudic reasoning occur in the Torah. However, it causes a different version of syllogistics. For instance, in the Torah there are only four genera of damages: ox, pit, spoliator, fire. Why just four, why only them, etc? Is there a culture where the same items are? Is there another jurisprudence where the same items are? No. Nobody without studying the Torah can invent these genera and their species. For instance, in recent jurisprudence we can find

out the following genera of damages: a) damage which happens in *objective sense* (it includes negative consequences which are described in the legal text) and in *subjective sense* (it includes negative consequences which were at own discretion considered); b) *physical damage* (damage for health of the person, e.g. mental damage) and *moral damage* (belittling of honour, advantage, business reputation, or personal non-property rights); c) *real damage* (it consists of expenses which the suffered person should pay for elimination of the caused property losses) and *nominal damage* (symbolic costs, when there were not real damage, e.g. kinds of trespass), etc.

As we see, modern jurisprudence accepts standard dichotomies of the Porphyrian tree: subjective and objective; physical and mental; real and nominal, etc. These dechotomies are contained in our thought, they are evident. The Torah typology is not evident; in order to see it we should study the Torah, e.g. nobody can understand what ox or pit is without studying the Torah. Thus, the first premises (genera) of Talmudic logic are contained in the Torah, but at the same time it implies another syllogistics that is not evident without studying the Torah.

4. Non-Aristotelian squares of opposition

Since Aristotle the best known logical pattern has been presented by square of opposition first aimed for setting up basic laws of syllogistics. These laws generate foundations of analytic a priori knowledge. Nevertheless, since Immanuel Kant and Heinrich Rickert many philosophers have paid attention that this square does not satisfy synthetic propositions (a priori and a posteriori as well).

Judaic (simple or complete) conjunctive logic (see definitions 4.1—4.2 of the previous chapter) assumes that we are able to deal with two kinds of semantic relations: analytic a priori (between the general, i.e. genus, and the partucular, i.e. species—H5, I4, I5, I6) and synthetic a priori (between extension and restriction—E1, E2, E3, E4). In Judaism it is prohibited to draw syllogistic conclusions

from genera to species, e.g. to build up deductions of Aristotelian syllogistics (i.e. to obtain analytic a priori syllogistic propositions). However, we should bear in mind why and how genera include species. For instance, we have to know that 'offering of the cattle' is more general than 'offering of the herd, and of the flock' (*Lev*. 1:2). The inclusion relation between genera (*kelal*) and species (*frat*) is studied in Aristotelian syllogisctics and its basic definition is given by the classical square of opposition, e.g. see Fig 4.1.

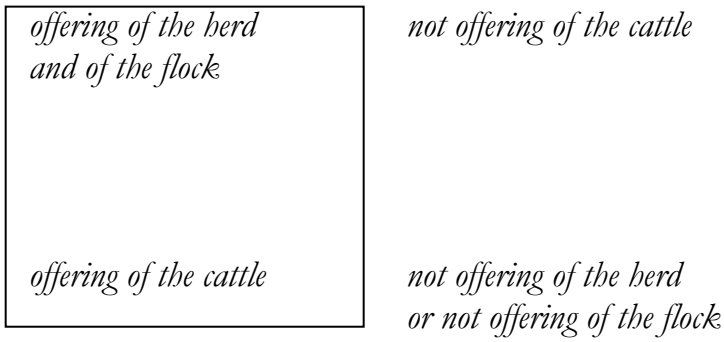

offering of the herd
and of the flock
not offering of the cattle
offering of the cattle
not offering of the herd
or not offering of the flock

Figure 4.1 Aristotle's square of opposition for the general and the particular from the verse of *Lev*. 1:2.

Nevertheless, the topic of synthetic a priori relations such as relations between extension (*ribuy*) and restriction (*mi'ut*) is out of Aristotelian syllogistics. Indeed, we cannot obtain Aristotle's square of opposition for the extension and the restriction. For example, let us consider the restriction 'bestow that money for sheep' and the extension 'bestow for whatsoever thy soul desireth' (*Deut*. 14:26). Then we cannot plot the classical square, because 'bestow that money for sheep' is not included into 'bestow for whatsoever thy soul desireth,' e.g. perhaps my soul does not desire sheep, but desire many other things.

We may prove that there are only two squares of opposition (one is for analytic a priori relations and the second is for synthetic

a priori relations) under following assumptions:

• we have the Boolean complement/negation (it satisfies definition 1.1);

• synthetic propositions cannot be reduced to manipulations with Venn diagrams, because they do not suppose including relations.

Aristotle proposed the following four oppositions: contradiction, contrariety, relation, and privation (*Categories*, Chapter 10, *Metaphysics*, Book I) that semantically underlay the square of opposition:

> We must next explain the various senses in which the term 'opposite' is used. Things are said to be opposed in four senses: (i) as correlatives to one another, (ii) as contraries to one another, (iii) as privatives to positives, (iv) as affirmatives to negatives (*Cathegories*, 10).

Aristotle himself described relations of the square of opposition to represent singular expressions (*Prior Analytics*, Chapter 46), see figure 4.2. He had an intuition that quantifiers (both universal and existential ones) satisfy the semantic relations of the square, too:

> An affirmation is opposed to a denial in the sense which I denote by the term 'contradictory,' when, while the subject remains the same, the affirmation is of universal character and the denial is not. The affirmation 'every man is white' is the contradictory of the denial 'not every man is white,' or again, the proposition 'no man is white' is the contradictory of the proposition 'some men are white.' But propositions are opposed as contraries when both the affirmation and the denial are universal, as in the sentences 'every man is white,' 'no man is white,' 'every man is just,' 'no man is just' (*On Interpretation*, 7).

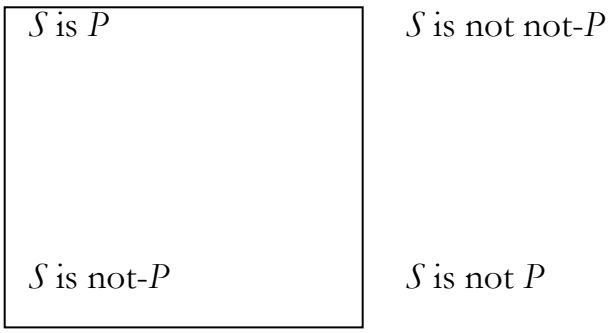


Figure 4.2 Aristotle's square of opposition.

However, for the first time, Apuleius explicitly claimed that quantified propositions satisfy the square. He wrote a short book, the *Peri Hermeneias*, about logic that was used for centuries in teaching. This book is the most famous in the history of logic for including the first appearance of the square of opposition, the best known logical schema for the pedagogic purpose (*Apuleius 1987*). He considered the four oppositions: contrary, subcontrary, contradictory, subalternation. First, he described that the two *incongruae* (contrary) propositions, on the left and right sides of the top of the square, never can be true at the same time and nonetheless are sometimes false at the same time. For example, when some pleasures are good, both universal propositions are false at the same time, since it is impossible that every pleasure is both a good and not a good. The two propositions along the bottom line (i.e. the mirror-image of the contrary) are called *subpares* (subcontrary). They are never false at the same time, but they can be true at the same time. Hence, to confirm that some pleasure is a good we cannot use an argument that some other pleasure is not a good. Further, we pair together the *alterutrae* (contradictory) propositions, if we add a negation to each of the pair of alternates, e.g. not every pleasure is a good means that some pleasure is not a good. Finally, the *subalternation* appears between universal and particular propositions, when the universal implies the particular,

e.g. if every pleasure is a good, then some pleasure is a good.

The meaning of propositions that satisfy the square of opposition can be checked on Venn diagrams. Recall that a Venn diagram is an ellipse that designates an extent of a concept *A*, i.e. a class of all real things that are denoted by *A*. These things are called denotations. By assumption, all inner points of ellipse designate appropriate real things. For instance, 'every man is mortal' is a true proposition, because the Venn diagram of 'man' is included into the Venn diagram of 'the mortal being' (i.e. all denotations of 'man' occur among denotations of 'the mortal being').

Kant first paid attention that there exists a true universal proposition like 'all bodies are heavy' such that Venn diagrams of its subject and predicate do not assume the including relation. So, the Venn diagram of 'body' just intersects the Venn diagram of 'heavy' (*Kant*).

Thus, in Kant's opinion, only analytic judgments satisfy the square of opposition. For synthetic judgments Venn diagrams lose any sense and, as a result, we cannot apply the square for them. In Aristotelian logic there is the inverse relation between content (class of all connotations) and extent (class of all denotations) of a concept. By continuing the Kant's ideas, Heinrich Rickert claimed that *for synthetic judgments (propositions) there is the direct relation between intension and extent of a concept* (please see definitions 2.1—2.2 and compare Rickert's claim with proposition 2.1). Therefore we cannot use Venn diagrams there at all.

Thus, according to Kant and Rickert, there are two logics (the Aristotelian for analytic propositions, where we can use Venn diagrams and the square of opposition, and the non-Aristotelian for synthetic propositions without Venn diagrams manipulations). This distinction entails another distinction (proposed first by Wilhelm Windelband and Heinrich Rickert) between two kinds of sciences: natural sciences (*Naturwissenschaften*) and cultural sciences (*Geisteswissenschaften*). In the first the Aristotelian logic is used by applying a nomothetic approach, in the second the non-Aristotelian

by applying an idiographic approach. The idiographic approach is concerned with individual phenomena, as in biography and much of history, while its opposite, the nomothetic approach, aims to formulate laws as general propositions (as analytic a priori).

In history as in an individualizing science (*eine individualisierende Wissenschaft*) we obtain the direct relation between intension and extent of historical concepts: the more general historical concept is more value relevant at the same time:

> It obviously remains true that in characterizing one part of historical activity, we can speak of a re-creative understanding of the "spiritual" [der geistigen] world. But the concepts of understanding and re-creation are also too imprecise and general to provide a fully autonomous and exhaustive characterization of the nature of all historical representation. As regards understanding, it is important, first, that the object of understanding in history is always something more than merely real; namely, it is value relevant and meaningful. And second, to remain within the domain of history, the value relevant and the meaningful are comprehended not in a generalizing fashion but in an individualizing fashion, even though their content may be only relatively historical. Finally, even the concept of the "re-creation" of historical individuality acquires its precise significance for the theory of the historical sciences only on the basis of the concept of the individualizing understanding of meaning (*Rickert*).

Under these conditions, the general does not imply the particular. For instance, as we saw, we cannot differ the universal proposition 'all bodies are heavy' from the particular one 'some bodies are heavy' by Venn diagrams, because the extents of their concepts (the extents of 'bodies' and 'heavy') are just intersected.

Rickert did not think of creating a new square of opposition that may become suitable for describing semantic oppositions between synthetic (historical, individualizing) propositions. If we set up such a problem, we will start in distinguishing between general and particular synthetic propositions.

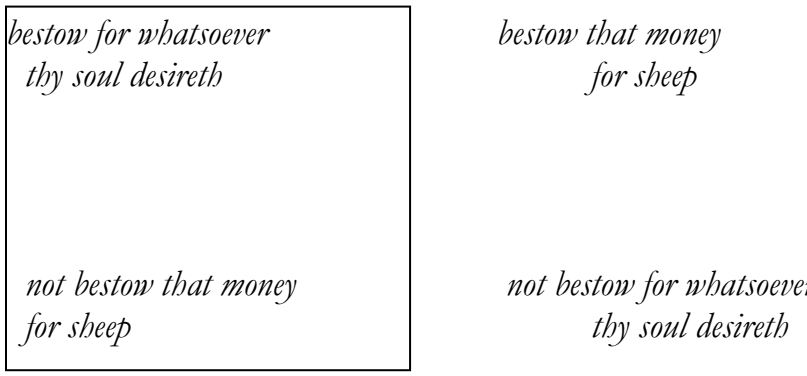

bestow for whatsoever thy soul desireth
bestow that money for sheep
not bestow that money for sheep
not bestow for whatsoever thy soul desireth

Figure 4.3 An example of the square of opposition for individualizing propositions from the verse of *Deut.* 14:26. Syntactically, *bestow for whatsoever thy soul desireth* is a universal affirmative proposition and *bestow that money for sheep* is a particular affirmative proposition. Nevertheless, I do not desire sheep and I do not know people who desire it, however they maybe are. Can we use Venn diagram for explicating relations between *bestow for whatsoever thy soul desireth* and *bestow that money for sheep*? No, because *thy soul desireth* has different meaning at different situations. Therefore at one and the same situation of utterance both statements may be simultaneously false, but cannot be simultaneously true.

For analytic propositions while we move from the general (i.e. the concept with the larger extent and the smaller content) to the particular (i.e. the concept with the smaller extent and the larger content), we are losing definiteness and certainty. For synthetic propositions, the general and the particular are two different points of view, because both have different extents and different contents with the same certainty, which satisfy a direct relation between them.

In the Apuleian square of opposition there is a duality between the general and the particular. Indeed, for the general there

is a contrary negation and for the particular a subcontrary negation, thereby the contrary negation tends to be maximized and the subcontrary negation tends to be minimized. In the new square of opposition (see figure 4.3) we could propose another *duality that takes place between the affirmation and the negation*. In this case the contrary negation holds between the general affirmative proposition and the particular affirmative proposition and the subcontrary negation between the general negative proposition and the particular negative proposition.

In order to explicate Windelband and Rickert's intuition, we can redefine logical content (intension).

Definition 4.1 *Let us take the set of all members of true conjunctions, where the name/proposition A occurs as well. Then this set is a non-well-founded intension (non-well-founded logical content) of A. For instance, let A be a human being. Then its non-well-founded intension is built up by the set* {'with smooth skin,' 'animate,' 'featherless bipeds,' etc.}.

Proposition 4.1 *Non-well-founded content (definition 4.1) includes the standard content as particular case (definition 2.1).*

Let us assume that an extent (logical scope) is defined in the standard way (definition 2.2). Then the following proposition can be readily proved:

Proposition 4.2 *There exists the law of the direct relation between intension and extent: the greater intension is, the greater extent is and the smaller intension is, the smaller extent is.*

From this proposition it follows that the duality that takes place between the affirmation and the negation is formally explicated as follows:

Proposition 4.3 *Let us take two different propositions A, B containing the same verb and satisfying definition 4.1. This means that there is a conjunction A&B that is true or false (i.e. A and B have the same or different non-well-founded content). Further, let us define the*

complement/negation ¬B satisfying definition 4.1 as well. Then in any case there is a duality relation between A and ¬B.

Proof. 1. *A* and *B* have the same non-well-founded content (*A*&*B* is true). Then $\neg B \Rightarrow A$, i.e. the extent of ¬*B* is smaller than *A*. 2. *A* and *B* have different non-well-founded contents (*A*&*B* is false). Then $A \Rightarrow \neg B$, i.e. the extent of ¬*B* is greater than *A*. In both cases we have a duality between *A* and ¬*B*.

Later we are formally proving that there exist two squares of opposition and, correspondingly, two syllogistics (the first for analytic a priori propositions and the second for synthetic a priori propositions).

In Aristotle's syllogistics, analytic propositions in Kant's words are formalized. In such propositions, a subject is thought within a predicate, the more common concept, for example: 'Socrates is a *man*' or 'All people are *animals*.' Therefore the predicate in formulas *S***a***P* ('every *S* is *P*'), *S***i***P* ('some *S* are *P*'), *S***e***P* ('no *S* is *P*'), *S***o***P* ('some *S* are not *P*') may be replaced by nouns, but not by adjectives. In other words, we have the following grammar: 'noun 1 (subject) + is + noun 2 (predicate),' and the concept of 'noun 1' is a kind (particular) of the concept of 'noun 2,' i.e. 'noun 2' is thought as general for 'noun 1.'

However, how far can we consider propositions like 'Socrates is white,' 'All bodies are heavy' within a conventional syllogistics formalizing just analytic propositions? At the first blush, these troubles might be deleted if we transformed an appropriate adjective into a noun. For example, the proposition 'Socrates is white' may be converted to the proposition 'Socrates is a white being,' and 'All bodies are heavy' to 'All bodies are something heavy.' However, such a transformation does not solve our problem, because the predicate is not general for the subject still. Thus, Socrates' whiteness is not his substantial attribute and the general of bodies is space, but not weight. Any body is thought first as space entity.

For the first time, Aristotle noticed that there are propositions that have been called synthetic since Kant and these propositions cannot be used in syllogistics. Aristotle's counterexample was as follows: "He who sits, writes and Socrates is sitting, then Socrates is writing" (*Topics* VIII, 10, 160 b 26 ff). This wrong syllogism was caused by using synthetic propositions. Kant's key example of synthetic propositions: 'All bodies are heavy.' They have the following grammar: 'noun 1 (subject) + is + adjective (attribute).' The connective 'is' of synthetic propositions is understood as follows:

$$A\,is\,B \equiv (\exists C(C\,is\,A) \wedge \forall C \forall D((C\,is\,A \wedge D\,is\,A) \Rightarrow C\,is\,D) \wedge \forall C(C\,is\,A \wedge C\,is\,B))$$

This understanding corresponds to the Kantian-Rickertian approach.

Assume that all the syllogistic synthetic propositions have the following meaning:

• *synthetic a priori*: 'All *S* are *P*' ('All bodies are heavy'): there exist *A* such that *A* is *S* and for any *A*, *A* is *S* and *A* is *P*;

• *synthetic a posteriori*: 'Some *S* are *P*' ('Socrates is white'): for any *A,* both '*A* is *S*' is false and '*A* is *P*' is false;

• *synthetic a priori*: 'No *S* are *P*' ('No bodies are angels'): there exist *A* such that if '*A* is *S*' is false, then '*A* is *P*' is true;

• *synthetic a posteriori*: 'Some *S* are not *P*' ('Socrates is not black'): for any *A*, '*A* is *S*' is false or there exist *A* such that '*A* is *S*' is false or '*A* is *P*' is false.

Let us propose now the syllogistic system formalizing synthetic propositions. This system is said to be *synthetic syllogistics*, while we are assuming that Aristotelian syllogistics is analytic. The basic logical connectives of synthetic syllogistics are as follows: a ('every + noun + is + adjective'), i ('some + noun + is + adjective'), e ('no + noun + is + adjective') and o ('some + noun + is not + adjective') that are defined in synthetic ontology in the following way:

$$S\mathsf{a}P := (\exists A(A\,is\,S) \wedge (\forall A(A\,is\,S \wedge A\,is\,P))); \qquad (1)$$

$$SiP := \forall A(\neg(A\,is\,S) \wedge \neg(A\,is\,P)); \tag{2}$$

$$SoP := \neg(\exists A(A\,is\,S) \wedge (\forall A(A\,is\,P \wedge A\,is\,S))),\textit{i.e.}\ (\forall A\neg(A\,is\,S) \vee \exists A(\neg(A\,is\,P) \vee \neg(A\,is\,S))); \tag{3}$$

$$SeP := \neg\forall A(\neg(A\,is\,S) \wedge \neg(A\,is\,P)),\textit{i.e.}\ \exists A(A\,is\,S \vee A\,is\,P). \tag{4}$$

Picture 5 Belarusian-Ukrainian landscape

Now let us formulate axioms of synthetic syllogistics:

$$SaP \Rightarrow SeP; \tag{5}$$

$$SaP \Rightarrow PaS; \tag{6}$$

$$SiP \Rightarrow PiS; \tag{7}$$

$$SaM \Rightarrow SeP; \tag{8}$$

$$MaP \Rightarrow SeP; \tag{9}$$

$$(MaP \wedge SaM) \Rightarrow SaP; \tag{10}$$
$$(MiP \wedge SiM) \Rightarrow SiP. \tag{11}$$

In synthetic syllogistics we have a novel square of opposition that we call the *synthetic square of opposition* (see figure 4.4), where the following theorems are inferred:

$SaP \Rightarrow \neg(SoP)$, $\neg(SoP) \Rightarrow SaP$, $SiP \Rightarrow \neg(SeP)$, $\neg(SeP) \Rightarrow SiP$, $SeP \Rightarrow \neg(SiP)$, $\neg(SiP) \Rightarrow SeP$, $SoP \Rightarrow \neg(SaP)$, $\neg(SaP) \Rightarrow SoP$, $SaP \Rightarrow \neg(SiP)$, $SiP \Rightarrow \neg(SaP)$, $\neg(SeP) \Rightarrow SoP$, $\neg(SoP) \Rightarrow SeP$, $SaP \Rightarrow SeP$, $SiP \Rightarrow SoP$, $SeP \vee SiP$, $\neg(SeP \wedge SiP)$, $SaP \vee SoP$, $\neg(SaP \wedge SoP)$, $\neg(SaP \wedge SiP)$, $SeP \vee SoP$.

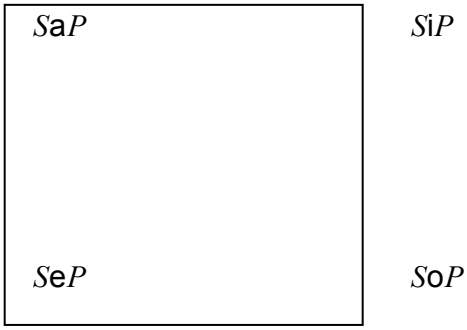


Figure 4.4 The synthetic square of opposition (for synthetic syllogistics, where synthetic propositions are formalized).

Suppose B is a complete Boolean algebra with the bottom element 0 and the top element 1 such that the cardinality of its domain $|B|$ is an infinite number. Build up the set B^B of all functions $f : B \mapsto B$. The set of all complements for finite subsets of B is a filter and it is called a Frechét filter, it is denoted by U. Further, define a new relation $\approx$ on the set B^B by

$f \approx g = \{a \in B : f(a) = g(a)\} \in \mathsf{U}$. It is easily proved that the relation $\approx$ is an equivalence. For each $f \in B^B$ let $[f]$ denote the equivalence class of f under $\approx$. The ultrapower B^B/U is then defined to be the set of all equivalence classes $[f]$ as f ranges over B^B. This ultrapower is called a *nonstandard* (or *non-Archimedean*) *extension of Boolean algebra* B, for more details see *Robinson* and *Schumann 2008*. It is denoted by ${}^*B$.

There exist two groups of members of ${}^*B$: (1) functions that are constant, e.g. $f(a) = m \in B$ on the set U, a constant function $[f = m]$ is denoted by ${}^*m$, (2) functions that are not constant. The set of all constant functions of ${}^*B$ is called standard set and it is denoted by ${}^\circ B$. The members of ${}^\circ B$ are called standard. It is readily seen that B and ${}^\circ B$ are isomorphic.

We can extend the usual partial order structure on B to a partial order structure on ${}^\circ B$:

• for any members $x, y \in B$ we have $x \le y$ in B iff ${}^*x \le {}^* y$ in ${}^\circ B$,

• each member ${}^*x \in {}^\circ B \setminus \{{}^*0\}$ (i.e. that is not a bottom element ^{*0} of ${}^\circ B$) is greater than any number $[f] \in {}^* B \setminus {}^\circ B$, i.e. ${}^*x > [f]$ for any $x \in B$, where $[f]$ is not constant function,

• ^{*0} is the bottom element of ${}^*B$.

Notice that under these conditions, there exist the top element ${}^*1 \in {}^* B$ such that $1 \in B$ and the bottom element ${}^*0 \in {}^* B$ such that $0 \in B$.

The ordering conditions mentioned above have the following informal sense: (1) the sets ${}^\circ B$ and B have isomorphic order structure; (2) the set ${}^*B \setminus \{{}^*0\}$ contains actual infinities that are less than any member of ${}^\circ B \setminus \{{}^*0\}$. These members are called *Boolean infinitesimals*.

Introduce three operations $\sup$, $\inf$, $\neg$ in the partial order structure of ${}^*B$:

$\inf([f],[g]) = [\inf(f,g)]$;

$\sup([f],[g]) = [\sup(f,g)]$;
$\neg[f] = [\neg f]$.

This means that a nonstandard extension ${}^*B$ of a Boolean algebra B preserves the least upper bound $\sup$, the greatest lower bound $\inf$, and the complement $\neg$ of B.

Consider the member $[h]$ of ${}^*B$ such that $\{a \in B : h(a) \le f(\neg a) \text{ or } h(a) > f(\neg a)\} \in \mathsf{U}$. Denote $[h]$ by $[f\neg]$. Then we see that $\inf([f],[f\neg]) \ge^* 0$ and $\sup([f],[f\neg]) \le^* 1$. Really, we have several cases.

• *Case 1*. The members $\neg[f]$ and $[f\neg]$ are incompatible. Then $\inf([f], [f\neg]) \ge^* 0$ and $\sup([f], [f\neg]) \le^* 1$,

• *Case 2*. Suppose $\neg[f] \ge [f\neg]$. In this case $\inf([f],[f\neg]) =^* 0$ and $\sup([f],[f\neg]) \le^* 1$.

• *Case 3*. Suppose $\neg[f] \le [f\neg]$. In this case $\inf([f],[f\neg]) \ge^* 0$ and $\sup([f],[f\neg]) =^* 1$.

• *Case 4*. The members $[f]$ and $\neg[f\neg]$ are incompatible. Then $\inf(\neg[f], \neg[f\neg]) \ge^* 0$ and $\sup(\neg[f],\neg[f\neg]) \le^* 1$,

• *Case 5*. Suppose $\neg[f\neg] \ge [f]$. In this case $\inf(\neg[f],\neg[f\neg]) \ge^* 0$ and $\sup(\neg[f],\neg[f\neg]) =^* 1$.

• *Case 6*. Suppose $\neg[f\neg] \le [f]$. In this case $\inf(\neg[f],\neg[f\neg]) =^* 0$ and $\sup(\neg[f],\neg[f\neg]) \le^* 1$.

• *Case 7*. The members $\neg[f\neg]$ and $\neg[f]$ are incompatible. Then $\inf([f], \neg[f\neg]) \ge^* 0$ and $\sup([f],\neg[f\neg]) \le^* 1$,

• *Case 8*. Suppose $\neg[f] \ge \neg[f\neg]$. In this case $\inf([f],\neg[f\neg]) =^* 0$ and $\sup([f],\neg[f\neg]) \le^* 1$.

• *Case 9*. Suppose $\neg[f] \le \neg[f\neg]$. In this case $\inf([f],\neg[f\neg]) \ge^* 0$ and $\sup([f],\neg[f\neg]) =^* 1$.

• *Case 10*. The members $[f]$ and $[f\neg]$ are incompatible. Then $\inf(\neg[f], [f\neg]) \ge^* 0$ and $\sup(\neg[f],[f\neg]) \le^* 1$,

• *Case 11*. Suppose $[f\neg] \geq [f]$. In this case $\inf(\neg[f],[f\neg]) \geq^* 0$ and $\sup(\neg[f],[f\neg]) =^* 1$.

• *Case 12*. Suppose $[f\neg] \leq [f]$. In this case $\inf(\neg[f],[f\neg]) =^* 0$ and $\sup(\neg[f],[f\neg]) \leq^* 1$.

Definition 4.2 *Now define hyperrational valued matrix logic* M_B *as the ordered system* $\langle {}^*B, \{{}^*1\}, \neg, \Rightarrow, \vee, \wedge \rangle$*, where*

• ${}^*B$ is the set of truth values,

• $\{{}^*1\}$ is the set of designated truth values,

• for all $[x] \in^* B$, $\neg[x] =^* 1 - [x]$,

• for all $[x],[y] \in^* B$, $[x] \Rightarrow [y] =^* 1 - \sup([x],[y]) + [y]$,

• for all $[x],[y] \in^* B$, $[x] \wedge [y] = \inf([x],[y])$,

• for all $[x],[y] \in^* B$, $[x] \vee [y] = \sup([x],[y])$.

Proposition 4.4 *In* M_B *there are only two squares of opposition.*

Proof. We have just eight cases: (1) $[f] \leq [f\neg]$, (2) $[f] \leq \neg[f\neg]$, (3) $[f\neg] \leq [f]$, (4) $[f\neg] \leq \neg[f]$, (5) $\neg[f\neg] \leq [f]$, (6) $\neg[f\neg] \leq \neg[f]$, (7) $\neg[f] \leq [f\neg]$, (8) $\neg[f] \leq \neg[f\neg]$. Taking into account that couples $[f]$ and $\neg[f]$ ($[f\neg]$ and $\neg[f\neg]$) are contradictory, we can claim that there exist two squares of opposition:

• if $[f] \leq \neg[f\neg]$ (resp. $[f\neg] \leq \neg[f]$), we have the conventional square of opposition (see figure 4.5); if $\neg[f\neg] \leq [f]$ (resp. $\neg[f] \leq [f\neg]$), we have its dual without changing meaning.

• if $[f\neg] \leq [f]$ (resp. $\neg[f] \leq \neg[f\neg]$), we have the synthetic square of opposition (see figure 4.6); if $\neg[f\neg] \leq \neg[f]$ (resp. $[f] \leq [f\neg]$), we have its dual without changing meaning;

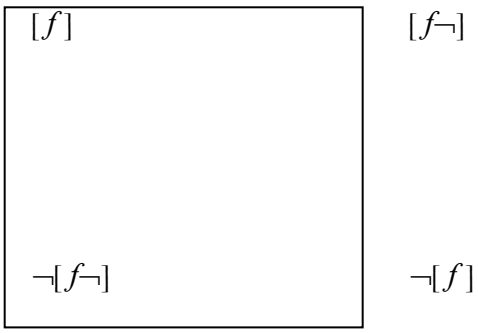


Figure 4.5 In case $[f\neg] \leq \neg[f]$, the square of oppositions for any members $[f]$, $[f\neg]$, $\neg[f]$, $\neg[f\neg]$ of ${}^{*}B$ holds true, i.e. $[f],[f\neg]$ are contrary, $[f],\neg[f]$ (resp. $\neg[f\neg],[f\neg]$) are contradictory, $\neg[f\neg],\neg[f]$ are subcontrary, $[f],\neg[f\neg]$ (resp. $[f\neg],\neg[f]$) are said to stand in the subalternation.

Figure 4.6 In case $\neg[f\neg] \leq \neg[f]$, the *synthetic* square of oppositions for any members $[f],[f\neg],\neg[f],\neg[f\neg]$ of ${}^{*}B$ holds true, i.e. $[f],\neg[f\neg]$ are contrary, $[f],\neg[f]$ (resp. $\neg[f\neg],[f\neg]$) are contradictory, $\neg[f],[f\neg]$ are subcontrary, $[f],[f\neg]$ (resp. $\neg[f\neg],\neg[f]$) are said to stand in the subalternation .

Now we can build models for atomic syllogistic formulas (i.e. syllogistic formulas without propositional connectives) due to algebra M_B.

Definition 4.3 *A structure* $\mathsf{B} = \langle O, I, \dot{\mathbf{a}}, \dot{\mathbf{e}}, \dot{\mathbf{i}}, \dot{\mathbf{o}}, \dot{\mathsf{a}}, \dot{\mathsf{e}}, \dot{\mathsf{i}}, \dot{\mathsf{o}} \rangle$ *is a non-Archimedean syllogistic model iff:*

• O is a restriction of the set M_B to an appropriate square of opposition (thereby the conventional square of opposition should hold true for Aristotelian syllogistics and the synthetic square of opposition holds for synthetic syllogistics).

• I is a mapping that associates a class of equivalence $[f] \in O$ with each atomic syllogistic formula $S \Diamond P$, where $\Diamond \in \{\mathbf{a}, \mathbf{e}, \mathbf{i}, \mathbf{o}, \mathsf{a}, \mathsf{e}, \mathsf{i}, \mathsf{o}\}$, so that $I(S \Diamond P) = |S| \dot{\Diamond} |P|$, where $\dot{\Diamond} \in \{\dot{\mathbf{a}}, \dot{\mathbf{e}}, \dot{\mathbf{i}}, \dot{\mathbf{o}}, \dot{\mathsf{a}}, \dot{\mathsf{e}}, \dot{\mathsf{i}}, \dot{\mathsf{o}}\}$ and

- $|S| \dot{\mathbf{a}} |P| = [f]$ (resp. $|S| \dot{\mathbf{a}} |P| = \neg[f\neg]$);
- $|S| \dot{\mathbf{e}} |P| = [f\neg]$ (resp. $|S| \dot{\mathbf{e}} |P| = \neg[f]$);
- $|S| \dot{\mathbf{i}} |P| = \neg[f\neg]$ (resp. $|S| \dot{\mathbf{i}} |P| = [f]$);
- $|S| \dot{\mathbf{o}} |P| = \neg[f]$ (resp. $|S| \dot{\mathbf{o}} |P| = [f\neg]$);
- $|S| \dot{\mathsf{a}} |P| = [f]$ (resp. $|S| \dot{\mathsf{a}} |P| = [f\neg]$);
- $|S| \dot{\mathsf{e}} |P| = [f\neg]$ (resp. $|S| \dot{\mathsf{e}} |P| = [f]$);
- $|S| \dot{\mathsf{i}} |P| = \neg[f\neg]$ (resp. $|S| \dot{\mathsf{i}} |P| = \neg[f]$);
- $|S| \dot{\mathsf{o}} |P| = \neg[f]$ (resp. $|S| \dot{\mathsf{o}} |P| = \neg[f\neg]$).

We now give the truth conditions of Boolean combinations of atomic syllogistic formulas in a non-Archimedean syllogistic model:

Definition 4.4

$\mathsf{M} \models \neg\phi$ iff $\mathsf{M} \not\models \phi$

$\mathsf{M} \models \phi \wedge \psi$ iff $\mathsf{M} \models \phi$ and $\mathsf{M} \models \psi$

$\mathsf{M} \models \phi \vee \psi$ iff $\mathsf{M} \models \phi$ or $\mathsf{M} \models \psi$

$\mathsf{M} \models \phi \Rightarrow \psi$ iff $\mathsf{M} \models \neg\phi$ or $\mathsf{M} \models \psi$

By using non-Archimedean models I have just proved that there are only two squares of opposition if we assume Boolean algebra as the basis of an appropriate non-Archimedean extension. The conventional square of opposition may be aimed for getting

analytic syllogistics (Aristotelian syllogistics) and the new one for getting synthetic syllogistics (Rickertian syllogistics). Aristotelian syllogistics can be used for analyzing semantic relations between the general (*kelal*) and the partucular (*frat*)—H5, I4, I5, I6. Rickertian syllogistics can be used for analyzing semantic relations between the extension (*ribuy*) and the restriction (*mi'ut*)—E1, E2, E3, E4. Hence, Judaic (simple or complete) conjunctive logic (see definitions 4.1—4.2 of the first chapter) supposes semantic relations of two kinds: analytic a priori treated within Aristotelian syllogistics and synthetic a priori treated within Rickertian syllogistics.

Judaic syllogistics is non-Aristotelian for different reasons. First, it is built up by studying genera and species occuring in the Torah (i.e. outside of the Porphyrian tree) and each Talmudic book proposes its own syllogisctic system. Therefore Judaic syllogistic knowledge should be regarded as analytic a posteriori. Second, its basic relations may be explicated by means of two syllogisctics: Aristotelian and Rickertian, with two squares of opposition: analytic and synthetic ones.

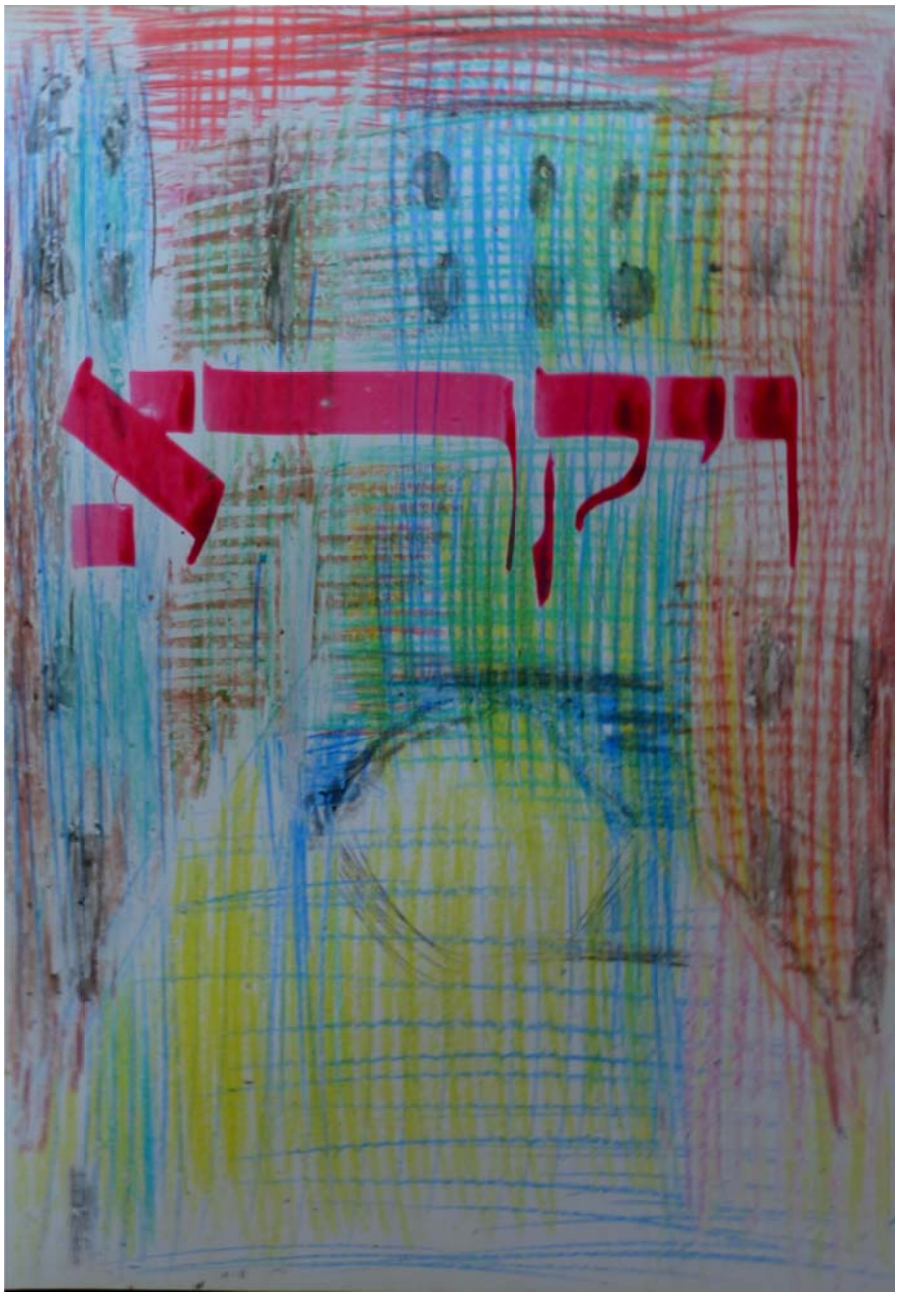
Chapter 3. Judaic theology of science

1. Judaic reasoning and the analytic a posteriori

Logic is the most abstract science, it is better formalized, than any another science. Actually, it is more abstract and better formalized even than mathematics. Perhaps, it is more mathematical than mathematics itself, since modern mathematics became modern thanks to that it has been formalizing in the logical frames. Logic allows us to formalize anything. The main destination of logic is in formalizing knowledge.

Why is formalization good? It allows us to make thought as much as possible transparent, to remove all double meanings and fogs, to convert intuitive indistinct knowledge into the language of "iron" algorithms. If you formalized a part of knowledge, this means you completely comprehended this part. The grade of formalizing is equal to the grade of obviousness.

Very few people not from logician communities correctly understand what this formalization is actually. It is not an invention of formulas as the majority think. It is an invention of unambiguous ways of how from one abstract object we can mechanically obtain other abstract objects. These ways are fixed in special definitions which are closely linked among themselves. These definitions as much as possible obvious and transparent, but they are so long so that it is much more convenient to think out for them abbreviations. These abbreviations for logical-mathematical definitions are formulas. The more composite formula, the bigger number of definitions compressed there. For this reason logical and mathematical texts are so sophisticated—if we are going to translate one page of the formalized text into the normal text, then we should understand that one page can pass into hundred pages, and even more. Therefore logicians are who are able not only to read, but also to make texts with improbable density of the compressed meanings, i.e. those who are dealing with formalizations.



Picture 6 The title page of the Book of Leviticus

Definition 1.1 *Formalized knowledge is analytic. Knowledge that for various reasons cannot be formalized is called synthetic.*

Among all logical methods of formalizing, Kant knew only Aristotle's syllogistics. In syllogistics it is supposed that we can check for each atomic proposition how its subject corresponds to its predicate. Here three cases are possible: (1) the extent (logical scope) of the subject is included into predicate's extent (it is supposed that this condition takes place for universal affirmative propositions), (2) the extent of the subject is not intersected with predicate's extent (it is for universal negative propositions), (3) the extent of the subject is intersected with predicate's extent and is not included in it (for particular propositions). According to Kant, a proposition is analytic if the meaning of its predicate and the meaning of its subject are reduced to above-mentioned operations on extends of subjects and predicates. If the meaning of the predicate and the meaning of the subject cannot be reduced to operations on their extents (e.g. the proposition is universal affirmative, but the extent of the predicate does not include the extent of the subject), then the proposition is synthetic. On this account, all analytic propositions may be modeled within manipulations with Venn diagrams. Synthetic ones may not.

There are several philosophical definitions to make clear what makes the a priori distinct from the a posteriori. For instance, there is an opinion that a priori justification is explicated by the difference between fallibility as the a posteriori and infallibility as the a priori, i.e. as "a mode of justification that always leads to truth" (*Goldman 1999*). However, many contemporary philosophers do not share the opinion that a priori justification is so characterized. An counterexample may be provided by the case of Euclidean geometry "regarded for centuries as describing the necessary character of space, but apparently refuted by … General Relativity" (*BonJour 1998*). This means that the Euclidean conception of space has been justified a priori up until the theory of

general relativity was made up. A modification of Goldman's distinction is presented by the idea that the a priori is justified by being unrevisable regardless of the agent's future experience (*Putnam 1979*).

Definition 1.2 *Knowledge that is given irrespective to an empirical research is a priori. Knowledge that we have due to an empirical research is a posteriori.*

Hence, on the one hand, scientific propositions systematically formalized should be qualified as analytic. If they are regardless of experience, then they are a priori. On the other hand, there are scientific propositions that require empirical investigation or experience to be known. For example, the proposition that water is equivalent to H_2O became a result of an appropriate chemical experiment. Nevertheless, it can be formalized within chemical equations. Therefore such a proposition known a posteriori should be regarded as analytic. As we see, the analytic (the necessary) can be either a priori or a posteriori, for more details see papers by Gareth Evans (*1979*), Timothy Williamson (*1986*), Albert Casullo (*2003*), John Hawthorne (*2002*), and Goldman (*1999*).

The synthetic can be either a priori or a posteriori as well. In most cases, contingent knowledge, which we cannot formalize (i.e. the synthetic), is a posteriori. However, there are cases, when it is a priori. For instance, Williamson (*1986*) exemplify this case by the following reasoning:

> Given a valid deduction from the premise that someone believes that *P* to the conclusion that *P*, believe that *P*.

If '*P*' is replaced by the proposition "There is at least one believer," then one can know a priori the synthetic proposition "There is at least one believer."

The Torah is not given a priori. According to Judaism, it came down to us as Revelation. Therefore this knowledge is not

contained in our thought, we know about due to an appropriate experience, the Revelation saved by the Judaic tradition originated by Moses and supported from generation to generation by the sages:

יח רב חיבר ספרא וספרי לבאר ולהודיע עיקרי המשנה ורבי חייא חיבר התוספתא לבאר ענייני המשנה וכן רבי הושעיה ובר קפרא חיברו ברייתות לבאר דברי המשנה ורבי יוחנן חיבר התלמוד הירושלמי בארץ ישראל אחר **יט** ומגדולי החכמים שקיבלו מרב חורבן הבית בקרוב משלוש מאות שנה רב הונא ורב יהודה ורב נחמן ורב כהנא ומגדולי החכמים שקיבלו ושמואל **כ** ומכלל רבה בר בר חנה ורבי אמי ורבי אסי ורב דימי וראבון מרבי יוחנן החכמים שקיבלו מרב הונא ומרב יהודה רבה ורב יוסף ומכלל החכמים שקיבלו מרבה ורב יוסף אביי ורבא ושניהם קיבלו גם מרב נחמן ומכלל החכמים שקיבלו מרבא רב אשי ורבינא ומר בר רב אשי קיבל מאביו ארבעים איש ואלו הן (א) רב **כא** נמצא מרב אשי עד משה רבנו ומרבינא אשי (ב) מרבא (ג) מרבה (ד) מרב הונא (ה) מרבי יוחנן ורב ושמואל (ו) מרבנו הקדוש (ז) מרבן שמעון אביו (ח) מרבן גמליאל אביו (ט) מרבן שמעון אביו (י) מרבן גמליאל הזקן אביו (יא) מרבן שמעון אביו (יב) מהילל אביו ושמאי (יג) משמעיה ואבטליון (יד) מיהודה ושמעון (טו) מיהושע וניתאי (טז) מיוסף ויוסף (יז) מאנטיגנוס (יח) משמעון הצדיק (יט) מעזרא (כ) מברוך (כא) מירמיה (כב) מצפניה (כג) מחבקוק (כד) מנחום (כה) מיואל (כו) ממיכה (כז) מישעיה (כח) מעמוס (כט) מהושע (ל) מזכריה (לא) מיהוידע (לב) מאלישע (לג) מאלייהו (לד) מאחייה (לה) מדויד (לו) משמואל (לז) מעלי (לח) מפינחס (לט) מיהושוע (מ) ממשה רבנו רבן של **כב** כל אלו החכמים הנזכרים הם גדולי כל הנביאים מעם ה' אלוהי ישראל מהם ראשי ישיבות ומהם ראשי גלייות ומהם מסנהדרי גדולה הדורות **כג** רבינא ורב ועימהם בכל דור ודור אלפים ורבבות ששמעו מהם ועימהם אשי הם סוף חכמי התלמוד ורב אשי הוא שחיבר התלמוד הבבלי בארץ **כד** ועניין שנער אחר שחיבר רבי יוחנן התלמוד הירושלמי בכמו מאה שנה הוא פירוש דברי המשנה וביאור עמוקותיה ודברים שני התלמודין שנתחדשו בכל בית דין ובית דין מימות רבנו הקדוש ועד חיבור התלמוד מכולם ומשני התלמודין ומן התוספתא ומספרא ומספרי ומן התוספתות יתבאר האסור והמותר והטמא והטהור והחייב והפטור והכשר והפסול כמו שהעתיקו איש מפי איש מפי משה מסיניי

Rav composed the *Sifra* and the *Sifre* to explain the sources for the Mishnah. Rabbi Chiyya composed the *Tosefta* to explain the subjects [discussed in] the Mishnah. Rabbi Hoshaia and bar Kafra composed *baraitot* to explain the matters [discussed in]

the Mishnah. Rabbi Yochanan composed the Jerusalem Talmud in *Eretz Yisrael* approximately three hundred years after the destruction of the Temple.

Among the great sages who received the tradition from Rav and Shemuel were: Rav Huna, Rav Yehudah, Rav Nachman, and Rav Kahana. Among the great sages who received the tradition from Rabbi Yochanan were: Ravvah bar bar Channah, Rav Ami, Rav Assi, Rav Dimi, and Rav Avin.

Among the Sages who received the tradition from Rav Huna and Rav Yehudah were Rabbah and Rav Yosef. Among the sages who received the tradition from Rabbah and Rav Yosef were Abbaye and Ravva. Both of them also received the tradition from Rav Nachman. Among the Sages who received the tradition from Ravva were Rav Ashi and Ravina. Mar bar Rav Ashi received the tradition from Rav Ashi, his father, and from Ravina.

Thus, there were forty generations from Rav Ashi back to Moses, our teacher, of blessed memory. They were: 1) Rav Ashi [received the tradition] from Ravva. 2) Ravva [received the tradition] from Rabbah. 3) Rabbah [received the tradition] from Rav Huna. 4) Rav Huna [received the tradition] from Rabbi Yochanan, Rav, and Shemuel. 5) Rabbi Yochanan, Rav, and Shemuel [received the tradition] from Rabbenu Hakadosh. 6) Rabbenu Hakadosh [received the tradition] from Rabbi Shimon, his father. 7) Rabbi Shimon [received the tradition] from Rabban Gamliel, his father. 8) Rabban Gamliel [received the tradition] from Rabban Shimon, his father. 9) Rabban Shimon [received the tradition] from Rabban Gamliel, the elder, his father. 10) Rabban Gamliel, the elder, [received the tradition] from Rabban Shimon, his father. 11) Rabban Shimon [received the Tradition] from Hillel, his father, and Shammai. 12) Hillel and Shammai [received the tradition] from Shemayah and Avtalion. 13) Shemayah and Avtalion [received the tradition] from Yehudah and Shimon [ben Shatach]. 14) Yehudah and Shimon [received the tradition] from Yehoshua ben Perachiah and Nittai of Arbel. 15) Yehoshua and Nittai [received the tradition] from Yosse ben Yo'ezer and Yosef ben Yochanan. 16) Yosse ben Yo'ezer and Yosef ben Yochanan [received the tradition] from Antignos. 17) Antignos [received the tradition] from Shimon the Just. 18) Shimon the Just

[received the tradition] from Ezra. 19) Ezra [received the tradition] from Baruch. 20) Baruch [received the tradition] from Jeremiah. 21) Jeremiah [received the tradition] from Tzefaniah. 22) Tzefaniah [received the tradition] from Chabbakuk. 23) Chabbakuk [received the tradition] from Nachum. 24) Nachum [received the tradition] from Yoel. 25) Yoel [received the tradition] from Michah. 26) Michah [received the tradition] from Isaiah. 27) Isaiah [received the tradition] from Amos. 28) Amos [received the tradition] from Hoshea. 29) Hoshea [received the tradition] from Zechariah. 30) Zechariah [received the tradition] from Yehoyada. 31) Yehoyada [received the tradition] from Elisha. 32) Elisha [received the tradition] from Elijah. 33) Elijah [received the tradition] from Achiah. 34) Achiah [received the tradition] from David. 35) David [received the tradition] from Shemuel. 36) Shemuel [received the tradition] from Eli. 37) Eli [received the tradition] from Pinchas. 38) Pinchas [received the tradition] from Joshua. 39) Joshua [received the tradition] from Moses, our teacher. 40) Moses, our teacher, [received the tradition] from the Almighty.

Thus, [the source of] all these people's knowledge is God, the Lord of Israel. All the sages who were mentioned were the leaders of the generations. Among them were heads of academies, heads of the exile, and members of the great Sanhedrin. Together with them in each generation, there were thousands and myriads that heard their [teachings].

Ravina and Rav Ashi were the final generation of the Sages of the Talmud. Rav Ashi composed the Babylonian Talmud in Shin'ar approximately one hundred years after Rabbi Yochanan composed the Jerusalem Talmud. The intent of both the Talmuds is to elucidate the words of the Mishnah, to explain its deeper points, and [to relate] the new matters that were developed by each court from the era of Rabbenu Hakadosh until the composition of the Talmud.

From the entire [body of knowledge stemming from] the two Talmuds, the *Tosefta,* the *Sifra,* and the *Sifre,* can be derived the forbidden and the permitted, the impure and the pure, the liable and those who are free of liability, the invalid and the valid as was received [in tradition], one person from another, [in a chain extending back] to Moses at Mount Sinai (Rambam, *Introduction to the Mishneh Torah*).

Thus, the Torah is complex data known a posteriori. However, at the same time this knowledge may be evaluated as being analytic. The feeling that the Judaic hermeneutics and exegesis closed under the Judaic inference rules (H1—H7, I1—I13, E1—E32 of chapter 1) of explicating and interpreting the Torah is not less analytic than strong sciences such as physics, astronomy or mathematics came over the sages as soon as they got acquainted with those subjects. In the 19th century, many well-educated rabbis drew a comparison between science and the Torah. For example, Adolf Schwarz (1846—1931), the Rabbi of Karlsruhe (Germany) and the brilliant scholar studied in Vienna and Breslau claimed that the Torah is like the heaven. While the first is investigated by the Judaic exegesis, the second is discovered by astronomy. Furthermore, on the one hand, the Torah is eternal as well as heaven. On the other hand, the Judaic exegesis is based on strong reasoning (*middot*) as well as astronomy is based on mathematics.

> Wer eine annähernd klare Vorstellung von der unermesslichen Bedeutung erlangen will, welche die, das ganze Leben des Israeliten umspannende Gesetzeskunde in der Wissenschaft des Judenthums besitzt, der muss diese Gesetzeskunde mit der erhabensten aller Wissenschaften, mit der Stern- oder Himmelskunde in Parallele stellen. Das auf der Höhe seiner weltgeschichtlichen Aufgabe stehende Volk Israel hat zu allen Zeiten in seiner Gotteslehre einen Himmel besessen; und das ist die Thorah uns auch thatsächlich, nicht allein vom Standpunkte des Glaubens, sondern auch vom Standpunkte der Forschung. Dem empfänglichen Gemüthe des gläubigen Menschen gewährt das Lesen in der hl. Schrift zum mindesten dieselbe Erbauung, wie der Anblick des gestirnten Himmels; aber auch dem Forscherauge enthüllen sich bei einem tieferen Eindringen in das Gottesbuch nicht viel weniger Wunder, als der Himmel sie für Jene hat, welche in seine Höhen emporzudringen verstehen. Die Sinailehre ist seit Jahrtausenden ein Forschungsgebiet, dessen Unermesslichkeit jener des Himmels gleicht; aber auch die Entdeckungen, welche man auf beiden Gebieten gemacht hat, gleichen einander wenigstens in

dem einen Punkte, dass die Kenntniss ihrer Geschichte die Kenntniss jener Mittel voraussetzt, mit deren Hilfe die Entdeckungen gemacht worden sind. In der Astronomie hat man das stets als etwas Selbstverständliches angesehen; da war eine Geschichte der astronomischen Entdeckungen ohne eine Geschichte der astronomischen Instrumente undenkbar. Bei uns jedoch glaubte man eine Geschichte der hermeneutischen Entdeckungen construiren und schreiben zu können, ohne es sich auch nur im Entferntesten einfallen zu lassen, dass die Hermeneutik selber ihre Geschichte hat, dass sie in erster Reihe eine Geschichte jener Instrumente ist, deren die Tradition sich bedient hat, am ihre Entdeckungen zu machen. Und weil man das nicht bedacht und erwogen, haben wir bis auf den heutigen Tag keine Geschichte der Halachah.
Die hermeneutischen Normen sind die astronomischen Instrumente der, die Thorah wie einen Himmel durchforschenden Halachah. Das ist ein Satz von fundamentaler Bedeutung; die Instrumente mögen noch so alt, so alt wie die Kinst oder Wissenschaft selber sein, welche sich ihrer bedient, sie mögen gleich von vornherein als äusserst vollkommen erscheinen; wenn sie ihren Zweck erfüllen, wenn sie der Kunst oder Wissenschaft zu einem gedeihlichen Fortschritt verhelfen, wenn sie als die, eine gesunde Entwickelung fördernden Mittel sich bewähren sollen, müssen sie selber als entwickelungs- und vervollkommnungsfähig sich erweisen. Die Halachah ist gleichsam die Astronomie der Thorah, und als solche eine Wissenschaft im eminentesten Sinne des Wortes (*Schwarz*).

If someone wants to achieve an approximately clear presentation of the immense importance, which the jurisprudence in Judaica (hold in the whole life of the Israelite peoples) has had, he should compare this jurisprudence with the highest science, with astronomy. Israel as the nation standing at the top of the world-historical mission has ever owned the heaven in the doctrine of God; and it is the Torah in fact, not only from the standpoint of faith, but also from the standpoint of investigation. Reading the Holy Writ given for the impressible nature of faithful people is at least of the same charachter as the sight of the starlit heaven; but also for the eye

> of researcher the same wonder is revealed in a deeper vision of the Book of God that is not less than the heaven for those who understand vision of their heights. For millennia the Sinai theory has been a research area whose immensity is like that of heaven, but also discoveries which have been made in both areas, which are similar at least at one point, according to that the knowledge of its history requires the knowledge of those funds that help making discoveries. In astronomy, it has always been regarded as something natural that the history of astronomical discovery without the history of astronomical instruments is unthinkable. For us, however, one claims that the history of hermeneutical discoveries might be constructed and written without dreaming up as the last that hermeneutics itself has its history that first of all it is a history of those instruments, which has served for the tradition to make its discoveries. And because it is not so considered and thought, we have no history of halakhah to this day.
>
> The hermeneutic rules are astronomical instruments of halakhah investigating the Torah as the heaven. This is a statement of fundamental importance that the instruments may be so old as old art or science is, which they are used for, they may appear simultaneously as perfect; if they fulfill their purpose, if they help art or science to achive a significant progress, if they are used as normal development-promoting means, then they should be regarded as able to develop and improve. The halakhah is like the astronomy of the Torah, and as such it is a science in the eminent meaning of the word.

Since the 19th century all sciences have been dichotomized into natural sciences (*Naturwissenschaften*) such as physics, chemistry, etc. investigating causal relations and spiritual sciences (*Geisteswissenschaften*) such as philosophy, philology, history, social science, etc. dealing with teleological relations. The spirit of natural sciences is satisfied by derivation of concepts through *generalization* (*die generalisierende Begriffsbildung*). Its source is in the reason tendency to fix iterated phenomena and common processes, though nothing in the world actually is iterated in facsimile. The generalization transmutes objects into copies of a genus, i.e. of a generic concept.

These copies are quite interchangeable, and without any damage to the intension (logical content) of the generic concept. However, there is also another approach to understanding the reality, namely when in a subject something especial, savor, is emphasized. This way of derivation of concepts is called by Rickert *individualization* (*die individualisierende Begriffsbildung*) and this way is dual to generalization as a whole and satisfies the spirit of historical knowledge.

Dr. Hirsch S. Hirschfeld (1812—1884) who was the Rabbi in Wollstein (Germany) traced this difference between *Naturwissenschaften* and *Geisteswissenschaften* in the Judaic exegesis, too. He paid heed to us that the Judaic exegesis is divided into the halakhic and haggadic exegesis and while the halakhic exegesis contains knowledge that is not less strong than natural sciences and builds up on the basis of genus-species relations as well (see proposition 2.1 of chapter 2), the haggadic exegesis is close to spiritual sciences, because also it contains contextual knowledge about individuality that is out of genus-species relations (see propositions 4.2—4.4 of chapter 2). The haggadic exegesis is historically mutable and reflects the history of Judaic hermeneutics, the spiritual life of Israel:

> Die Exegese ist ein Ausdruck des Geistes jeder Zeit, insofern dieser mit der Bibel sich beschäftigt. Die Exegese nämlich ändert sich mit der Zeit, und ihre Resultate sind sehr verschiedentlich. Da nun aber die Bibel unveränderlich dieselbe bleibt, und die aufgefundenen Varianten oder vorgenommenen Textconjecturen keineswegs berechtigt erscheinen, Einfluss auf die Auffassung des biblischen Inhalts zu üben, so kann die Verschiedenheit der einzelnen Auslegungen nur dem Geiste der Zeit oder des Individuums beigemessen werden, der sich natürlich nach der Individualität und den Verhältnissen verschieden gestaltet; die Exegese vermittelt und somit die charakteristische Eigenthümlichkeit des Exegeten. Eine Geschichte der Exegese ist demnach eine Geschichte des menschlichen Geistes, in welcher dieser sich zwar nicht frei

> entfaltet, also auch nicht in ungebundener Art seine ganze Eigenthümlichkeit verräth, dagegen aber gerade durch den Widerstand der Bibel und die Benutzung und Verarbeitung derselben die Intensität der wesentlichen innern Ktraft um so deutlicher darlegt (*Hirschfeld 1847*).
>
> The exegesis is an expression of the spirit of each time, inasmuch as this deals with the Bible itself. The exegesis has changed in length of time, and their results are very different. But since the Bible remains unaltered and the same, and the found variants or forwarded textual assumptions hold, which have an influence on the understanding of the Biblical content, so the diversity of the various interpretations can be ascribed just to the spirit of time or individual, which naturally is formed differently by individuality and relationships; the exegesis shows thus the characteristic peculiarity of the exegete. A history of exegesis is therefore a history of human spirit in which this develops not free as well as not in the unbound specy the whole peculiarity teaches, however just by the resistance of the Bible and the use and processing of the same the intensity of the main inner force is treated more clearly.

The halakhic exegesis concerns the Biblical laws and Judaic hermeneutic principles. Halakhic rules are similar to physical laws, because they have all features of laws of nature, i.e. in the same measure they are typically theoretically reversible in time, unchanged since first discovered, applied everywhere where there are appropriate cases, typically expressed in terms of mathematical equations, etc. In Hirschfeld's opinion, we can generalize empirical data of the Torah and obtain laws in terms of genus-species relations as well as we have generalized empirical data of natural sciences:

> Wie der Naturforscher die Gesetze der Natur betrachtet und untersuht, so behandelten und erforschten die Talmudisten die biblischen Gesetze. Üeber die Erscheinung gehet Nichts; was die Empirie lehrt, hat volle Gültigkeit. Der Mensch kann dabei nicht den Grund der Wirkung warum Dieses ein Anderes

> bedingt oder neutralisirt, angeben, er schaut nur seinen Gegenstand an, untersucht seine Einwirkungen und generalisirt die einzelnen Kräfte zu einer Grundkraft, auf die er alle zurückführt, und aus der er alle Erscheinungen sich erklärt. Eine Einzelerscheinung als Ausnahme, lässt er nur als Wunder insofern gelten, als sie in einer gläubigen Ueberzeugung gerechtfertigt ist; sonst schliesst er immer aus der Erscheinung auf die veranlassende Ursache. Ganz so wurden die biblischen Gesetze und Gebrauche von Seiten des Talmud's behandelt. Gott hat sie eingesetzt, wie er Kräfte in die Natur gelegt, sie sind ewig und wirken ewig. Wodurch sie wirken, wie das was sie erzielen sollen, mit der Wirkung in Verbindung stehet; auf welche Weise die Wirkung aus der That nach der Vorschrift entspringt; dies liegt ausser dem Gebiete menschlichen Denkvermögens. Der Mensch mag es sich erklären, darf aber nie wähnen, er habe die wirkliche Verbindung erfasst (*Hirschfeld 1840*).

> In the way as the scientist considers and seeks the laws of nature, so Talmudists treate and study the Biblical laws. Nothing is over phenomena, what the empiricism teaches has full force and effect. The human being cannot point out the cause for effect why this another serves or neutralizes, he only looks at his object, examining its effects and generalizing some forces into a ground force to which he reduces all, and from which he explains all phenomena. An isolated phenomenon as an exception can only be considered by him as miracle as if it is justified in a faithful belief; nevertheless he ever deduce from the phenomenon to the cause. In the same way the Biblical laws and habits are interpreted from the viewpoint of the Talmud. God has made them, when He placed forces into nature, they are eternal and hold forever. How they work, how things which they must attain stand in connection with the effect, how the effect arises from the fact according to order, is outside of the field of human intellectual capacities. The human being may explain it, but never imagine that he captures the real connection.

In view of Hirsch Hirschfeld's demonstration, the halakhic exegesis is knowledge evaluated as analytic a posteriori, namely, on the one hand, it can be formalized within mathematical equations

of a special form and, on the other hand, all the data come to hand by investigating the Torah as something outside of our thought.

The first initial mathematical equation set up by Hirsch Hirschfeld for formalizing *middot* (Judaic inference rules) is presented by the so-called 'combinatory deduction' (*Combinationsschluss*). It symbolically explicates the rule *binyan 'av* [בנין אב] (I3). As we see, I3 is evaluated by Hirschdeld as basis of Judaic reasoning. Its formalization is being obtained as follows. Let A and B be media (e.g. member, thing, relation) for defining the law (*die Träger der Gesetzbestimmung*), β and γ be their particulars/features (*Eigenthümlichkeiten*), α be their general (*die gemeinsame Eingenschaft*), C be a conclusion. Then the Judaic deduction is of the form:

$$A + \alpha + \beta—\gamma = A + x \textbf{ and } B + \alpha—\beta + \gamma = \qquad (1)$$
$$B + x :: \beta—\gamma = —\beta + \gamma = o$$

Symbols A, B are either neutral in respect to x, i.e. they equal to zero, o, or both are x. Then from B it follows that β does not satisfy x and from A that γ does not satisfy x (i.e. both β and γ is equal to o in respect to x). Hence, the only reason for x is presented just by α. If C bears α, then it bears x as well. No matter if C bears β and γ also or not, because β and γ are equal to o in respect to x.

Rule (1) concerns a combination from one verse [בנין אב מכתוב אחד] but it is applied as well to a combination from two verses [בנין אב משני כתובים]. The second case is called *mah maẓinu* [מה מצינו], as Hirsch S. Hirschfeld notices. Its difference consists in that (1) assumes only one verse for defining the law and *mah maẓinu* supposes two verses. An example of the first case is as follows:

> Every bed, whereon he lieth that hath the issue, is unclean: and every thing, whereon he sitteth, shall be unclean (*Lev.* 15: 4)

Bed and chair are used for rest (it is their general, α, אב). Then each thing used by a patient for rest is unclean according to this verse and rule (1).

The second significant mathematical equation, which is regarded as initial for deriving other equations, tells us about the way of objection. If we take a general α such that a media A is not included into α and a media B is included into α, then it is possible that there is x that is satisfied both by A and by B:

$$A—\alpha = B + \alpha :: A + x = B + x. \qquad (2)$$

Objection [פירכא] a priori (*Widerlegung a priori*), our second initial mathematical equation, is formulated as follows:

$$A—\alpha + \beta = A + x :: B + \alpha —\beta = B + (?), \qquad (3)$$

where β is a new particular found in the Scripture.

Formula (3) says that if there are α, β such that α holds on B and does not hold on A and β holds on A and does not hold on B, then we have not enough information to deduce x in respect to A, B. Symbolically:

$$\alpha—A = \alpha + B \textbf{ and } \beta + A = \beta—B \textbf{ then} \qquad (4)$$
$$x + A = x + (?)$$

Now let us take a new medium C, then formula (3) may be transformed into the following formula of objection a posteriori (*Widerlegung a posteriori*):

$$A—\alpha = A + x \textbf{ and } C + \alpha = C—x \textbf{ then} \qquad (5)$$
$$B + \alpha = B + (?)$$

As we see, it cannot be solved whether *B* corresponds to either *A* or *C*.

The aim of Judaic reasoning is to reconstruct genus-species relations of the Torah by using mathematical equations among which formulas (1), (3) are initial. For realizing this aim each concept of the Torah should be denoted by a sign and then we should construct mathematical equations for all signs. Let us consider a simple example, in which we build up mathematical equations for the following signs: G.Sch. (violent action of damage, *Gewaltschaden*), Bs.Bsch. (possession of the injured, *Besitz des Beschädigten*), O.Pl. (public place, *öffentlicher Platz*), W.Sch. (intentional action of damage, *Willenschaden*), Gz.Ers. (100% cost of damage, *ganzer Schadenersatz*), Hlb.Ers. (50% cost of damage, *halber Ersatz*). The formula

W.Sch. O.Pl.—Hlb.Ers. = G.Sch. O.Pl. + Hlb.Ers. :: (6)
W.Sch. Bs.Bch. + Gz.Ers. = G.Sch. Bs.Bch. + Hlb.Gz.Ers.

shows that an intentional action at a public place does not require 50% cost of damage, while a violent action at the same place requires 50% cost of damage. Furthermore, an intentional action at a private place requires 100% cost of damage, while a violent action at the same place requires 50% or 100% cost of damage. Formula (6) is logically equivalent to the following formula:

W.Sch. O.Pl.—Hlb.Ers. = W.Sch. Bs.Bch. + Gz.Ers. ::
G.Sch. O.Pl. + Hlb.Ers. = G.Sch. Bs.Bch. + Gz. Hlb.Ers.

Let us take another set of signs: Scl. (slave, *Sclavin*), Fr. (free woman, *freie Frau*), Bsch. (the acquisition by sexual act, *die Acquisition durch Beischlaf*), Gld. (the acquisition by money, *die Acquisition durch Geld*), then according to the Torah, the mathematical equation is as follows:

Scl.—Bsch. = Scl. + Gld. :: Fr. + Bsch. = Fr. + Gld. (7)

We can add a new sign, Vkf. (sale, *Verkauf*), to (7). We know that while a slave can be sold, a woman cannot, symbolically:

Scl.—Bsch. + Vkf. = Scl. + Gld. :: Fr. + Bsch.—Vkf. = Fr. + (?) (8)

From (8) the following objection a posteriori follows:

Bsch.—Scl. = Bsch. + Fr. **and** Vkf. + Scl. = Vkf.—Fr. **then** Gld. + Scl. = Gld. + (?)

Further, we have more precise information if we take a sign, Schw. (sister-in-law, *Schwägerin*), because we know that if a sister-in-law becomes widow, she is devoted to brothers of her former husband still:

Scl.—Bsch. = Scl. + Gld. **and** Schw. + Bsch. = Schw.—Gld. **then** Fr. + Bsch. = Fr. + (?)

Another example concerns kashrut. Our initial data are as follows. It is forbidden to cook flesh in milk and to eat the cooked (Mfl.). Also, each pleasure from the cooked is forbidden, not only the pleasure from eating (vbt.N.R.). Not only cooking, but also preparing flesh in milk is prohibited (vbt.Zbr.). The first fruits (i.e. fruits of tree that is not older than three years) cannot be eaten (Bfr.). Symbolically:

Bfr.—vbt.Zbr. = Bfr. + vbt.N.R. :: Mfl. + vbt.Zbr. = Bfr. + vbt.N.R.

First fruits are prohibited since the beginning (v. Abg., *von Anbeginn*):

Bfr.—vbt.Zbr. + v.Abg. = Bfr. + vbt.N.R. ::
Mfl. + vbt.Zbr.—v.Abg. = Mfl. + (?)

The leaven (*Brod*) on Passover (Gs.Ost., *Ostertagen*) is forbidden:

Bfr.—vbt.Zbr. + v.Abg. = Bfr. + vbt.N.R. **and** Gs.Ost. + vbt.Zbr.—v.Abg. = Gs.Ost. + vbt.N.R. **then** Mfl. + vbt.Zbr.—v.Abg. = Mfl. + vbt.N.R. :: Mfl. + vbt.Zbr.—v.Abg. = Mfl. + (?)

To sum up, we see that the more signs are being used in equations, the more precise information is being investigated and formalized. Hirsch Hirschfeld was one of the first rabbis and scholars who proposed to formalize the Torah by assuming that the halakhic exegesis is analytic a posteriori knowledge. In his opinion, all Judaic inference rules can be formalized by initial equations (1) and (3). To exemplify Hirschfeld's approach for formalizing the inference rules, let us quote his way of explication of *qal wa-ḥomer*:

> Gesetze und Gebräuche stehen aber auch zuweilen, ohne dass sie mit einander gleich sind, in ihren Bestimmungen in einem gewissen Verhältniss zu einander. Die bei dem einen ausgesprochenen Bestimmungen werden auf ein anderes übertragen; weil dieses in einem gewissen Gegensätze zu dem ändern sich befindet. Es kann nämlich der eine Gegenstand oder das eine Verhältniss eine Eigenschaft oder eine Eigenthümlichkeit haben, die es für eine Bestimmung, die anderwärts ausgesprochen ist, um so mehr berechtigt. Durch die Schrift theils, theils durch eine verständige Ansicht zeigt es sich nämlich, dass eine Gesetzbestimmung, wenn eine gewisse Berechtigung sich vorfindet, anzuwenden sei. Es wird nämlich geschlossen, wenn diese Gesetzbestimmung ohne die Eigenthümlichkeit, Eigenschaften oder Bedingungen, die sie zu veranlassen berechtigt wären, ihre Anwendung hat, um so viel eher musste sie dort, wo dieselben sich vorfinden, statt haben. Die Formel lautet:

$A = o—\alpha = A + x :: B = o + \alpha = B + x + (?)$.

Ausser dem α nämlich wird A und B für sich gegen das x indifferent gehalten, gleich Null. Das also, was in A das x veranlasst, ist das Allgemeinste, das jeden zukömmlich ist, ist nichts Besonderes, und darum nichts Hervorragendes. Es haftet an dem A keine Eigenthümlichkeit dem B gegenüber, die in Bezug auf x Werth hätte, und es ist darum gleich Null. Dagegen hat das B eine Eigenthümlichkeit oder Eigenschaft, die dem A abgebet, und in Beziehung auf x von Bedeutung ist. Das B muss daher gewiss besitzen das x und noch weit mehr, was wir auch mit dem Fragezeichen vermerkt haben. Da A und B an und für sich gegen das x keine Bedeutung haben und beide gleich Null sind, so werden wir in Zukunft immer die Formel stellen $A—\alpha$ für $A = o—\alpha$ und eben so für $B = o + \alpha$ bloss $B + \alpha$; weil nämlich A, B etc. blosse Namen sind, und für sich keinen Werth haben.

Das Wesen dieses Schlusses bestehet darin, dass man einem jeden Gegenstände und einem jeden Verhältnisse eine seiner Eigenthümlichkeit und seinen Bedingungen entsprechende Gesetzbestimmung vindiciren will. Es liegt dieses in der Natur einer jeden Sache; denn nach den Berechtigungen, die ein jeder Gegenstand hat, sind auch seine Ansprüche. Wenn daher von zweien Gegenständen oder Verhältnissen das eine berechtigter ist für eine Gesetzbestimmung, so hat es auch mehr Ansprüche darauf. Diese Gesetzbestimmung muss darum in sich Etwas haben, wodurch sie bald mehr bald minder für einen Gegenstand geeignet erscheint. Sie muss nämlich entweder erschwerender Art sein, wodurch sie einem ohnehin wichtigen Gegenstand oder wichtigem Verhältniss zukömmlich ist, oder erleichternder Art, wodurch sie sich eher für minder wichtige Dinge eignet. Wenn daher von zweien Gegenständen oder Verhältnissen das eine minder wichtig erscheint, als das andere, und gleichwohl eine Bestimmung erschwerender Art hat, so muss dieses um so eher diese Bestimmung theilen; und eben so umgekehrt. Dieser Schluss beruhet demnach auf dem nicht correspendirenden Verhältniss zwischen der in der Schrift bezeichneten Bestimmung und der sich vorfindendes Eigenthümlichkeit oder Bedingung; mit einem Worte: auf dein Gegensatz zwischen leicht and schwer. Er heisst daher auch in

der talmudischen Sprache קל וחומר „leicht und schwer“. Entweder nämlich ist die Bestimmung zu schwer für die Eigenthümlichkeit, oder die Eigenthümlichkeit zu schwer für die Bestimmung. Immer aber constatirt, dass dasjenige, bei welchem in der Bibel die Bestimmung verzeichnet ist, Leichtes und Schweres in sich vereinigt. Da קל das kleinere Wort ist, so wird es in der Verbindung dieser beiden Wörter vorgesetzt.
Wir betrachten nun die einzelnen Glieder des Schlusses, und geben ihnen die gebührlichen Benennungen. Der Schluss bestehet aus zweien Gegenständen oder zweien Verhältnissen, von deren einem die Bibel eine Gesetzbestimmung ausgesagt hat, die dem ändern vindicirt werden soll. Der erste heisst darum מלמד „Lehrer“, der andere למד „Lehrling“ „Schüler“. Diese beiden Glieder stehen in einem gegensätzlichen Verhältniss zu einander, welches die rabbinischen Commentatoren נגוד „Gegensatz“ nennen. Der Gegensatz bestehet darin, dass eine Eigenschaft oder Bedingung mit der Gesetzbestimmung in einem Causalnexus stehet, und bei dem למד sich vorfindet, bei dem מלמד nicht. Der Gegenstand oder das Verhältniss, bei dem sich die Eigentümlichkeit vorfindet, wird חומר „das Gewichtige“ genannt, weil sie den Gegenstand oder das Verhältniss gewichtiger und berechtigter macht für die fragliche Bestimmung. Die Gesetzbestimmung selbst, die deducirt werden soll, heisst דין „Gesetz“. Dort, wo sie sich vorfindet, heisst sie נדון „das Abzuleitende", und wohin sie übertragen werden soll, בא מין הדין „das aus dem Schlüsse Hervorgehende“. Streng genommen, wenn man die Gegenstände, die an und für sich doch keine Bedeutung haben, nicht beachtete sind die Theile, die den eigentlichen Schluss bedingen, nur zwei, nämlich: die sich vorfindende Gesetzbestimmung, und die Eigenthümlichkeit, die sie herbeizuführen berechtigt ist; denn die Eigenthümlichkeit findet sich an dem einen nicht vor, und die Gesetzbestimmung s o l l erst auf den ändern übertragen werden. Das zweite heisst, von diesem Standpunct aus, סוף דינא „das Ende des Schlusses“, weil es an dem Gliede sich vorfindet, das in dem Schlusse später auftritt, und das zu deducirende ist; das erste תחילת דינא „Anfang des Schlusses“.—Wir werden, im Laufe dieser Abhandlung, den Gegenstand oder das Verhältniss sammt

seiner Eigenthümlichkeit, von dem die Bestimmung abgeleitet werden soll, den Vordersatz, das andere den Hintersatz nennen; die zu deducirende Gesetzbestimmung den Mittelsatz, und die übertragene und deducirte den Schlusssatz.—Die obigen Buchstaben damit benannt, so ist $A = o$—α מלמד, Vordersatz, תחילת דינא; $B = o + \alpha$ למד, Hintersatz, סוף דינא; $A + x$ נדון, Mittelsatz; $B + x + (?)$ דין, Schlusssatz; die Beziehung des Vorder- zum Hintersatz, das Verhältniss.
Wir wiederholen die Form der Deutlichkeit wegen in anderer Gestalt:
$A = B :: A$—$\alpha = A + x :: B + \alpha = B + x + (?)$.

Vorder-,	Mittel-,	Hinter-,	Schlusssatz
מלמד	**נדון**	**למד**	**דין**
נגוד Verhältniss		**חומר**	**בא מין דין**
תחלת דין oder	**עיקר**	**סוף דין**	

(*Hirschfeld. 1840*).

Laws and habits stand also sometimes, without that they are equal, in their definitions in a certain relation between each other. These definitions expressed in a relation are transferred to another, because this comes out in a certain opposition to another. Namely one object or relation have a property or peculiarity, which is more applied for a definition that is elsewhere expressed. What is said through the Holy Scriptures and by an understanding view shows that a definition of law, if one finds some justification, is to be applied. Indeed, it is deduced that if this definition of law has its application without a peculiarity, property or condition that this definition would justify to cause, more likely these must appear anywhere, where there are themselves. The formula is:

$A = o$—$\alpha = A + x :: B = o + \alpha = B + x + (?)$.

Out of α, both A and B, considered indifferent in respect to x, are equal to zero. So that what causes x in A is the most general that is suitable for anyone is nothing special, and therefore nothing outstanding. It relates to A no peculiarity compared to B, which in respect to x would have a value, and it is therefore

equal to zero. In contrast, *B* has a peculiarity or property that is given to *A* and in relation to *x* it is of importance. *B* must have *x* therefore certainly and much more, which we denoted by question mark also. Since *A* and *B* have no meaning in and for themselves in respect to *x* and both are equal to zero, we will set always in the future the formula $A—\alpha$ for $A = o—\alpha$ and the same for $B = o + \alpha$ only $B + \alpha$; namely because *A*, *B* etc. are mere names, and have no value for themselves.

The essence of this conclusion is in that one wants to apply the law definition for any object and any relation, its peculiarity and its conditions. This lies in the nature of each thing, because according to possibilities to apply that each object has they bacome their claims. Therefore, if two objects or conditions has more possibilities to apply for the law definition, it also has more claims for it. This law definition must therefore have something in itself that allows them to appear sometimes more, sometimes less suitable for an object. They must be either the kind of more heavy cases, making them always suitable for an important object or for an important relation, or the kind of more light cases, making them more suitable for less important things. Therefore, if two objects or conditions of the less importance appear, when something other has however a definition of the kind of more heavy cases, it must share that definition as well, and vice versa. This conclusion is therefore based on an incommunicable relation between the definition specified in the Holy Scriptures and the peculiarity or condition found; in one word: on the opposition between light and heavy. It is said, hence, in the Talmudic language קל וחומר "light and heavy." Namely, the definition is either too heavy for the peculiarity, or the peculiarity is too heavy for the definition. Always, nevertheless, it is ascertained that the same what has a definition in the Bible, contains both light and heavy too. Since the word קל is smaller, it is put before the conjunction of these two words.

We now consider some members of the conclusion, and give them proper names. The conclusion consists of two objects or two conditions, about one of both the Bible states a law definition that will ascribe to another. The first is called מלמד "teacher" and the other למד "apprentice," "student." These two terms are in an opposing relation to each other, which the

Rabbinic commentators call נגוד "opposition." The opposition is that a property or condition with the law definition stands in a causal nexus, where למד takes place and מלמד does not. The object or relation, where there is the peculiarity, is called חומר "heavy," because the peculiarity is more heavy and harder than the object or the relation for the debatable definition. The law definition as a whole that is to be deduced is called דין "law." Anywhere, where there is it, it is called נדון "the derivable," and places where it is transmitted to, בא מין הדין "what follows the conclusion." Strictly, if the objects that have in and for themselves no meaning are not concerned, they are parts that determine the actual conclusion, only two, namely: the law definition found and the peculiarity that can cause them; then there is no peculiarity and the law definition must be transmitted to another. The second is called, from this standpoint, סוף דינא "the end of conclusion," because it occurs at the member that appears later in the conclusion and it is to deduce; the first תחילת דינא "the beginning of conclusion."—We will call in the course of this treatise the object or the relation together with its peculiarity, which the definition is to be derived from, the before-proposition, the second the behind-proposition; what is to deduce the law definition the middle-proposition, and the transmitted and deduced the conclusion.—If to use the above letter, $A = o$—α מלמד, before-proposition, תחילת דינא; $B = o + \alpha$ למד, behind-proposition, סוף דינא; $A + x$ נדון, middle-proposition; $B + x +$ (?) דין, conclusion; the link between before- and behind-proposition, the relation.

We repeat the form for clarity in another way:

$A = B :: A$—$\alpha = A + x :: B + \alpha = B + x +$ (?).

before-	middle-	behind-	conclusion
מלמד	**נדון**	**למד**	**דין**
נגוד relation		**חומר**	**בא מין דין**
תחלת דין or	**עיקר**		**סוף דין**

2. The Judaic proof of God's existence

As we see, Judaic logic is well developed and can be formalized as a kind of formal system where knowledge is regarded as the analytic a posteriori. Has this logic ever been used for proving God's existence?

In Christianity many arguments for the existence of God have been proposed since the Church Fathers. The majority of them were studied by Thomas Aquinas, O.P., in his *Summa Theologia*. Following him let us quote some arguments:

> I answer that it can be proved in five ways that God exists.
> The first and plainest is the method that proceeds from the point of view of motion. It is certain and in accord with experience, that things on earth undergo change. Now, everything that is moved is moved by something; nothing, indeed, is changed, except it is changed to something which it is in potentiality. Moreover, anything moves in accordance with something actually existing; change itself, is nothing else than to bring forth something from potentiality into actuality. Now, nothing can be brought from potentiality to actual existence except through something actually existing: thus heat in action, as fire, makes fire-wood, which is hot in potentiality, to be hot actually, and through this process, changes itself. The same thing cannot at the same time be actually and potentially the same thing, but only in regard to different things. What is actually hot cannot be at the same time potentially hot, but it is possible for it at the same time to be potentially cold. It is impossible, then, that anything should be both mover and the thing moved, in regard to the same thing and in the same way, or that it should move itself. Everything, therefore, is moved by something else. If, then, that by which it is moved, is also moved, this must be moved by something still different, and this, again, by something else. But this process cannot go on to infinity because there would not be any first mover, nor, because of this fact, anything else in motion, as the succeeding things would not move except because of what is moved by the first mover, just as a stick is not moved except through what is

moved from the hand. Therefore it is necessary to go back to some first mover, which is itself moved by nothing—and this all men know as God.

The second proof is from the nature of the efficient cause. We find in our experience that there is a chain of causes: nor is it found possible for anything to be the efficient cause of itself, since it would have to exist before itself, which is impossible. Nor in the case of efficient causes can the chain go back indefinitely, because in all chains of efficient causes, the first is the cause of the middle, and these of the last, whether they be one or many. If the cause is removed, the effect is removed. Hence if there is not a first cause, there will not be a last, nor a middle. But if the chain were to go back infinitely, there would be no first cause, and thus no ultimate effect, nor middle causes, which is admittedly false. Hence we must presuppose some first efficient cause—which all call God.

The third proof is taken from the natures of the merely possible and necessary. We find that certain things either may or may not exist, since they are found to come into being and be destroyed, and in consequence potentially, either existent or non-existent. But it is impossible for all things that are of this character to exist eternally, because what may not exist, at length will not. If, then, all things were merely possible (mere accidents), eventually nothing among things would exist. If this is true, even now there would be nothing, because what does not exist, does not take its beginning except through something that does exist. If then nothing existed, it would be impossible for anything to begin, and there would now be nothing existing, which is admittedly false. Hence not all things are mere accidents, but there must be one necessarily existing being. Now every necessary thing either has a cause of its necessary existence, or has not. In the case of necessary things that have a cause for their necessary existence, the chain of causes cannot go back infinitely, just as not in the case of efficient causes, as proved. Hence there must be presupposed something necessarily existing through its own nature, not having a cause elsewhere but being itself the cause of the necessary existence of other things—which all call God.

The fourth proof arises from the degrees that are found in things. For there is found a greater and a less degree of

goodness, truth, nobility, and the like. But more or less are terms spoken of various things as they approach in diverse ways toward something that is the greatest, just as in the case of hotter (more hot) which approaches nearer the greatest heat. There exists therefore something that is the truest, and best, and most noble, and in consequence, the greatest being. For what are the greatest truths are the greatest beings, as is said in the *Metaphysics* Bk. II. 2. What moreover is the greatest in its way, in another way is the cause of all things of its own kind (or genus); thus fire, which is the greatest heat, is the cause of all heat, as is said in the same book (cf. Plato and Aristotle). Therefore there exists something that is the cause of the existence of all things and of the goodness and of every perfection whatsoever—and this we call God.

The fifth proof arises from the ordering of things for we see that some things which lack reason, such as natural bodies, are operated in accordance with a plan. It appears from this that they are operated always or the more frequently in this same way the closer they follow what is the Highest; whence it is clear that they do not arrive at the result by chance but because of a purpose. The things, moreover, that do not have intelligence do not tend toward a result unless directed by some one knowing and intelligent; just as an arrow is sent by an archer. Therefore there is something intelligent by which all natural things are arranged in accordance with a plan—and this we call God.

In response to the first objection, then, I reply what Augustine says; that since God is entirely good, He would permit evil to exist in His works only if He were so good and omnipotent that He might bring forth good even from the evil. It therefore pertains to the infinite goodness of God that he permits evil to exist and from this brings forth good.

My reply to the second objection is that since nature is ordered in accordance with some defined purpose by the direction of some superior agent, those things that spring from nature must be dependent upon God, just as upon a first cause. Likewise, what springs from a proposition must be traceable to some higher cause which is not the human reason or will, because this is changeable and defective and everything changeable and liable to non-existence is dependent upon some unchangeable

> first principle that is necessarily self-existent as has been shown (*Thatcher*).

In the Medieval Hebrew philosophy many such arguments were adopted and reinterpreted. For instance, Maimonides writes:

> Our faith begins with the traditions that have been transmitted to us by our ancestors and in our sacred literature. This is alluded to in the verse, 'Hear O Israel, God is our Lord, God is one.' However, the highest level of faith comes from philosophical proof, and those who have the ability are required to prove the foundations of our faith (*Moreh Nevuchim* 3:51).

Nevertheless, in the way of Thomas Aquinas Judaic sages have never proposed own proofs. In order to see why, let us remind Kant's objections against philosophical arguments for the existence of the Lord:

> The ontological argument aims at asserting the possibility of conceiving the idea of an ens realissimum, of being possessed of all reality. But the idea of existence and the fact of existence are two very different things. Whatever I conceive, or sensibly imagine, I necessarily conceive as though it were existing. Though my pocket be empty, I may conceive it to contain a 'hundred thalers.' If I conceive them there, I can only conceive them as actually existing there. But, alas, the fact that I am under this necessity of so conceiving by no means carries with it a necessity that the coins should really be in my pocket. That can only be determined by experience.
>
> The cosmological argument contends that if anything exists, there must also exist an absolutely necessary being. Now, at least I myself exist. Hence there exists an absolutely necessary being. The argument coincides with that by which the thesis of the fourth antinomy is supposed. The objections to it are summed up in the proof of the antithesis of the fourth antinomy. As soon as we have recognized the true conception of causality, we have already transcended the sensible world.

> The physico-theological or teleological argument is what is often styled the argument from design. It proceeds not from general, but particular experience. Nature discloses manifold signs of wise intention and harmonious order, and these are held to betoken a divine designer. This argument deserves always to be treated with respect. It is the oldest and clearest of all proofs, and best adapted to convince the reason of the mass of mankind. It animates us in our study of nature. And it were not only a cheerless, but an altogether vain task to attempt to detract from the persuasive authority of this proof. There is naught to urge against its rationality and its utility.
> All arguments, however, to prove the existence of God must, in order to be theoretically valid, start from specifically and exclusively sensible or phenomenal data, must employ only the conceptions of pure physical science, and must end with demonstrating in sensible experience an object congruous with, or corresponding to, the idea of God. But this requirement cannot be met, for, scientifically speaking, the existence of an absolutely necessary God cannot be either proved or disproved. Hence room is left for faith in any moral proofs that may present themselves to us, apart from science. With this subject ethics, the science of practice or of practical reason, will have to deal (*Kant*).

Kant maintains that it is impossible to prove the divine existence from the standpoint of empirical sciences. Does it mean that the Lord does not exist? According to Kant, the Lord is an indemonstrable entity, the ultimate end of the pure use of reason. He is the synthetic a priori that is already out of the phenomenal world, therefore our reasoning about Him faces limits of pure reason. In other words, He exists as ultimate pragmatic limits of our thought.

In Judaism there is an own proof of God's existence said to be a pragmatic argument, which is close to Kant's approach. This argument is expressed in the following motto:

> קודשא בריך הוא ואורייתא וישראל חד הוא

The Lord and the Torah and Israel are all one

This statement comes from the *Zohar*, the best known Kabbalistic book:

> ג' דרגין אינון מתקשרן דא בדא קב"ה אורייתא וישראל וכל חד דרגא על דרגא סתים וגליא קב"ה דרגא על דרגא סתים וגליא. אורייתא הכי נמי סתים וגליא ישראל הכי נמי דרגא על דרגא הה"ד מגיד דבריו ליעקב חקיו ומשפטיו לישראל תרין דרגין אינון יעקב וישראל חד גליא וחד סתים
>
> There are three levels bound together—the Holy One, blessed be He, the Torah and [the people of] Israel. Each of them [comprises] level upon level, hidden and revealed. The Holy One, blessed be He, level upon level, is hidden and revealed. The Torah, too, is hidden and revealed. And so too Israel is level upon level, as it is written, 'He tells His words [of Torah] to Jacob, His statutes and ordinances to Israel' [*Psalms* 147:19]. There are two levels [of the Jewish people mentioned in the verse]: Jacob and Israel. One [Jacob] is revealed and the other [Israel] is hidden (*Zohar* vol. III, 73a).

According to the motto mentioned above there is only our everyday life, the life-world (*die Lebenswelt*) that is organized by some pragmatic rules, i.e. by everyday reasoning (including all nonverbal aspects of speech acts) which fully influences our behavior. Not any reasoning belongs to a complex of pragmatic rules. This complex may be regarded as everyday logic that defines the higher reality for us. All senses, meanings, evidences of the life-world are implicitly or explicitly defined by this logic.

Judaic logic contains different sophisticated circuits of massive-parallel reasoning. According to Judaism, this reasoning is an outcome of revelation, this has not been thought up by human beings. For preserving Judaic reasoning there is a Judaic religious practice setting Jewish everyday life. Thanks to this practice Judaic logic makes the pragmatic complex defining the life-world of Jewish people which follow the Torah and Judaic hermeneutics. As a result, Judaic logic becomes the logic of the life-world for the

Jewish people, i.e. the logic of the higher evidence, incorporated into behaviors of Jews.

Therefore the *Lord* (the higher evidence, i.e. the pragmatic limits of everyday practice), the *Torah* (the religious doctrine, i.e. an appropriate reasoning and speech acts) and *Israel* (an appropriate everyday practice) *are all one* within pragmatic limits of the everyday life. Judaic logic, as we see, may be understood as one of the bright versions of everyday logic.

Hence, the Lord exists as pragmatic limits of the life-world. Beyond the life-world there is no God, there is no meaning at all:

> For the grave cannot praise thee, death can not celebrate thee: they that go down into the pit cannot hope for thy truth. The living, the living, he shall praise thee, as I do this day: the father to the children shall make known thy truth (*Isaiah* 38:18—19).

While Kant claims that the Lord is a transcendental ideal of synthetic a priori reasoning, we could claim that in Judaism the Lord is the widest pragmatic limits of analytic a posteriori reasoning. Notice that the analytic a posteriori takes place in our knowledge when, on the one hand, it is inferable (analytic) and, on the other hand, has everyday meaning within limits of our everyday life (a posteriori). God exists just in our existences within the life-world. Søren Aabye Kierkegaard was right. The Lord is the highest analytic a posteriori.

3. Theology of science: Christianity and Judaism

In philosophy of science, many aspects of science such as epistemological and metaphysical assumptions, logical foundations and methods, implications in society are regarded. However, there are many other philosophical aspects of sciece that have been ignored still, among them religious presuppositions of scientists that can be explicated, as well. In order to focus on some new philosophical ideas in science that have been out of analysis, Michał

Kazimierz Heller, the Catholic priest, physicist and philosopher of science, proposed a new interdicsiplinary branch first of all combining philosophy of science and theology, the so-called theology of science, where the physical world studied within science is concerned in the theological perspective, also.

> The fundamental theological truth concerning the universe is the statement that establishes its creation by God. Is necessary to state right away the tautology that for theologians the universe is all of that which has been created by God. Clearly the universe of science is only a part of the universe of theology. This is true not only in the sense that the universe of science contains only that which is defined as the "material world," while the universe of theology goes beyond the domain of the material, but also in the sense that theology can pronounce statements about the "material world" which go beyond the empirical method, that is, which do not pertain to the universe of science. Thus, the material world in the theological perspective is, in a sense, richer than the material world as seen in the perspective of the natural sciences. It is precisely here that the possibility arises for a theology of science. As a reflection of the sciences the theology of science would study the consequences of the fact that the natural sciences explore the world created by God.
>
> Above all else the theology of science should be an integral part o f theology and be a part of its peculiar character. If it is correct that science is a great value for humanity, then the theology of science should be considered as a subheading of the theology of earthly values. A specific function of the subheading would be, among other matters, to have frequent recourse to the results of the philosophy of science. The philosophy of science, as we have said, examines the boundaries of the possibilities of the natural sciences, but by its very nature it cannot go beyond them. On the contrary, theology pushes on beyond those boundaries. The awareness that the world has been created by God permits the theology of science to glimpse aspects and dimensions of the world which cannot be reached by modern empirical investigations. In other words the theology of science can look at the boundaries of the

> empirical domain and, therefore, at the boundaries of the natural sciences, by putting itself in a certain sense on the other side, that is, on the side not accessible to the sciences themselves (Heller, *The New Physics and a New Theology*).

The fact is that some scientific ideas and hypotheses in natural sciences may be treated as theological, indeed. Let us consider an example. Michał Kazimierz Heller claims that mathematics is a science of structure aimed to elicit hidden structures of reality:

> No serious contemporary mathematician will agree with a claim that mathematics is a science of quantity. It is [rather] a science of deduction ..., it is more and more frequently formulated as a science of structure, of how particular elements of same structures can be deduced from others, or how these structures are related to one another by means of various relations of inference. If one looks at mathematics in this light and applies it to explore the world, then ... it will really be excavating ... hidden structures of reality, penetrating the deep structure of the world which the "naked eye" is unable to see ... What physics truly encloses is much broader than what is traditionally understood as quantity (Heller, *Philosophy is an adventure of a man who is on his way*).

This claim of Heller is based on several assumptions, among which the most significant are that, on the one hand, there should exist only one mathematics and, on the other hand, this mathematics can fully explore the physical world. Both key assumptions are coming out in believing that it can be invented a theory of everything (or final theory), a theory of theoretical physics that links together all known physical phenomena (e.g. phenomena of general relativity and quantum mechanics) and predicts the outcome of any physical experiment.

Let us suppose that the theory of everything is obtained. Obviously, then it is the only true mathematical theory that is completely applied for explaining, discovering and predicting all the

physical phenomena. Heller believes that this mathematical theory exists in fact, but did not come down still, not invented. If the final theory existed, it could be evaluated from the theological viewpoint as the divine Logos that has been active in creation, revelation, and redemption since the very beginning:

> τῷ λόγῳ τοῦ κυρίου οἱ οὐρανοὶ ἐστερεώθησαν καὶ τῷ πνεύματι τοῦ στόματος αὐτοῦ πᾶσα ἡ δύναμις αὐτῶν
>
> By the word [logos] of the Lord were the heavens made; and all the host [dynamis] of them by the breath [pneuma] of his mouth (*Psalm* 33:6).
>
> In the beginning, was the Word; and the Word was with God; and the Word was God [*Greek*: θεὸς ἦν ὁ λόγος, *Aramaic*: w-'lh' 'ythy hw' hw mlt] (*John* 1:1).

The divine Logos manifests himself as Sophia, the divine wisdom, the theory of everything. According to Sergei Bulgakov, the Russian philosopher, economist, theologian, God creates the world by Logos in Sophia and in its sophianic foundation the world is divine (*Bulgakov*). Vladimir Solovyov, another Russian philosopher and theologian, maintains that in the divine organism of Christ, the first acting, unifying principle that expresses the unity of that is the Word or Logos and the second principle producing the unity is called Sophia, God's body, the matter of Divinity, permeated with the principle of divine unity (Soloviev, *Lectures on Divine Humanity*).

As we see, the physical hypothesis that there may exist the final theory has a theological treatement and in case this hypothesis is true, it is possible to enunciate that the final theory in physics is a divine Logos and Sophia creating the world.

Hence, the religious thought contains plenty of scientific and philosophical insights and presuppositions. Though these intuitive statements are formulated in the prereflexive form, they have been

used as a heuristic potential in scientific researches till now. Different religions represent various "thread balls" of logical, philosophical, scientific ideas in the twisted non-discursive form. These balls might be compared to a single point before the Big Bang and the Universe expansion. In these balls such a density of meanings could be detected that allows then to cultivate cultures and civilizations only due to the interpretation and untwisting of those first meanings (i.e. due to their "expansion"). It is possible to believe that this density of meanings could originate only as a result of revelation in real.

For example, in Christianity one could find out a source of ideas of totalitarian metaphysics and holistic viewpoint (in Vladimir Solovyov's words, the philosophy of total unity) that is followed by Michał Kazimierz Heller, also. These totalitarian assumptions are caused by the assertion of necessity of eliminating ethnic, religious and social differences towards the uniting belief in Christ:

> Where there is neither Greek nor Jew, circumcision nor uncircumcision, Barbarian, Scythian, bond nor free: but Christ is all, and in all (*Colos.* 3:11, *The King James Version*).

> Where there is neither Jew nor Gentile [Aramean], neither circumcision nor uncircumcision, neither Greek nor barbarian, neither bond nor free; but the Messiah is all, and in all (*Colos.* 3:11, *Murdock translation of Peshitta*).

The metaphysics of total unity plays now an appreciable role in natural sciences, synergetics, ecology. In particular, science adapted ideas of Christian totalitarian and holistic standpoint thanks to Russian cosmism, the philosophical movement that emerged in Russia in the early 20th century. Its main representatives are as follows: Nikolai Fyodorov (1827—1903, who originated this movement), Konstantin Tsiolkovsky (1857—1935, the father of Soviet cosmonautics), Vladimir Vernadsky (1863—1945, the pioneer of biogeochemistry and radiogeology), Alexander

Chizhevsky (1897—1964, the founder of heliobiology, astrobiology, historiometry). All of them fairly refered to religious presuppositiuons in their scientific investigations, first of all to Orthodox Christianity. For instance, Alexander Chizhevsky as specialist in the study of the Sun's relationship to life (e.g. he showed the influence of the eleven year solar cycles upon human mass behavior) follows religious assumptions of the ancient Chaldean and Hebrew astrologers who supposed influence of celestial bodies upon human affairs.

In the movement of Russian cosmism, anthropic elements of religion and ethics with a natural philosophy of the origin, evolution and future existence of the cosmos were being united. Cosmism as philosophy of active evolutionism was influenced by some ideas of the Russian Orthodox Church. In this philosophy one has been speaking of the possibility and necessity for the human mind to regulate and transform the laws of nature. For example, Nikolai Fyodorov wrote about the universal resurrection of the dead, physical immortality, and the technological transformation of the cosmos. For Russian philosophers the main idea is that the world is an absolute in the process leading to renewal of the unity of all after fracturing in the first days of creation.

The Christian roots of totalitarian metaphysics can be detected as well in a universal evolution, a theory formulated by Pierre Teilhard de Chardin (1881—1955, a French philosopher and Jesuit priest) and Julian Huxley (1887—1975, an English evolutionary biologist and humanist) that was aimed to describe the gradual development of the Universe from subatomic particles to human society. Notice that Teilhard de Chardin was directly influenced by Vladimir Vernadsky's ideas contributed to Gaia theory. Teilhard de Chardin (*1959*) tried to integrate Darwinian evolution and Christianity and to foreshow the further global evolution toward the noosphere (the society with the global nature reorganised by using human mind) in which the biosphere is being

transformed just as the biosphere was a result of geosphere's transformations.

In modern physics the Christian standpoint of total unity first of all comes out in the anthropic principle, the statement that the conditions of the physical Universe must allow the carbon life and the observer to exist. This principle was first formulated by Brandon Carter, the Australian theoretical physicist (born 1942):

> we must be prepared to take account of the fact that our location in the universe is *necessarily* privileged to the extent of being compatible with our existence as observers (*Carter 1974*).

According to these ideas, there is a total unity combining the fundamental laws of physics, the carbon life and the human beings as observers.

The totalitarian and holistic thought inspired by Christianity has been causing the use of Aristotelian deductive syllogistics as the main tool of argumentation both in the Christian catechization and theology and in the West culture in general. In Aristotle's syllogistics we are moving from general knowledge to particular conclusions. For example, in the following syllogism

> All men are mortal,
> Socrates is a man;
> therefore Socrates is mortal

we are concluding the facts, which can be verified by experience, from the less obvious general knowledge. Indeed, 'Socrates is a man' and 'Socrates is mortal' are singular facts which we can know from the direct observation or from testimonies borne by reliable witnesses. However, the statement that all people are mortal cannot be verified on experience. What is its truth then grounded on? Is that all people we knew did not live longer than 120 years? But we know not all people and on a broader scale it is a popular induction of type 'all swans are white' and 'all ravens are black.'

The logical pattern used in deductive syllogisms is represented by modus ponens in the most general view:

> If he is a man, he is mortal;
> He is a man,
> hence, he is mortal.

For the first time Karl Popper (*1934*) paid attention that there is an asymmetry between verification (modus ponens) and falsification (modus tollens): no general (empirical) statement can be proved or even definitively confirmed, but only it can be falsified. Thus, according to Popper giving priority to confirmation before falsification is not grounded logically. On the contrary, in scientific theories we should prefer modus tollens, instead of modus ponens. Nevertheless, in the totalitarian thought we detect the priority of confirmation (modus ponens) and deductive syllogism, this priority undoubtedly speaks of appropriate prereflexive presuppositions.

Nonrandomly, in totalitarian cultures schemata of confirmation and deductive syllogisms are preferred, because in these cultures thinking is totalitarian and holistic. For instance, in the Soviet Union of Stalin's epoch the following modus ponens was often used:

> Enemies of Soviet people must be taken into camp,
> Non-content with Stalin's policy are enemies of Soviet people;
> Therefore, non-content with Stalin's policy must be taken into camp.

Premisses of syllogisms in the political discourse of the Soviet Union were compiled in the communistic ideology.

Thus, the Christian culture has endowed the scientific thought with ability to be holistic. An absolutely other picture is observed in the Judaic culture. While the Christian thinking is totalitarian and holistic, the Judaic thinking is massive-parallel and concurrent, it prefers a singular differentiation. For example, in

Judaism there is differentiating between those animals which may be eaten and those which are unclean, between things devoted to the Temple and not devoted, etc. This feature of the Judaic thinking has entailed the impossibility of using Aristotelian deductive syllogistics and in fact the latter has been never used in Judaism.

Let us notice that besides syllogistics, Aristotle developed another logical system—dialectics or topics in which traductive syllogisms are used (in them output is moving from general to general or from particular to particular). Traductive syllogisms might be exemplified by any mathematical proposition without quantifiers: if A is greater than B and B is greater than C, then A is greater than C. The latter proposition is said to be *topos* in the Aristotelian terminology.

In Judaism very difficult schemata of logical reasoning adequate for massive-parallelism and concurrency of Jewish religious thought have been developed. Recall that the Tannaitic tradition recognises mainly three collections of logical inference rules:

- the seven rules of Hillel;
- the thirteen rules of Rabbi Ishmael (they include the seven rules of Hillel) for inferring new legislative (halakhic) aspects of the Torah;
- the thirty-two rules of Rabbi Eliezer ben Yosi ha-Gelili for inferring new narrative (haggadic) aspects of the Scripture including the Prophets (*Nevi'im*) and the Writings (*Ketuvim*) also.

An important fact is that Maimonides (1135—1204), one of the most authoritative rabbis of all periods, asserted that Jewish legal arguments belong to the sphere of dialectics, in particular the thirteen hermeneutic principles of Rabbi Ishmael by which the Torah is expounded, might be treated as Aristotelian *topoi* and never as deductive syllogisms. Only Karaites suggested to use Aristotelian deductive syllogisms as legal arguments, thus they denied the significance of Judaic logical inference rules.

The point is that in Judaism something like Ecumenical Councils of the Christian Church has never appeared and the plurality of opinions has ever supposed in respect to plenty of subjects and in many cases it has been even in advance. It is possible to act legitimately even if we follow contrary viewpoints such as Shammai and Hillel's opposite viewpoints who ever debated:

> תני רב יחזקאל עשה כדברי בית שמאי עשה כדברי בית הלל עשה
>
> Rav Yeḥezk'el learnt: If one acts in accordance with the opinion of Shammai's party he has acted [legitimately]; if he acts in accordance with the opinion of Hillel's party he has acted [legitimately] (*Berakhot* 11a).

Another example, it is allowed to obey the law either according to Rabbi or according to other sages even in cases both opinions are contrary:

> אמר רבה בר חנא אמר ר' חייא עשה כדברי רבי עשה כדברי חכמים עשה
>
> Rabbah bar Ḥannah said in the name of R. Ḥiyya', "If one acted according to Rabbi he has acted [legitimately]; [If one acted] according to the sages he has acted [legitimately]" (*Baba Bathra* 124a—b).

Whithin the plurality of opinions we cannot use deductive syllogisms as legal arguments, because syllogistics is developed by means of a holistic standpoint that stands in contradiction to Judaic presuppositions of plurality and to dialectics as a whole. The Christian culture inherits the Roman approach to making decision in disputes in respect to legal subjects, when a final decision should be sole:

> The answers of jurists are the decisions and opinions of persons authorized to lay down the law. If they are unanimous their

> decision has the force of law; if they disagree, the judge may follow whichever opinion he chosses, as is ruled by a rescript of the late emperor Hadrian (Gaius, *Institutes,* 1.7).

Adopting the Roman totalitarian viewpoint allowed the Church Fathers to implement the Roman legal approach in organizing the Ecumenical Councils for making ultimate decisions in religious subjects. On contrary, Judaic reasoning has been presented as a massive-parallel and concurrent proof system that is traditionally called dialectics.

Recall that from the viewpoint of modern science the dialectical plurality in the nature is explicated as massive-parallel locally-connected mathematical machines that cannot have a single centralised source exercising precise control over vast numbers of heterogeneous devices (*Adamatzky* et al. *2005*). Interactive-computing paradigm is able to describe concurrent/parallel computations whose configuration may change during the computation and is decentralised as well. Within the framework of this paradigm, one proposed a lot of so-called concurrency calculi also called process algebras or process calculi. They are typically presented using systems of equations. These formalisms for concurrent systems are formal in the sense that they represent systems by expressions and then reason about systems by manipulating the corresponding expressions.

Unlike models of computation with object-oriented semantics, the Judaic logic assumes actor-oriented (concurrency) semantics adequate for decentralised massive-parallel computing. Actor-oriented semantics encompass algorithms used to implement concurrent systems.

The Judaic logic should provide complete sets of possible deduction behaviors of the Talmudic reasoning. For verifying this logic we need to construct a process calculus (process semantics) that could test the soundness and completeness of process-deductive exchange of input data. Evidently, this logic with non-well-founded (massive-parallel and concurrent) proof system can

have unexpected metalogical properties that are not studied yet. The conventional computing is set up in the frameworks of well-founded logics and well-founded proof systems. We can assume that setting up a computation using the obtained behavioral logic and an appropriate process semantics can be provided with new horizons.

In turn, supposing massive parallelism in Judaic reasoning has affected creating an unusual holistic view presented in Kabbalah. According to the Kabbalistic standpoint, the world is a stream of information which has both a cogitative and calculative aspect. The cogitative aspect is presented by meaning and using of Hebrew words and the calculative aspect by arithmetical operations on numerical values of Hebrew letters. In Kabalah's view, only information objectively exists. Any physical process is reduced to massive-parallel Kabbalistic calculations over Hebrew words. Such a point is very close to unconventional computing, modern theories in which any natural processes—biological (*Adamatzky 2007*), chemical (*Berry and Boudol 1992*, *Adamatzky* et al. *2005*), mechanical—might be regarded as a physical substratum of computing. All these processes are interpreted as massive-parallel computations. In unconventional computing one asserted too that only information/computing objectively exists. For example, unconventional substratum of computation can be exemplified by the $\approx 6 \times 10^{13}$ cells of a human body, each one of relatively simple elements, working in parallel to accomplish extremely complex tasks. For such systems there is no operational semantics. The reason is that initial notions of that semantics were developed within the sequential model of computation, then they were extended to concurrency, but not to massive-parallelism.

Unconventional computing may be understood as an alternative to physics. The point is that natural processes are being analyzed in unconventional computing without assuming the existence of universal concepts such as matter, energy, gravitation. Natural processes are observed contextually. If we compare

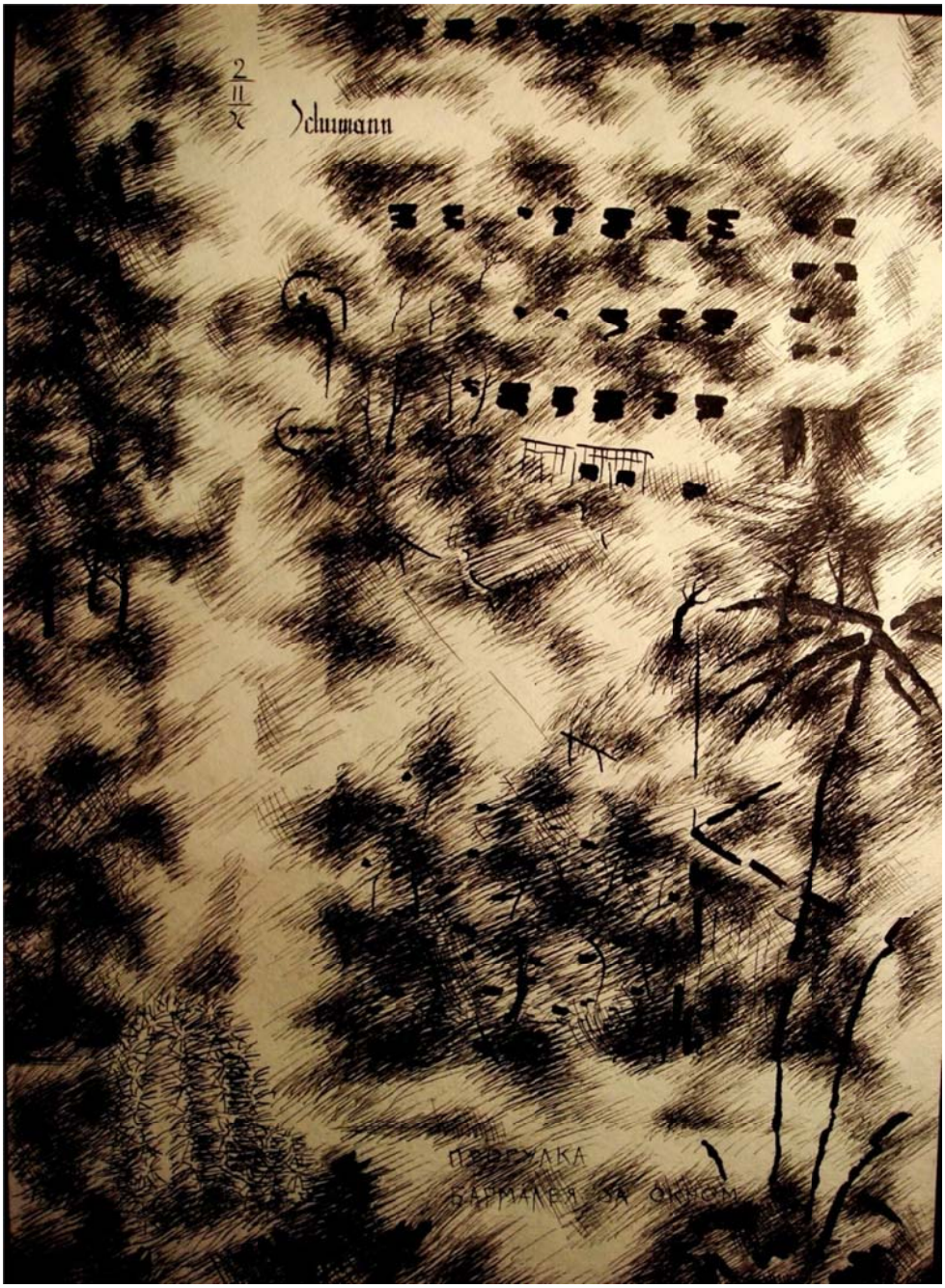
unconventional computing with modern physics, then we can call to mind the analogy of distinction between intuitionistic mathematics and classical mathematics. In the same way as in intuitionism, in unconventional computing, abstract theoretical terms which are not dealing with concrete modelling of physical processes (i.e. not having constructive meaning), are discarded. Otherwise in physics.

Philosophical premises of unconventional computing undoubtedly originate in Kabbalah. It is necessary to add that beginnings of information science, computation theory and mathematical logic ascends through Leibniz and Llull to Kabbalistic ideas. Ramon Llull (1232—1315) (Raymond Lully, Raymundus Lullius) was a Majorcan writer and philosopher who invented the method (called the Lullian Circle) of using logical and combinatorial means to produce knowledge. No doubts that he was influenced by Kabbalah and its calculation tools. Leibniz continued Llull's idea and gave it the name *ars combinatoria*.

Following Kabbalistic presuppositions, Leibniz defined a character as any written sign, speaking of a "real" character as one that represents an idea directly and not simply as the word embodying the idea. Real characters have Hebrew characters (letters) as their prototypes. These real characters form an alphabet of human thought in which each fundamental concept is represented by a unique real character. The calculus in which composition rules on that alphabet and their arithmetic meanings are described is called a *characteristica universalis* or 'universal characteristic.' Complex thoughts would be represented by combining characters for simpler thoughts.

Thus, the analysis of Judaic logic and logical assumptions in Kabbalistic calculations can make a considerable contribution to the development and understanding of logical and computation theories linked with massive-parallel computing.

The unconventional computing paradigm is an alternative to the final theory in physics. While the second should be holistic, the



Picture 7 Bar-Maley's walk behind the window

first pluralistic. Furthermore the second should be built up on the basis of standard mamthematics (e.g. it assumes the conventional approach to algorithms), the first on the basis of non-standard mathematics (e.g. interactive, concurrent and massive-parallel computing). And finally, theological aspects which may be explicated in the unconventional computing paradigm and in the physical theory of everything are too different. We can claim that while the theological dimension of the unconventional computing paradigm is close to Judaism, that of the physical theory of everything is close to Christianity.

To sum up, we notice that Michał Kazimierz Heller introducing the term 'theology of science' thinks of that this theology is rather Christian. Nevertheles, we can detect Judaic theological aspects and roots in some scientific ideas too. This means that theology of science cannot be reduced only to one theological system; it is the study of religious presuppositions in science as such. Theology of science may consider Christian, Judaic as well as many other approaches to the nature and science in the theological perspective. Hence, there cannot be only Christian theology of science as thinks Heller.

4. Pluralistic reasoning in Judaism and Judaic argumentation theory

Due to the thirteen rules of Rabbi Ishmael, the Torah can be studied from the logical point of view well and easily. The hermeneutic approach supported by these rules allows us to develop logical investigations of the Torah, even symbolic-logical ones. However, considering the Torah from the pragmatic point of view is a not-so-easy task. Such a consideration is centered on the linking of practice and theory including theoretical extraction from practice and practical applications of theory. The pragmatic approach should satisfy the so-called pragmatic maxim proposed by Charles Sanders Peirce, according to that we should regard just

effects having practical bearings and the object is being constructed from these effects.

The pragmatic standpoint is most inclusive and contains logical investigations as well. Within this approach we postulate practice as human activity fulfilled by means of speech acts and utterances also, typically involving logical tools, e.g. Rabbi Ishmael's inference rules have been used not only in the Judaic theological argumentation and disputation, but also in Judaic forms of presenting political viewpoints. Pragmatic investigations assume that we are dealing with the effects of context. Therefore Judaic argumentation constructions can vary from utterance to utterance depending upon contexts and situations. The Judaic pragmatics should set up all types of situations that can be faced in everyday's practice including speech activity.

While on the base of Rabbi Ishmael's rules explicitly involved in Judaic reasoning we could reconstruct the Judaic logical standpoint as a whole, it is much more sophisticated to conceive the pragmatic standpoint of Judaism. One of the possible ways to reconstruct the Judaic pragmatic approach may be presented by the Judaic treatment of controversy and political confrontation.

In Judaism the following two basic kinds of controversy are distinguished: 'controversy in the name of heaven' and 'controversy not in the name of heaven.' The first kind is traditionally exemplified by the controversy between Hillel and Shammai and the second kind by the controversy of Korah and his congregation resisting Moses' leadership. The second kind was related against the Torah and the whole Jewish community as such, therefore it is strongly condemned in Judaism.

כל מחלוקת שהיא לשם שמיים סופה להתקיים ושאינה לשם שמיים אין סופה להתקיים איזו היא מחלוקת שהיא לשם שמיים זו מחלוקת הלל ושמאי ושאינה לשם שמיים זו מחלוקת קורח ועדתו

> Every controversy that is in the name of heaven, the end thereof is [destined] to result in something permanent; but one that is not in the name of heaven, the end thereof is not [destined] to result in something permanent. Which is the [kind of] controversy that is in the name of heaven? Such as was the controversy between Hillel and Shammai; and which is the [kind of] controversy that is not in the name of heaven? Such as was the controversy of Korah and all his congregation (*Pirqe Abot* 5:17).

Nevertheless, the opposition between Hillel's party and Shammai's party was not less tense and even sometimes with murderous effects (see the *Talmud Yerushalmi*, *Shabbat* 1, 3, 4), but both were related to supporting an eternal tradition in the same measure. The sages claimed that Hillel and Shammai received the Judaic tradition in its purity from the same source originated at Mount Sinai. This means that their controversy is contained in the Torah itself:

> Hillel and Shammai and their court received the tradition from Shemayah and Avtalion and their court. Rabban Yochanan ben Zakkai and Rabbi Shimon, the son of Hillel the elder, received the tradition from Hillel [and Shammai] and his [their] court[s] (Rambam. *Introduction to the Mishneh Torah*).

The controversy between Hillel and Shammai may be examined as a prototype of all political confrontations that might occur in Judaic communities and might be explicated within the Judaic pragmatic approach. The main feature of the controversy between Hillel and Shammai regarded as significant in Judaism consists in expressing friendship towards one another as a rule:

> שנחלקו ב"ש וב"ה בצרות מודים שאין ממזר אלא ממי שאיסורו איסור ערוה וענוש כרת מאן מודים אילימא ב"ש לב"ה פשיטא בני חייבי לאוין כשרים נינהו אלא ב"ה לב"ש היא גופא חייבי כריתות היא לעולם ב"ש לב"ה ולאפוקי מדר' עקיבא דאמר יש ממזר מחייבי לאוין קמ"ל דאין ממזר מחייבי לאוין ת"ש אע"פ שנחלקו ב"ש וב"ה בצרות ובאחיות בגט ישן

ובספק אשת איש ובמגרש את אשתו ולנה עמו בפונדק בכסף ובשוה כסף בפרוטה ובשוה פרוטה לא נמנעו ב"ש מלישא נשים מבית הלל ולא ב"ה מבית שמאי ללמדך שחיבה וריעות נוהגים זה בזה לקיים מה שנאמר (זכריה ח) האמת והשלום אהבו

Come and hear: Although Beth Shammai and Beth Hillel are in disagreement on the questions of rivals, sisters, an old bill of divorce, a doubtfully married woman, a woman whom her husband had divorced and who stayed with him over the night in an inn, money, valuables, a *perutah* and the value of a *perutah*, Beth Shammai did not, nevertheless, abstain from marrying women of the families of Beth Hillel, nor did Beth Hillel refrain from marrying those of Beth Shammai. This is to teach you that they shewed love and friendship towards one another, thus putting into practice the Scriptural text, Love ye truth and peace [*Zech*. 8:19] (*Yebamoth* 14b).

In Judaism we can find out different political viewpoints from the Judaic left wing supporting liberal causes or even radical anarchism to the Judaic right wing supporting conservative causes or radical nationalism. For instance, on the one hand, the Bible indicates that the pre-monarchic Israelite society was anarchistic in the libertarian meaning: "In those days there was no king in Israel: every man did that which was right in his own eyes" (*Judges* 21:25). As a result, many Hasidic and Kabbalistic authorities have described the future society as libertarian anarchistic community. Thereby they often referred to their contemporary radical thought, e.g. Rabbi Abraham Chen (1878—1958) who belonged to the Chabad movement deeply respected the Russian anarchist thinker Petr Kropotkin, whom he called a 'saint of the new world,' whose 'soul is as pure as crystal.' There were also Tolstoyan rabbis in Russia. All those Judaic leftists pointed out that the Talmud contains clearly anti-authoritarian passages such as "Love labor, hate mastery over others, and avoid a close relationship with the government" (*Pirqe Abot* 1:10). However, it is just the one possible point. On the other hand, the Bible maintains that David was chosen by God as king of the united monarchy and his descendant

should become the Messiah, the king of the future society. This reference shows a potency of rightist ideas and conservative standpoint and states that traditional institutions may have only minimal change in society. The less the world has changed the better it is.

As we see, the Torah may support some leftist ideas as well as some rightist ones, but not any political viewpoints. This means that in applying to politics the Torah has rigid pragmatic limits. For instance, the controversy between Hillel and Shammai's parties did not cross beyond these pragmatic limits, but the controversy between Korah and Moses fell out of limits.

In controversies, the Talmud recommends to share the side that presented by more kindly and modest people as in the case of Hillel and Shammai:

> א"ר אבא אמר שמואל שלש שנים נחלקו ב"ש וב"ה הללו אומרים הלכה כמותנו והללו אומרים הלכה כמותנו יצאה בת קול ואמרה אלו ואלו דברי אלהים חיים הן והלכה כב"ה וכי מאחר שאלו ואלו דברי אלהים חיים מפני מה זכו ב"ה לקבוע הלכה כמותן מפני שנוחין ועלובין היו ושונין דבריהן ודברי ב"ש ולא עוד אלא שמקדימין דברי ב"ש לדבריהן כאותה ששנינו מי שהיה ראשו ורובו בסוכה ושלחנו בתוך הבית בית שמאי פוסלין וב"ה מכשירין
>
> R. Abba stated in the name of Samuel: For three years there was a dispute between Beth Shammai and Beth Hillel, the former asserting, 'The halakhah is in agreement with our views' and the latter contending, 'The halakhah is in agreement with our views'. Then a bath kol issued announcing, '[The utterances of] both are the words of the living God, but the halakhah is in agreement with the rulings of Beth Hillel.' Since, however, both are the words of the living God' what was it that entitled Beth Hillel to have the halakhah fixed in agreement with their rulings? Because they were kindly and modest, they studied their own rulings and those of Beth Shammai, and were even so [humble] as to mention the actions of Beth Shammai before theirs, as may be seen from what we have learnt: If a man had his head and the greater part of his body within the sukkah but

> his table in the house, Beth Shammai ruled [that the booth was] invalid but Beth Hillel ruled that it was valid (*Erubin* 13b).

Thus, Judaism presupposes a plurality of opinions within appropriate pragmatic limits. If we reconstruct the Judaic argumentation theory, i.e. pragmatic limits in using logical tools, we will find out that all proofs of Judaic sages may be classified at least into the following three groups from the pragmatic standpoint:

• *Proofs as trees*. Some premises in reasoning are compatible, i.e. they can be combined in the same deductions. Therefore it has no sense to destinguish, whom (which autority) the premises belong to.

• *Proofs as parallel processes*. Some premises are incompatible, they never occur in the same deductions, but they are concurrent, i.e. used in parallel, disjoint deductive exchanging of input data. For example, the sages maintained that the inference rule *heqeš* (see definition 3.1 of chapter 5) cannot be involved in the same deduction more than two times: "that which is learnt through a *heqeš* does not in turn teach through a *heqeš*" (*Zevachim* 49b). In other words, the inference rule *heqeš* may be used only concurrently.

• *Proofs as competitive communication*. Some inference rules of Judaic logic are competitive, i.e. they have a communication among themselves, though they are incompatible and give different results. In this case we should destinguish, whom (which autority) the premises belong to.

As we see, only the first group of Judaic inference rules could find a standard formalisation within the framework of conventional logic. The two others can be formalized only within a version of behavioral logic containing proof techniques for massive-parallel and concurrent deductions. Evidently, this logic should have non-well-founded syntax and non-well-founded semantics (*Schumann 2008*). Thus, it is possible to aim obtaining a behavioral (non-well-founded) logic, formalizing the Judaic reasoning. For verifying this formalisation of Judaic proof system it is possible to build up a new version of process calculus and process semantics.

The development of behavioral logic is very promising today. There are lots of its versions suited to discourse and reasoning about sequential behavior: Hoare logic, algorithmic logic, process logic, and dynamic logic. However, extending these logics to concurrent behavior has proved problematic. These logics are unapplied also to analyzing massive-parallel process. Therefore formalizing a Judaic version of behavioral logic could find a lot of practical applications in modern computer sciences.

Nevertheless, we will not build up the Judaic behavioral logic, we limit ourselves just to considering logical cornerstones of Judaic argumentation theory. First of all, recall that a proof (or derivation) of a well-formed formula S from a set of premises U is a finite tree such that:

- S is the root of the tree and it is called the end-formula.
- The leaves of the tree are all axioms or members of U.
- Each child node of the tree is obtained from its parent nodes by an inference rule, i.e. if S is a child node of S_1, … , S_n, then $\frac{S_1,\ldots,S_n}{S}$ is an instance of a rule.

In the Judaic argumentation theory, the set of all propositions (laws or prohibitions) of the Torah is regarded as set U of premises. Thereby in drawing derivation trees, the following inference rules are most demanded: *qal wa-ḥomer*, *gezerah šawah*, *heqeš, binyan 'av*. Let us denote them by q, g, h, b respectivelly.

Definition 4.1 *The derivation,* $\mathrm{Der}^{\mathrm{I}}(A, B)$, *in Judaic reasoning is a two-placed predicate such that A is a child node and B is a parent node of an appropriate inference rule* I *(where* I *is one of the rules from the set* {q, g, h, b}*), whereas either B belongs to U or B is a child node of another inference rule* I_0 *(from the same set* {q, g, h, b}*). In the second case we can write* $\mathrm{Der}^{\mathrm{I}}(A, \mathrm{Der}^{\mathrm{I}_0}(B, C))$, *where C either belongs to U or is a child node of another inference rule* I_1 *(from* {q, g, h, b}*) and then* $\mathrm{Der}^{\mathrm{I}}(A, \mathrm{Der}^{\mathrm{I}_0}(B, \mathrm{Der}^{\mathrm{I}_1}(C, D))$, *and so on. The predicate* $\mathrm{Der}^{\mathrm{I}}(A, B)$ *is read as follows: "the*

standpoint A is provable by the argument B through I*;" in the Talmudic phraseology: "B teaches A through* I*" or "A is learnt from B through* I*."*

By this definition, the Judaic derivation supposed in proof argumentation is a reflexive and transitive relation. Indeed, the standpoint *A* is provable by the argument *A* and if the standpoint *A* is provable by the argument *B* and *B* is provable by the argument *C*, then *A* is provable by the argument *C*. Formally, the binary relation $\mathrm{Der}_0^{\mathrm{I}}$ of derivation is defined as the least relation satisfying: whenever there is a proof tree containing *P* in the conclusion and *Q* among the premises, then $\mathrm{Der}_0^{\mathrm{I}}(P, Q)$ holds. Further, define $\mathrm{Der}^{\mathrm{I}}$ to be the reflexive and transitive closure of $\mathrm{Der}_0^{\mathrm{I}}$.

Definition 4.1 allows us to commit all derivations trees to writing. There is no end of possible combinations taking into account the sets of premises and inference rules and, as a result, there is a plenty of derivation predicates. However, not all combinations are acceptable from the standpoint of Judaic pragmatic limits. According to the Talmud, the following derivation predicates are strongly forbidden:

$\mathrm{Der}^{\mathrm{h}}(A, \mathrm{Der}^{\mathrm{h}}(B, C))$ is rejected (1)

i.e. that which is learnt through a *heqeš* does not in turn teach through a *heqeš*.

According to *Rabbi Yoḥanan bar Nappaḥa'* [רבי יוחנן בר נפחא]:

$\mathrm{Der}^{\mathrm{g}}(A, \mathrm{Der}^{\mathrm{h}}(B, C))$ is rejected (2)

i.e. that which is learnt by a *heqeš* does not in turn teach through a *gezerah šawah*.

$\mathrm{Der}^{\mathrm{b}}(A, \mathrm{Der}^{\mathrm{h}}(B, C))$ is rejected (3)

i.e. that which is learnt through a *heqeš* cannot in turn teach through a *binyan 'av*.

$\mathrm{Der}^{h}(A, \mathrm{Der}^{g}(B, C))$ is rejected (4)

i.e. that which is learnt by a *gezerah šawah* does not in turn teach through a *heqeš*.

Nevertheless, *Rav Papa'* [רב פפא] negates (4) as follows:

$\mathrm{Der}^{h}(A, \mathrm{Der}^{g}(B, C))$ is allowed (5)

i.e. that which is learnt by a *gezerah šawah* can in turn teach through a *heqeš*.

In the meantime, the universality of *qal wa-ḥomer* in respect to *heqeš* is asserted by all sages:

$\mathrm{Der}^{q}(A, \mathrm{Der}^{h}(B, C))$ is allowed (6)

i.e. that which is learnt through a *heqeš* teaches in turn by a *qal wa-ḥomer*.

Because of rejecting some derivation predicates in the Talmud, we can define the following four new predicates:

Definition 4.2 *The predicate* $W_l(I_i, I_j)$, *where* I_i, I_j *from the set* {q, g, h, b} *means that* I_i *has a weak learning for* I_j, *i.e. if we apply the inference rule* I_j, *we cannot just after apply the rule* I_i.

Hence, propositions (1)—(4) are expressed by predicates W_l(h, h), W_l(g, h), W_l(b, h), W_l(h, g), respectively.

Definition 4.3 *The predicate* $W_t(I_i, I_j)$, *where* I_i, I_j *belong to the set* {q, g, h, b}, *means that* I_i *has a weak teaching for* I_j, *i.e. if we apply the inference rule* I_i, *we cannot just after apply the rule* I_j.

Obviously that if I_i has a weak teaching for I_j, then I_j has a weak learning for I_i, i.e. $W_l(I_i, I_j) \Rightarrow W_t(I_j, I_i)$. The conditions (1)—

(4) can be formulated by predicates W_t(h, h), W_t(h, g), W_t(h, b), W_t(g, h), respectively.

Definition 4.4 *The predicate* $S_t(I_i, I_j)$, *where* I_i, I_j *from the set* {q, g, h, b} *means that* I_i *has a strong learning for* I_j, *i.e. if we apply the inference rule* I_j, *we can always apply the rule* I_i.

This kind of predicates can be exemplified by S_t(h, g) satisfying condition (5) and by S_t(q, h) satisfying condition (6).

Definition 4.5 *The predicate* $S_t(I_i, I_j)$, *where* I_i, I_j *from the set* {q, g, h, b} *means that* I_i *has a strong teaching for* I_j, *i.e. if we apply the inference rule* I_i, *we can always apply the rule* I_j.

For instance, S_t(g, h) satisfies (5) and S_t(h, q) satisfies condition (6).

Following the Judaic sages, we may propose now an ordering relation over the set {q, g, h, b}. On the basis of propositions (1)—(6), it is possible to order the Judaic inference rules q, g, h, b by predicates of definitions 4.2—4.5. The tool of ordering is presented by a traditional Judaic way, *qal wa-ḥomer*. In the Talmudic tractate *Zevachim* we face the following six reasoning by *qal wa-ḥomer*, which order the inference rules q, g, h, b.

Q1 If that which is learnt by a *heqeš*, which cannot teach by a *heqeš* (1), can teach by a *qal wa-ḥomer* (6); then what is learnt through a *gezerah šawah*, which can in turn teach by a *heqeš*, as follows from *Rav Papa'* (5), can surely teach in turn by a *qal wa-ḥomer*:

('Der^h(*A*, Der^h(*B*, *Y*)) is rejected' & 'Der^q(*C*, Der^h(*B*, *Y*)) is allowed') ⇒ ('Der^h(*A*, Der^g(*D*, *X*)) is allowed' & 'Der^q(*E*, Der^g(*D*, *X*)) is allowed').

	weak teaching	**weak learning**	**strong teaching**	**strong learning**
heqeš	for h	for h	for q	for g
gezerah šawah	—	—	for h	—
qal wa-ḥomer	—	—	—	for h
Has *qal wa-ḥomer* a strong learning for *gezerah šawah*?				

This table shows that, on the one hand, q has the property to be a strong learning for h, g has the property to be a strong teaching for h. On the other hand, h has the properties to be a strong learning for g and a strong teaching for q. Taking into account that due to (1) and (6) h is weaker than q, we can infer from minor to major that q has the property of h to be a strong learning for g.

Q2 If a *heqeš*, which cannot in turn teach by a *heqeš* (1), can teach in turn by a *qal wa-ḥomer* (6); then a *gezerah šawah*, which does teach by a *gezerah šawah* like itself, can surely teach through a *qal wa-ḥomer*:

('$\mathrm{Der}^{h}(A, \mathrm{Der}^{h}(B, Y))$ is rejected' & '$\mathrm{Der}^{q}(C, \mathrm{Der}^{h}(B, Y))$ is allowed') ⇒ ('$\mathrm{Der}^{g}(A, \mathrm{Der}^{g}(D, X))$ is allowed' & '$\mathrm{Der}^{q}(E, \mathrm{Der}^{g}(D, X))$ is allowed').

	weak teaching	**weak learning**	**strong teaching**	**strong learning**
heqeš	for h	for h	for q	—
gezerah šawah	—	—	for g	for g
qal wa-ḥomer	—	—	—	for h
Has *gezerah šawah* a strong teaching for *qal wa-ḥomer*?				

According to this table, on the one hand, g has the property to be a strong teaching for g. On the other hand, h has the properties to be a strong teaching for q. Taking into account that

due to (1) h is weaker than g, we may deduce from minor to major that g has the property of h to be a strong teaching for q.

Q3 If a *gezerah šawah*, which cannot be learnt from a *heqeš*, as follows from *Rabbi Yoḥanan bar Nappaḥa'* dictum (2), can nevertheless teach by a *heqeš*, in accordance with *Rav Papa'* (5); then a *qal wa-ḥomer*, which can be learnt from a *heqeš* (6), can surely teach by a *heqeš*:

('Der^{g}(*A*, Der^{h}(*B*, *Y*)) is rejected' & 'Der^{h}(*C*, Der^{g}(*A*, *X*)) is allowed') ⇒ ('Der^{q}(*D*, Der^{h}(*B*, *Y*)) is allowed' & 'Der^{h}(*E*, Der^{q}(*D*, *Z*)) is allowed').

	weak teaching	**weak learning**	**strong teaching**	**strong learning**
heqeš	for g	—	for q	for g
gezerah šawah	—	for h	for h	—
qal wa-ḥomer	—	—	—	for h
Has *qal wa-ḥomer* a strong teaching for *heqeš*?				

As follows from this table, we assume that g is weaker than q (g is a weak learning for h, although q is a strong learning for h). Further, g has the property to be a strong teaching for h. Then taking into account that g is weaker than q, we can entail from minor to major that q has the property of g to be a strong teaching for h.

Q4 If a *gezerah šawah*, which cannot be learnt from a *heqeš*, in accordance with *Rabbi Yoḥanan bar Nappaḥa'* dictum (2), can teach by a *gezerah šawah*; then a *qal wa-ḥomer*, which can be learnt by a *heqeš* (6), can teach by a *gezerah šawah*:

('Der^{g}(*A*, Der^{h}(*B*, *Y*)) is rejected' & 'Der^{g}(*C*, Der^{g}(*A*, *X*)) is allowed') ⇒ ('Der^{q}(*D*, Der^{h}(*B*, *Y*)) is allowed' & 'Der^{g}(*E*, Der^{q}(*D*, *Z*)) is allowed').

	weak teaching	**weak learning**	**strong teaching**	**strong learning**
heqeš	—	for g	for q	—
gezerah šawah	for h	—	for g	for g
qal wa-ḥomer	—	—	—	for h
Has *qal wa-ḥomer* a strong teaching for *gezerah šawah*?				

In the table we see that g is weaker than q (indeed, g is a weak teaching for h, but q is a strong learning for h), at the same time g has the property to be a strong teaching for g. From minor to major we conclude that q has the same property to be a strong teaching for g.

Q5 If a *gezerah šawah*, which cannot be learnt by a *heqeš*, in accordance with *Rabbi Yoḥanan bar Nappaḥa'* dictum (2), can teach by a *qal wa-ḥomer*; then a *qal wa-ḥomer*, which can be learnt from a *heqeš* (6), can teach by a *qal wa-ḥomer*:

('$\mathrm{Der}^{g}(A, \mathrm{Der}^{h}(B, Y))$ is rejected' & '$\mathrm{Der}^{q}(C, \mathrm{Der}^{g}(A, X))$ is allowed') ⇒ ('$\mathrm{Der}^{q}(D, \mathrm{Der}^{h}(B, Y))$ is allowed' & '$\mathrm{Der}^{q}(E, \mathrm{Der}^{q}(D, Z))$ is allowed').

	weak teaching	**weak learning**	**strong teaching**	**strong learning**
heqeš	—	for g	for q	—
gezerah šawah	for h	—	for q	—
qal wa-ḥomer	—	—	—	for g, for h
Has *qal wa-ḥomer* a strong teaching for *qal wa-ḥomer*?				

Since g is weaker than q (because g is a weak teaching for h, but q is a strong learning for h) and g has the property to be a strong teaching for q. From minor to major we obtain that q has the property of g to be a strong teaching for q.

Q6 If a *heqeš*, which cannot be learnt through a *heqeš* (1), can teach by a *qal wa-ḥomer*; then a *qal wa-ḥomer*, which is learnt through a *heqeš* (6) can surely teach through a *qal wa-ḥomer*:

('Der^{h}(*A*, Der^{h}(*B*, *Y*)) is rejected' & 'Der^{q}(*C*, Der^{h}(*A*, *X*)) is allowed') ⇒ ('Der^{q}(*D*, Der^{h}(*B*, *Y*)) is allowed' & 'Der^{q}(*E*, Der^{q}(*D*, *Z*)) is allowed').

	weak teaching	**weak learning**	**strong teaching**	**strong learning**
heqeš	for h	for h	for q	—
gezerah šawah	—	—	—	for h
qal wa-ḥomer	—	—	—	—
Has *qal wa-ḥomer* a strong learning for *qal wa-ḥomer*?				

We know that h is weaker than q (indeed, h is a weak learning for h, but q is a strong learning for h) and h has the property to be a strong teaching for q. From minor to major we infer that q has the property of h to be a strong teaching for q.

Reasoning from Q1 to Q6 shows that *heqeš* is the weakest inference rule and *qal wa-ḥomer* the strongest. As we see, in the Talmud inference rules have different pragmatic limits. The widest limits are borne by *qal wa-ḥomer* that can be applied everywhere, the most restricted by *heqeš*.

The whole text of the Talmud devoted to pragmatic limits of the Judaic inference rules is quoted below:

> למה לי לאקשויי לעולה אמר רבינא איצטריך אי אקשיה לחטאת ולא אקשיה לעולה הוה אמינא חטאת מהיכן למדה מעולה דבר הלמד בהיקש חוזר ומלמד בהיקש א"ל מר זוטרא בריה דרב מרי לרבינא וניקשיה לעולה ולא ניקשיה לחטאת הוה אמינא דבר הלמד בהיקש חוזר ומלמד בהיקש וכי תימא ניקשי אקושי לחטאת ניחא ליה דמקיש ליה לעיקר ולא נקיש ליה לטפל להכי אקשיה לחטאת ואקשיה לעולה למימר דבר הלמד בהיקש שאינו חוזר ומלמד בהיקש רבא אמר מהכא דכתיב (ויקרא ד) כאשר יורם משור

זבח השלמים למאי הלכתא אי ליותרת הכבד ושתי הכליות בגופיה כתיב <אמר רב פפא> משום דבעי אגמורי יותרת הכבד ושתי הכליות מפר העלם דבר של צבור לשעירי <עבודת כוכבים> {עבודה זרה} בגופיה לא כתיב ומפר כהן משיח הוא דגמר להכי איצטריך כאשר יורם דניהוי כמאן דכתב בגופיה ולא ניהוי דבר הלמד בהיקש חוזר ומלמד בהיקש אמר ליה רב פפא לרבא וליכתביה בגופיה ולא נקיש אי כתב בגופיה ולא אקיש הוה אמינא דבר הלמד בהיקש חוזר ומלמד בהיקש וכי תימא נקשי אקושי ניחא ליה דכתביה בגופיה מדאקיש ליה אקושי להכי כתביה ואקשיה למימרא דבר הלמד בהיקש אין חוזר ומלמד בהיקש: <היק"ש וגזיר"ה שו"ה ק"ל וחומ"ר סימן>: דבר הלמד בהיקש אין חוזר ומלמד מהיקש אי מדרבא אי מדרבינא דבר הלמד בהיקש מהו שילמד בגזירה שוה תא שמע רבי נתן בן אבטולמוס אומר מנין לפריחה בבגדים שהיא טהורה נאמר קרחת וגבחת בבגדים ונאמר קרחת וגבחת באדם מה להלן פרח בכולו טהור אף כאן פרח בכולו טהור והתם מנא לן דכתיב (ויקרא יג) מראשו ועד רגליו ואיתקש ראשו לרגל מה להלן כולו הפך לבן פרח בכולו טהור אף כאן כולו הפך לבן פרח בכולו טהור אמר ר' יוחנן בכל התורה כולה למידין למד מלמד חוץ מן הקדשים שאין דנין למד מלמד דאם כן לא יאמר צפונה באשם ותיתי בג"ש דקדשי קדשים מחטאות לאו למימרא דדבר הלמד בהיקש אין חוזר ומלמד בגזירה שוה ודלמא משום דאיכא למיפרך מה לחטאת שכן מכפרת על חייבי כריתות קדשי קדשים יתירי כתיבי: דבר הלמד מהיקש חוזר ומלמד בקל וחומר מדתנא דבי ר' ישמעאל דבר הלמד בהיקש מהו שילמד בבנין אב אמר רבי ירמיה לא לכתוב צפונה באשם ותיתי מבנין אב מחטאת למאי הלכתא כתביה לאו למימרא דדבר הלמד בהיקש אין חוזר ומלמד בבנין אב וליטעמיך תיתי מבנין אב מעולה מאי טעמא לא אתי משום דאיכא למיפרך מה לעולה שכן כליל חטאת נמי איכא למיפרך מה לחטאת שכן מכפרת על חייבי כריתות חדא מחדא לא אתיא תיתי חדא מתרתי מהי תיתי לא נכתוב רחמנא עולה ותיתי מחטאת ואשם מה להנך שכן מכפרין לא נכתוב רחמנא בחטאת ותיתי מהנך מה להנך שכן זכרים לא נכתוב באשם ותיתי מהנך מה להנך שכן ישנן בציבור כביחיד דבר הלמד בגז"ש מהו שילמד בהיקש אמר רב פפא (ויקרא ו) וזאת תורת זבח השלמים [וגו'] אם על תודה למדנו לתודה שבא מן המעשר מדאשכחן שלמים דאתו ממעשר שלמים גופייהו מנא לן דכתיב (דברים כז) שם (דברים יד) שם א"ל מר זוטרא בריה דרב מרי לרבינא מעשר דגן חולין בעלמא הוא א"ל אמר אמרה למד קדש ומלמד קדש דבר הלמד בג"ש מהו שילמד בג"ש אמר רמי בר חמא תניא (ויקרא ז) סלת מרבכת למדנו לרבוכה שבאה סולת חלות מניין ת"ל חלות חלות רקיקין מניין ת"ל מצות מצות אמר ליה רבינא ממאי דמצות מצות מחלות גמר דלמא ממאפה תנור גמר אלא אמר רבא תניא (ויקרא ד) וקרבו ופרשו והוציא מלמד שמוציאו שלם יכול ישרפנו שלם נאמר כאן (ויקרא ד) ראשו וכרעיו ונאמר להלן (ויקרא א) ראשו וכרעיו מה להלן ע"י ניתוח אף כאן על

ידי ניתוח אי מה להלן בהפשט אף כאן נמי בהפשט תלמוד לומר וקרבו ופרשו מאי תלמודא א"ר פפא כשם שפרשו בקרבו כך בשרו בעורו ותניא רבי אומר נאמר כאן עור ובשר [ופרש] ונאמר להלן עור ובשר ופרש מה להלן ע"י ניתוח שלא בהפשט אף כאן ע"י ניתוח שלא בהפשט דבר הלמד בגזירה שוה מהו שילמד בק"ו ק"ו ומה היקש שאינו מלמד בהיקש אי מדרבא אי מדרבינא מלמד בקל וחומר מדתנא דבי רבי ישמעאל גז"ש המלמדת בהיקש מדרב פפא אינו דין שתלמד בק"ו הניחא למאן דאית ליה דרב פפא אלא למאן דלית ליה דרב פפא מאי איכא למימר אלא קל וחומר ומה היקש שאין מלמד בהיקש אי מדרבא אי מדרבינא מלמד בק"ו מדתנא דבי רבי ישמעאל גזירה שוה המלמדת בגז"ש חבירתה מדרמי בר חמא אינו דין שתלמד בק"ו דבר הלמד בגז"ש מהו שילמד בבנין אב תיקו. דבר הלמד בקל וחומר מהו שילמד בהיקש ק"ו ומה גזירה שוה שאינה למדה בהיקשא מדר' יוחנן מלמד בהיקש מדרב פפא קל וחומר הלמד מהיקש מדתנא דבי ר' ישמעאל אינו דין שילמד בהיקש הניחא למאן דאית ליה דרב פפא אלא למאן דלית ליה דרב פפא מאי איכא למימר תיקו. דבר הלמד בקל וחומר מהו שילמד בגזירה שוה קל וחומר ומה גזירה שוה שאינה למידה בהיקשא מדרבי יוחנן מלמד בגזירה שוה מדרמי בר חמא קל וחומר הלמד בהיקש מדתנא דבי רבי ישמעאל אינו דין שתלמד בגז"ש. דבר הלמד בקל וחומר מהו שילמד בקל וחומר ק"ו ומה גזירה שוה שאינה למידה בהיקש מדר' יוחנן מלמד בק"ו כדאמרן ק"ו הלמד מהיקש מדתנא דבי רבי ישמעאל אינו דין שילמד בקל וחומר וזהו ק"ו בן ק"ו בן בנו של ק"ו הוא אלא ק"ו ומה היקש שאינו למד בהיקש אי מדרבא אי מדרבינא מלמד בק"ו מדתנא דבי רבי ישמעאל ק"ו הלמד מהיקש מדתני דבי רבי ישמעאל אינו דין שילמד בק"ו וזהו ק"ו בן קל וחומר. דבר הלמד בקל וחומר מהו שילמד בבנין אב אמר רבי ירמיה תא שמע מלק ונמצאת טריפה ר"מ אומר אינה מטמאה בבית הבליעה רבי יהודה אומר מטמאה בבית הבליעה אמר ר' מאיר קל וחומר ומה נבלת בהמה שמטמאה במגע ובמשא שחיטתה מטהרת טריפתה מטומאתה נבילת עוף שאין מטמא במגע ובמשא אינו דין שתהא שחיטתה מטהרת טריפתה מטומאתה מה מצינו בשחיטה שמכשרתה באכילה מטהרת טריפתה מטומאתה אף מליקה שמכשרתה באכילה תטהר טריפתה מטומאתה ר' יוסי אומר דיה כנבלת בהמה טהורה ששחיטתה מטהרתה ולא מליקתה ולא היא התם תיהוי היא משחיטה דחולין קאתיין. דבר הלמד בבנין אב מהו שילמד בהיקש ובג"ש ובק"ו ובבנין אב פשוט מהא חדא מפני מה אמרו לן בדם כשר שהרי לן באימורין כשר לן באימורין כשר שהרי לן בבשר כשר יוצא הואיל ויוצא כשר בבמה טמא הואיל והותר בעבודת ציבור חוץ לזמנו הואיל ומרצה לפיגולו חוץ למקומו הואיל והוקש לחוץ לזמנו שקיבלו פסולין וזרקו דמן בהנך פסולין דחזו לעבודת ציבור וכי דנין דבר שלא בהכשירו מדבר שבהכשירו תנא מזאת תורת העולה ריבה סמיך ליה

Why must it be likened to both a sin-offering and a guilt-offering?—Said Rabina, It is necessary: if it were likened to a sin-offering and were not likened to a guilt-offering I would say, Whence did we learn [that] a sin-offering [is slaughtered in the north]? from a burnt-offering: thus that which is learnt through a hekkesh in turn teaches through a hekkesh. Mar Zutra the son of R. Mari said to Rabina: Then let it be likened to a burnt-offering and not likened to a sin-offering?—Then I would say, [that elsewhere] that which is learnt through a hekkesh in turn teaches through a hekkesh; and if you object, Then let it be likened to a sin-offering, [I could reply:] It [Scripture] prefers to liken it to the principal rather than to the secondary. Therefore it likened it to a sin-offering and it likened it to a burnt-offering, thus intimating that that which is learnt through a hekkesh does not in turn teach through a hekkesh.

Raba said: [It is learnt] from the following, for it is written, As is taken off from the ox of the sacrifice of peace-offerings. For what purpose [is this written]? if for the lobe of the liver and the two kidneys, [surely] that is written in the body of the text! But because [Scripture] wishes to intimate that [the burning of] the lobe of the liver and the two kidneys of the he-goats [brought as sin-offerings] for idolatry shall be learnt by analogy from the community's bullock [for a sin-offering on account] of [sinning in] unawareness, whereas this law is not explicitly stated in the passage on the bullock of unawareness, but is learnt from the anointed priest's bullock: therefore 'as is taken off' is required, so that it might count as written in that very passage and not as something which is learnt through a hekkesh and then in turn teaches through a hekkesh. Said R. Papa to Raba: Then let [Scripture] write it in its own context, and not assimilate [it to the anointed priest's bullock]?—If [Scripture] wrote it in its own context, and did not teach it by assimilation, I would say, That which is learnt through a hekkesh can in turn teach through a hekkesh; and if you object, Then let Scripture assimilate it? [I could answer that Scripture] prefers to write it [explicitly] in its own context rather than to teach it through a hekkesh. Therefore [Scripture] wrote it and assimilated it, in order to teach that that which is learnt through a hekkesh does not in turn teach through a hekkesh.

(Mnemonic: Hekkesh and gezerah shawah; kal wa-homer.) [It is agreed that] that which is learnt through a hekkesh does not in turn teach through a hekkesh, [this being learnt] either by Raba's or by Rabina's [exegesis]. Can that which is learnt through a hekkesh teach through a gezerah shawah?—Come and hear: R. Nathan b. Abtolemos said: Whence do we know that a spreading outbreak [of leprosy] in garments [covering the whole] is clean? Karahath [baldness of the back of the head] and gabbahath [baldness of the front] are mentioned in connection with garments, and also in connection with man: just as in the latter, if [the plague] spread over the whole skin, he is clean; so in the former too, if it spread over the whole [garment], it is clean. And how do we know it there? Because it is written, [And if the leprosy ... cover all the skin ...] from his head even to his feet, and [thereby] his head is assimilated [through a hekkesh] to his feet: as there, when it is all turned white, having broken out all over him, he is clean; so here too, when it breaks out all over him, he is clean. Said R. Johanan: In the whole Torah we rule that whatever is learnt can teach, save in the case of sacrifices, where we do not rule that whatever is learnt can teach. For if it were so [that we did rule thus], let 'northward' not be said in connection with a guilt-offering, and it could be inferred from sin-offerings by the gezerah shawah of 'it is most holy.' Surely then its purpose is to teach that that which is learnt by a hekkesh does not in turn teach through a gezerah shawah. But perhaps [we do not learn it there] because one can refute it: as for a sin-offering, [it requires north] because it makes atonement for those who are liable to kareth?—A superfluous 'most holy' is written.

That which is learnt through a hekkesh teaches in turn by a kal wa-homer. [This follows] from what the school of R. Ishmael taught.

That which is learnt through a hekkesh, can it teach through a binyan ab?—Said R. Jeremiah: Let 'northward' not be written in connection with a guilt-offering, and it could be inferred from a sin-offering by a binyan ab. For what purpose then is it written? Surely to intimate that that which is learnt through a hekkesh cannot in turn teach through a binyan ab. Yet according to your reasoning, let it be inferred from a burnt-offering by a binyan ab? Why then is it not so inferred? Because you can refute it: as

for a burnt-offering, [it requires the north] because it is altogether burnt. So in the case of a sin-offering too, you can refute it: as for a sin-offering, [it requires the north] because it makes atonement for those who are liable to kareth!
One cannot be learnt from one; [but] let one be learnt from [the other] two?—From which could it be derived? [Will you say,] Let the Divine Law not write it in the case of a burnt-offering, and it could be derived from a sin-offering and a guilt-offering; [then you can argue,] as for these, [they require the north] because they make atonement. Let not the Divine Law write it in respect of a sin-offering, and let it be derived from the others; [then you can argue,] as for those, the reason is because they are males. Let not the Divine Law write it in connection with a guilt-offering and let it be derived from the others; [then you can argue,] the reason is because they operate in the case of a community as in the case of an individual.
That which is learnt by a gezerah shawah, can it in turn teach through a hekkesh?—Said R. Papa, It was taught: And this is the law of the sacrifice of peace-offerings ... if he offers it for a thanksgiving: [from this] we learn that a thanksgiving can be brought from tithe, since we find that a peace-offering can be brought from tithe. And how do we know [this of] a peace-offering itself?—Because 'there' is written in each case. Said Mar Zutra the son of R. Mari to Rabina: But corn tithe is merely hullin?—Said he to him: Who says that which is learnt must be holy, and that which teaches must be holy?
Can that which is learnt by a gezerah shawah teach by a gezerah shawah?—Said Rami b. Hama, It was taught: Of fine flour soaked [murbeketh]: this teaches that the rebukah [soaked cake] must be of fine flour [soleth]. How do we know [the same of] halloth? Because halloth is stated in both places. How do we know it of rekikin [thin wafers]? Because mazzoth [unleavened bread] is written in connection with each. Said Rabina to him: How do you know that he learns [the gezerah shawah of] mazzoth, mazzoth, from halloth; perhaps he learns it from oven-baked [cakes]? Rather said Raba: It was taught: And its inwards, and its dung, [even the whole bullock] shall he carry forth [without the camp]: this teaches that he carries it forth whole. You might think that he burns it whole; [but] 'its head and its legs' is stated here, and 'its head and its legs' is stated

elsewhere: as there it means after cutting up, so here too it means after cutting up. If so, as there it is after the flaying [of the skin], so here too it means after the flaying? Therefore it says, 'and its inwards and its dung'. How does this teach [the reverse]?—Said R. Papa: Just as its dung is within it, so must its flesh be within its skin. And it was [further] taught, Rabbi said: Skin and flesh and dung are mentioned here, and skin and flesh and dung are mentioned elsewhere: as there [it was burnt after] being cut up, but without flaying, so here too [it is burnt after being] cut up, but without flaying.

Can that which is learnt by a gezerah shawah teach in turn by a kal wa-homer?—[It can, and we learn this by a] kal wa-homer: If [that which is learnt by] a hekkesh, which cannot teach by a hekkesh, as follows from either Raba's or Rabina's [proof], can teach by a kal wa-homer, which follows from what the school of R. Ishmael taught; then [what is learnt through] a gezerah shawah, which can [in turn] teach by a hekkesh, as follows from R. Papa, can surely teach [in turn] by a kal wa-homer! That is well according to him who accepts R. Papa's teaching; but what can be said on the view that rejects R. Papa's teaching?—Rather [this is the] kal wa-homer: if [what is learnt by] a hekkesh, which cannot [in turn] teach by a hekkesh, as follows either from Raba or from Rabina, can teach [in turn] by a kal wa-homer, which follows from what the school of R. Ishmael taught; then a gezerah shawah, which does teach by a gezerah shawah like itself, which follows from Rami b. Hama, can surely teach through a kal wa-homer.

Can that which is learnt by a gezerah shawah subsequently teach by a binyan ab?—The question stands.

Can that which is learnt by a kal wa-homer teach in turn by a hekkesh?—[Yes, and we learn this by a] kal wa-homer: if a gezerah shawah, which cannot be learnt from a hekkesh, as follows from R. Johanan's [dictum], can nevertheless teach by a hekkesh, in accordance with R. Papa; then a kal wa-homer, which can be learnt from a hekkesh, in accordance with the school of R. Ishmael, can surely teach by a hekkesh! That is well on the view that accepts R. Papa's [dictum], but what can be said on the view that rejects R. Papa's [dictum]? Then the question stands.

Can that which is learnt by a kal wa-homer teach in turn by a gezerah shawah?—[Yes, for this follows by a] kal wa-homer: if a gezerah shawah, which cannot be learnt from a hekkesh, in accordance with R. Johanan, can teach by a gezerah shawah, in accordance with Rami b. Hama; then is it not logical that a kal wa-homer, which can be learnt by a hekkesh, in accordance with the school of R. Ishmael, can teach by a gezerah shawah?
Can that which is learnt by a kal wa-homer teach in turn by a kal wa-homer? [Yes, for this follows from a] kal wa-homer: if a gezerah shawah, which cannot be learnt by a hekkesh, in accordance with R. Johanan, can teach by a kal wa-homer, as we have [just] said; then a kal wa-homer which can be learnt from a hekkesh, in accordance with the school of R. Ishmael, is it not logical that it can teach by a kal wa-homer? And this is a kal wa-homer derived from a kal wa-homer. Surely this is a secondary derivation from a kal wa-homer?—Rather, [argue thus: Yes, and this follows from a] kal wa-homer: if a hekkesh which cannot be learnt through a hekkesh, in accordance with either Raba or Rabina, can teach by a kal wa-homer, in accordance with the school of R. Ishmael; then a kal wa-homer, which is learnt through a hekkesh, in accordance with the school of R. Ishmael, can surely teach through a kal wa-homer! And this is a kal wa-homer derived from a kal wa-homer.
Can that which is learnt by a kal wa-homer teach in turn through a binyan ab?—Said R. Jeremiah, Come and hear: If one wrung the neck [of a bird sacrifice] and it was found to be a terefah, R. Meir said: It does not defile in the gullet; R. Judah said: It does defile in the gullet. Said R. Meir: It is a kal wa-homer: if the shechitah of an animal cleanses it, even when terefah, from its uncleanness, yet when it is nebelah it defiles through contact or carriage; is it not logical that shechitah cleanses a bird, when terefah, from its uncleanness, seeing that when it is nebelah it does not defile through touch or carriage? Now, as we have found that shechitah which makes it [a bird of hullin] fit for eating, cleanses it, when terefah, from its uncleanness; so wringing [the neck], which makes it [a bird sacrifice] fit for eating, cleanses it, when terefah, from its uncleanness. R. Jose said: It is sufficient that it be like the nebelah of a clean [i.e., edible] animal, which is cleansed by shechitah, but not by wringing its neck. Yet that is not so: even

> granted there that it is so, yet it is deduced from the shechitah of hullin.
> Can that which is learnt by a binyan ab teach by a hekkesh or by a gezerah shawah or by a kal wa-homer or by a binyan ab?—Solve one [of the questions] from the following: Why did they say that if the blood is kept overnight [on the altar] it is fit? Because if the emurim are kept overnight they are fit. Why are the emurim fit if kept overnight? Because the flesh is fit if kept overnight. [Flesh that] goes out? Because [flesh that] goes out is fit at the high place [bamah]. Unclean [flesh]? Because it was permitted in public service. [The emurim of a burnt-offering intended to be burnt] after time? Because it propitiates in respect of its piggul status. [The emurim of a burnt-offering intended to be burnt] out of bounds? Because it was likened to [the intention to burn it] after time. Where unfit [persons] received [the blood] and sprinkled it—in the case of those unfit persons who are eligible for public service. Can you then argue from what is its proper way to that where the same is not the proper way?—The Tanna relies on the extension indicated by This is the law of the burnt-offering (*Zevachim* 49b—51a).

This Talmudic reasoning tells us about pragmatic aspects of logical inference rules involved for inferring Judaic laws. It is prohibited to use inference rules without regarding these aspects. For example, we cannot use a *heqeš* in one and the same deduction if we face cases (1)—(4). According to these aspects, many Judaic proofs may be drawn up only as parallel processes.

The Judaic argumentation theory also contains pragmatic limits for proofs as competitive communication when different Rabbis claim different opinions in respect to the same subject. In order to define these limits we build up a special kind of syllogistics where it is defined whose opinion should be choosen in a dispute. This system is called Judaic pragmatic-syllogistics.

Definition 4.6 *The alphabet of Judaic pragmatic-syllogistics is an ordered system* $\mathbf{A}_P = \langle V, Q_1, Q_2, L_1, L_2, L_3, L_4, K \rangle$, *where*

1. V is the set of propositional variables p, q, r, ...;

2. $\mathcal{Q}_1$ is a set of pragmatic-syllogistic variables *P*, *Q*, *R*, … running over different sages (Tannaim as well as Amoraim);

3. $\mathcal{Q}_2$ is a set of pragmatic-syllogistic constants including the following 28 sages:

Mar bar Rav 'Aši [מר בר רב אשי], the sage of Babylon, Amora of the seventh generation, the son of *Rav 'Aši*, he headed the academy of Sura city.

Rab [רבי אבא אריכא], the Amora of the first generation, who lived in Babylonia, where he established an academy at Sura.

Rabbah [רבה בר נחמני], the sage of Babylon, Amora of the third generation.

Rabban Šim'on ben Gamli'el [רבן שמעון בן גמליאל], the Tanna of the third generation and president of the Great Sanhedrin, the father of *Rabbi Yehudah ha-Naśi'*.

Rabbi 'Eli'ezer [רבי בן אליעזר הורקנוס], the Israelite, the Tanna of the second generation, he descended from a priestly family.

Rabbi 'Eli'ezer ben Ya'qob [רבי אליעזר בן יעקב], the Tanna of the fourth generation, the student of *Rabbi 'Akiba'*.

Rabbi Gamali'el [רבי גמליאל דיבנה], the Israelite, the Tanna of the second generation, the first person to lead the Sanhedrin as president after the fall of the second temple.

Rabbi Ḥiyy'a [רבי חייא], the Israelite, the Amora belonging to the first generation.

Rabbi Me'ir [רבי מאיר], the Tanna of the fourth generation, his father was a descendant of the Roman Emperor Nero who had converted to Judaism.

Rabbi Natan [רבי נתן הבבלי], the Tanna of the third generation, the son of a Babylonian exilarch, then he settled in the land of Israel, where he was made chief of the school at Usha.

Rabbi Neḥemiah [רבי נחמיה], the Israelite, the Tanna of the fourth generation, attributed as the author of the *Mišnat ha-Middot*, the earliest known Hebrew text on geometry.

Rabbi Šim'on [רבי שמעון בר יוחאי], the Israelite, the Tanna of the fourth generation, attributed as the author of the *Zohar*, the

chief work of Kabbalah, he was also one of the most eminent disciples of *Rabbi 'Akiba'*.

Rabbi Šim'on ben 'Ele'azar [רבי שמעון בן אלעזר], the Tanna of the fourth generation, the student of *Rabbi Me'ir*.

Rabbi Yehošu'a [רבי יהושע בן חנניה], the Israelite, the Tanna of the second generation, the Levitical descent, he served in the sanctuary as a member of the class of singers.

Rabbi Yehudah [רבי יהודה בר מערבא], the Israelite, the Tanna of the fourth generation and son of *Rabbi 'Ila'i* [רבי אלעאי].

Rabbi Yehudah ha-Naśi' [רבי יהודה הנשיא], the Israelite, the Tanna of the fifth generation, the chief editor of the Mishnah, he was of the royal line of King David.

Rabbi Yišma'el [רבי ישמעאל], the Israelite, the Tanna of the third generation, he formalized a set of 13 hermeneutic rules by which halakha was derived from the Torah.

Rabbi Yose [רבי יוסי בן חלפתא], the Israelite, the Tanna of the fourth generation, the student of *Rabbi 'Akiba'*.

Rav 'Aḥa' [רבי אחא], the Israelite, Amora of the fourth generation.

Rav 'Aši [רב אשי], the Babylonian Amora of the sixth generation, one the first editors of the Babylonian Talmud, he reestablished the academy at Sura.

Rav Ḥisda' [רב חסדא], the Babylonian Amora of the third generation, he descended from a priestly family, the disciple of *Rab* and *Rav Huna'*.

Rav Huna' [רב הונא], the Amora of the second generation who lived in Babylonia and headed the academy of Sura, the disciple of *Rav*, he descended from a priestly family.

Rav Naḥman [רב נחמן בר יעקב], the Amora of the third generation, who lived in Babylonia, the student of *Šmu'el* and then the head of the school of Nehardea.

Rav Šešet [רב ששת], the Babylonian Amora of the third generation and colleague of *Rav Naḥman*.

Rav Yehudah [רב יהודה בן יחזקאל], the Babylonian Amora of the second generation, the disciple of *Rab*, he founded an academy at Pumbedita.

Rav Yosef [רב יוסף בר חייא], the Amora of the third generation, who lived in Babylonia, the disciple of *Rav Yehudah*, the head of the academy of Pumbedita.

Ravina' [רבינא], the Babylonian Amora of the sixth generation, one of the first editors of the Babylonian Talmud.

Šmu'el [שמואל], one of the leading Babylonian Amoraim of the first generation, who succeeded his father as head of the academy of Nehardea, he mastered the contemporary sciences such as medicine and astronomy.

4. $L_1 = \{\neg\}$;

5. $L_2 = \{\wedge, \vee, \Rightarrow\}$;

6. L_3 is a set containing two unary pragmatic-syllogistic connectives A(…), H(…) meaning "the statement… is anonymous," "… is the final decision (*halakha*)" respectively;

7. L_4 is a set of binary pragmatic-syllogistic connectives containing five elements …=$_b$…, T(…, …), t(…, …), …*…, exc$_n$(…) called the functors "the statement… belongs to the sage …," "the sage… is a disciple of the sage …," "the sage… is later than the sage …," "the statement … is followed by the statement…," "the opinion of the sage … prevails (or is rejected), but not in these n named cases" respectively.

8. L_5 is a set consisting of the only ternary pragmatic-syllogistic connective agst$_{...}$(…, …) called the functor "in respect to the statement… the sage … is against the sage …"

9. K is the set of auxiliary symbols containing two brackets:(,).

Definition 4.7 *The language of Judaic pragmatic-syllogistics is an ordered system* $\mathbf{L}_P = \langle \mathbf{A}_P, \mathbf{F}_P \rangle$ *, where*

1. $\mathbf{A}_P$ is the alphabet of Judaic pragmatic-syllogistics;

2. $\mathbf{F}_P$ is a set of all formulas formed by means of symbols in $\mathbf{A}_P$; this set $\mathbf{F}_P$ contains all formulas defined by the rules (a), (b), and (c) of definition 3.2 of chapter 2 and additionally by the following rules:

(d1) if φ is propositional formula, then the expression A(φ) is a formula of pragmatic-syllogistics;

(d2) if φ is propositional formula, then the expression H(φ) is a formula of pragmatic-syllogistics;

(d3) if φ is a propositional formula and Q is a pragmatic-syllogistic variable (or a pragmatic-syllogistic constant), then the expression $\varphi =_b Q$ is a formula of pragmatic-syllogistics;

(d4) if P, Q are pragmatic-syllogistic variables (or pragmatic-syllogistic constants), then the expression T(P, Q) is a formula of pragmatic-syllogistics;

(d5) if P, Q are pragmatic-syllogistic variables (or pragmatic-syllogistic constants), then the expression t(P, Q) is a formula of pragmatic-syllogistics;

(d6) if φ, ψ are propositional formulas, then the expression $\varphi * \psi$ is a formula of pragmatic-syllogistics;

(d7) if Q is a pragmatic-syllogistic variable (or a pragmatic-syllogistic constant), then the expression $\mathrm{exc}_n(Q)$ is a formula of pragmatic-syllogistics;

(d8) if φ is propositional formula and P, Q are pragmatic-syllogistic variables (or pragmatic-syllogistic constants), then the expression $\mathrm{agst}_{\varphi}(P, Q)$ is a formula of pragmatic-syllogistics.

Formulas that are defined by means of rules (d1)—(d8) are called formulas of Judaic pragmatic-syllogistics in the strict sense. They are denoted by P.

Definition 4.8 *Judaic pragmatic-syllogistics is an ordered system* $\mathbf{S}_P = \langle \mathbf{A}_P, \mathbf{F}_P, C \rangle$, *where*

1. $\mathbf{A}_P$ is the alphabet of Judaic pragmatic-syllogistics;

2. $\mathbf{F}_P$ is a set of all formulas formed by means of symbols in $\mathbf{A}_P$;

3. C is the inference operation in $\mathbf{F}_P$.

The *inference rules of Judaic pragmatic-syllogistics* are as follows:

1. the *substitution rule*, we replace a propositional variable p_j of formula $\alpha(p_1, \ldots, p_n)$, containing propositional variables $p_1, \ldots, p_n$, by a propositional formula $\beta(q_1, \ldots, q_k)$, containing propositional variables $q_1, \ldots, q_k$ (as well as by a pragmatic-syllogistic formula $\beta(\sigma, \tau)$ or $\beta(\sigma)$), containing pragmatic-syllogistic variables or pragmatic-syllogistic constants), and we obtain a new propositional formula $\alpha'(p_1, \ldots, p_{j-1}, \beta(q_1, \ldots, q_k), p_{j+1}, \ldots, p_n)$ (as well as a new pragmatic-syllogistic formula $\alpha'(p_1,\ldots,p_{j-1},\beta(\sigma,\tau),p_{j+1},\ldots,p_n)$ or $\alpha'(p_1,\ldots,p_{j-1},\beta(\sigma),p_{j+1},\ldots,p_n)$):

$$\frac{\alpha(p_1,\ldots,p_j,\ldots,p_n)}{\alpha'(p_1,\ldots,p_{j-1},\beta(q_1,\ldots,q_k),p_{j+1},\ldots,p_n)}$$

or

$$\frac{\alpha(p_1,\ldots,p_j,\ldots,p_n)}{\alpha'(p_1,\ldots,p_{j-1},\beta(\sigma,\tau),p_{j+1},\ldots,p_n)}, \quad \frac{\alpha(p_1,\ldots,p_j,\ldots,p_n)}{\alpha'(p_1,\ldots,p_{j-1},\beta(\sigma),p_{j+1},\ldots,p_n)}$$

In the same way, from any pragmatic-syllogistic formula $\beta(p,P)$, $\beta(p)$, $\beta(P,Q)$, $\text{agst}_p(P,Q)$ or $\beta(P)$ a new formula $\beta'(\alpha(q_1,\ldots,q_k),P)$, $\beta'(\alpha(q_1,\ldots,q_k))$, $\beta'(R,Q)$, $\beta'(P,R)$, $\text{agst}_{\alpha(q_1,\ldots,q_k)}(P,Q)$, $\text{agst}_p(R,Q)$, $\text{agst}_p(P,R)$ or $\beta'(R)$ follows if we replace a propositional variable p by a propositional formula

$\alpha(q_1,...,q_k)$, containing propositional variables $q_1,...,q_k$, or a pragmatic-syllogistic variable *P* by *R* and *Q* by *R*:

$$\frac{\beta(p,P)}{\beta'(\alpha(q_1,...,q_k),P)}, \quad \frac{\beta(p)}{\beta'(\alpha(q_1,...,q_k))}, \quad \frac{\beta(P,Q)}{\beta'(R,Q)},$$
$$\frac{\beta(P,Q)}{\beta'(P,R)}, \quad \frac{\text{agst}_p(P,Q)}{\text{agst}_{\alpha(q_1,...,q_k)}(P,Q)}, \quad \frac{\text{agst}_p(P,Q)}{\text{agst}_p(R,Q)},$$
$$\frac{\text{agst}_p(P,Q)}{\text{agst}_p(P,R)}, \quad \frac{\beta(P)}{\beta'(R)}.$$

2. *modus ponens*, according to that if two formulas of pragmatic-syllogistics α and $\alpha \Rightarrow \beta$ hold, then we deduce a formula β.

Now let us consider the axioms of Judaic pragmatic-syllogistics. First of all, they consist of axioms of propositional logic (e.g., axioms (1), (2), (3) of the propositional system **S**, see the previous chapter), and furthermore of the following additional expressions:

The axioms concerning the general properties to be a conflict between sages:

$$\neg\text{agst}_\varphi(Q,Q), \tag{7}$$
$$\text{agst}_\varphi(Q,R) \Rightarrow \text{agst}_\varphi(R,Q), \tag{8}$$
$$\text{agst}_\varphi(Q,R) \Rightarrow (\varphi =_b Q \vee \varphi =_b R), \tag{9}$$
$$\text{agst}_\varphi(Q,R) \Rightarrow (\varphi =_b Q \Rightarrow \neg(\varphi =_b R)), \tag{10}$$
$$((\varphi \equiv \neg\psi) \wedge \varphi =_b Q \wedge \psi =_b R) \Rightarrow \text{agst}_\varphi(Q,R), \tag{11}$$
$$\text{A}(\varphi) \Rightarrow \neg\text{agst}_\varphi(Q,R). \tag{12}$$

The axioms concerning the general properties to be a teacher of another sage:

$$\neg \mathrm{T}(Q,Q), \tag{13}$$
$$\mathrm{T}(Q,R) \Rightarrow \neg \mathrm{T}(R,Q), \tag{14}$$
$$(\mathrm{T}(P,Q) \wedge \mathrm{T}(Q,R)) \Rightarrow \mathrm{T}(P,R), \tag{15}$$
$$(\varphi =_b P \wedge \neg \mathrm{agst}_{\varphi}(P,Q) \wedge \mathrm{T}(P,Q)) \Rightarrow \varphi =_b Q. \tag{16}$$

The axioms in respect to the general properties to be later than another sage:

$$\neg t(Q,Q), \tag{17}$$
$$t(Q,R) \Rightarrow \neg t(R,Q), \tag{18}$$
$$(t(P,Q) \wedge t(Q,R)) \Rightarrow t(P,R). \tag{19}$$

The general properties to be followed by another statement are expressed by the following axioms:

$$\neg(\varphi * \varphi), \tag{20}$$
$$(\varphi * \psi) \Rightarrow \neg(\psi * \varphi), \tag{21}$$
$$((\varphi * \psi) \wedge (\psi * \chi) \Rightarrow (\varphi * \chi). \tag{22}$$

The general properties to be an opinion of a sage are formulated by the axioms:

$$(\psi =_b Q \wedge (\varphi \Rightarrow \psi)) \Rightarrow \varphi =_b Q, \tag{23}$$
$$\varphi =_b Q \Rightarrow \neg(\neg\varphi =_b Q), \tag{24}$$
$$(\varphi \wedge \psi) =_b Q \Rightarrow ((\varphi =_b Q) \wedge (\varphi =_b Q)). \tag{25}$$

The axioms regarding the general properties to be a final decision (*halakhah*):

$$(\mathrm{H}(\psi) \wedge (\varphi \Rightarrow \psi)) \Rightarrow \mathrm{H}(\varphi), \tag{26}$$
$$\mathrm{H}(\varphi) \Rightarrow \neg(\mathrm{H}(\neg\varphi)), \tag{27}$$
$$\mathrm{H}(\varphi \wedge \psi) \equiv (\mathrm{H}(\varphi) \wedge \mathrm{H}(\psi)). \tag{28}$$

The next axioms describing more specific properties of pragmatic-syllogistic connectives were formulated by Rabbi Samuel ha-Nagid, the Spanish poet and Talmudist, born at Cordova 993 and died at Granada 1055, in his famous book, the *Introduction to the Talmud* [מבוא התלמוד]. These axioms are compiled by him from the Talmud and completely accepted in Judaism.

D1 One sage against many: the final decision is like the many, i.e. when the opinion of an individual conflicted with that of the majority, we should choose the opinion of the majority:

$$(\neg(\varphi =_b Q) \wedge \varphi =_b R_1 \wedge \varphi =_b R_2 \wedge \ldots \wedge \varphi =_b R_n) \quad \wedge$$
$$(\mathrm{agst}_\varphi(Q, R_1) \wedge \mathrm{agst}_\varphi(Q, R_2) \wedge \ldots \wedge \mathrm{agst}_\varphi(Q, R_n)) \Rightarrow \mathrm{H}(\varphi).$$

D2 The final decision is never like the disciple when in dispute with his teacher:

$$(\varphi =_b Q \wedge \mathrm{T}(Q, R) \wedge \mathrm{agst}_\varphi(Q, R)) \Rightarrow \neg\mathrm{H}(\varphi).$$

D3 If a later sage is in dispute with an earlier sage, the final decision is like the later sage:

$$(\varphi =_b Q \wedge t(Q, R) \wedge \mathrm{agst}_\varphi(Q, R)) \Rightarrow \mathrm{H}(\varphi).$$

D4 Within one tractate of the Mishnah, a dispute followed by an anonymous statement, representing one of the views, means that the final decision is in accordance with the latter:

$$(\varphi * \psi \wedge (\varphi \equiv \neg\psi) \wedge \mathrm{agst}_\varphi(Q, R) \wedge \mathrm{A}(\psi)) \Rightarrow \mathrm{H}(\psi).$$

D5 Within one tractate of the Mishnah, an anonymous statement followed by statements containing a dispute means that the final decision is not like the anonymous statement:

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ספר מבוא התלמוד לרבי שמואל הנגיד ז"ל
עם קיצור כללי התלמוד מלוקט מספרי הכללים
ראשי תיבות הבאים בכללים
קיצור כללי המשנה
מבוא התלמוד
קיצור כללי המשנה
מבוא התלמוד לרבי שמואל הנגיד
הגהות בן אריה
הגהות הריעב"ץ

Figure 4.1 *The Introduction to the Talmud* by Rabbi Samuel ha-Nagid in the *Brakhot*. The Vilna Edition of the Talmud, printed in Vilna.

$$(\varphi * \psi \wedge (\varphi \equiv \neg \psi) \wedge \text{agst}_{\psi}(Q, R) \wedge \text{A}(\varphi)) \Rightarrow \neg \text{H}(\varphi) .$$

D6 If there is a dispute in the Barayta and an anonymous statement in the Mishnah following one view, the final decision is like the latter:

$$(\varphi \textit{ from the Barayta } \wedge \psi \textit{ from the Mishnah } \wedge (\varphi \equiv \neg \psi) \wedge \text{agst}_{\varphi}(Q, R) \wedge \text{A}(\psi)) \Rightarrow \text{H}(\psi) .$$

D7 If there is a dispute in the Mishnah and an anonymous statement in the Barayta, we do not say the final decision is like the Barayta, because we say "If *Rabbi Yehudah ha-Naśi'* did not teach it, how could *Rabbi Ḥiyy'a*, the editor of the Barayta, know it?"

$$(\varphi \textit{ from the Barayta } \wedge \psi \textit{ from the Mishnah } \wedge (\varphi \equiv \neg \psi) \wedge \text{agst}_{\psi}(Q, R) \wedge \text{A}(\varphi)) \Rightarrow \neg \text{H}(\varphi) .$$

D8 The compilers of the Talmud were *Rav 'Aši* and *Ravina'* and their colleagues:

$$(\varphi \textit{ from the Gemara} \wedge \text{A}(\varphi)) \Rightarrow (\varphi =_b \textit{ Rav 'Aši } \vee \varphi =_b \textit{ Ravina'}).$$

D9 An anonymous statement in the Mishnah belongs to *Rabbi Me'ir*:

$$(\varphi \textit{ from the Mishnah} \wedge \text{A}(\varphi)) \Rightarrow (\varphi =_b \textit{ Rabbi Me'ir}).$$

D10 An anonymous statement in the Tosefta belongs to *Rabbi Neḥemiah*:

$$(\varphi \textit{ from the Tosefta} \wedge \text{A}(\varphi)) \Rightarrow (\varphi =_b \textit{ Rabbi Neḥemiah}).$$

D11 An anonymous statement in the Sifra belongs to *Rabbi Yehudah*:

$(\varphi$ *from the Sifra* $\wedge \mathrm{A}(\varphi)) \Rightarrow (\varphi =_b$ *Rabbi Yehudah*$)$.

D12 An anonymous statement in the Sifre belongs to *Rabbi Šim'on*:

$(\varphi$ *from the Sifre* $\wedge \mathrm{A}(\varphi)) \Rightarrow (\varphi =_b$ *Rabbi Šim'on*$)$.

Notice that by using axioms (12), (16), D12, we can prove that if the statement φ from the Sifre is anonymous, then it belongs to *Rabbi 'Akiba'*, because *Rabbi Šim'on* is a disciple of *Rabbi 'Akiba'*.

D13 "Some say" [יש אומרים] means *Rabbi Natan*:

"Some say φ" $\Rightarrow \varphi =_b$ *Rabbi Natan*.

D14 "Others say" [אחרים אומרים] means *Rabbi Me'ir*:

"Others say φ" $\Rightarrow \varphi =_b$ *Rabbi Me'ir*.

D15 Where *Rabbi Me'ir* is named in a source, and his decision is disputed, either by *Rabbi Yehudah*, *Rabbi Yose*, *Rabbi Šim'on* or *Rabbi 'Eli'ezer ben Ya'qob*, the final decision is like his opponent:

$(\varphi =_b$ *Rabbi Yehudah* $\wedge \mathrm{agst}_\varphi($*Rabbi Me'ir*, *Rabbi Yehudah*$)) \Rightarrow \mathrm{H}(\varphi)$,
$(\varphi =_b$ *Rabbi Yose* $\wedge \mathrm{agst}_\varphi($*Rabbi Me'ir*, *Rabbi Yose*$)) \Rightarrow \mathrm{H}(\varphi)$,
$(\varphi =_b$ *Rabbi Šim'on* $\wedge \mathrm{agst}_\varphi($*Rabbi Me'ir*, *Rabbi Šim'on*$)) \Rightarrow \mathrm{H}(\varphi)$,
$(\varphi =_b$ *Rabbi 'Eli'ezer ben Ya'qob* $\wedge \mathrm{agst}_\varphi($*Rabbi Me'ir*, *Rabbi 'Eli'ezer ben Ya'qob*$)) \Rightarrow \mathrm{H}(\varphi)$.

D16 *Rabbi Yehudah* against *Rabbi Šim'on*: the final decision is like *Rabbi Yehudah*:

$(\varphi =_b$ *Rabbi Yehudah* $\wedge$ agst$_\varphi$(*Rabbi Šim'on*, *Rabbi Yehudah*)) $\Rightarrow$ H(φ).

D17 The final decision is always like *Rabbi Yose*, even against more than one named Tanna:

$(\varphi =_b$ *Rabbi Yose* $\wedge$ agst$_\varphi$(*Rabbi Yose*, Q)) $\Rightarrow$ H(φ).

D18 The Mishnah of *Rabbi 'Eli'ezer ben Ya'qob* is "small but pure"

[משנת ראב"י בק ונקי], i.e. he is not mentioned often, but when he is the final decision is always like him:

$(\varphi =_b$ *Rabbi 'Eli'ezer ben Ya'qob* $\wedge$ agst$_\varphi$(*Rabbi 'Eli'ezer ben Ya'qob*, Q)) $\Rightarrow$ H(φ).

D19 *Rabbi Yehudah ha-Naśi'* against *Rabbi Šim'on ben 'Ele'azar*: the final decision is like *Rabbi Yehudah ha-Naśi'*:

$(\varphi =_b$ *Rabbi Yehudah ha-Naśi'* $\wedge$ agst$_\varphi$(*Rabbi Yehudah ha-Naśi'*, *Rabbi Šim'on ben 'Ele'azar*)) $\Rightarrow$ H(φ).

D20 *Rabbi Yehudah ha-Naśi'* against *Rabbi Yehudah*: the final decision is like *Rabbi Yehudah ha-Naśi'*:

$(\varphi =_b$ *Rabbi Yehudah ha-Naśi'* $\wedge$ agst$_\varphi$(*Rabbi Yehudah ha-Naśi'*, *Rabbi Yehudah*)) $\Rightarrow$ H(φ).

D21 *Rabbi Yehudah ha-Naśi'* against *Rabbi Me'ir*: the final decision is like *Rabbi Yehudah ha-Naśi'*:

($\varphi =_b$ *Rabbi Yehudah ha-Naśi'* $\wedge$ agst$_\varphi$(*Rabbi Yehudah ha-Naśi'*, *Rabbi Me'ir*)) $\Rightarrow$ H(φ).

D22 *Rabbi Yehudah ha-Naśi'* against *Rabbi Yose* in words of *Rabbi Yehudah*: the final decision is like *Rabbi Yehudah ha-Naśi'*:

($\varphi =_b$ *Rabbi Yehudah ha-Naśi'* $\wedge$ agst$_\varphi$(*Rabbi Yehudah ha-Naśi'*, *Rabbi Yose* in words of *Rabbi Yehudah*)) $\Rightarrow$ H(φ).

D23 *Rabbi Yehudah ha-Naśi'* against *Rabbi Yišma"el* in words of *Rabbi Yose*: the final decision is like *Rabbi Yehudah ha-Naśi'*:

($\varphi =_b$ *Rabbi Yehudah ha-Naśi'* $\wedge$ agst$_\varphi$(*Rabbi Yehudah ha-Naśi'*, *Rabbi Yišma'el* in words of *Rabbi Yose*)) $\Rightarrow$ H(φ).

This condition that *Rabbi Yehudah ha-Naśi'* is not against *Rabbi Yišma'el* directly because of the restriction defined in D2, D3.

D24 *Rabbi Yehudah ha-Naśi'* against *Rabban Šim'on ben Gamli'el*: the final decision is like *Rabban Šim'on ben Gamli'el*:

($\varphi =_b$ *Rabban Šim'on ben Gamli'el* $\wedge$ agst$_\varphi$(*Rabbi Yehudah ha-Naśi'*, *Rabban Šim'on ben Gamli'el*)) $\Rightarrow$ H(φ).

From this statement, we can assume that *Rabbi Yehudah ha-Naśi'* is a disciple of *Rabban Šim'on ben Gamli'el*, taking into account D2.

D25 Wherever *Rabban Šim'on ben Gamli'el* appears in the Mishnah and there is no dispute, the final decision is like him:

($\varphi =_b$ *Rabban Šim'on ben Gamli'el* $\wedge$ $\neg$agst$_\varphi$(*Rabban Šim'on ben Gamli'el*, Q)) $\Rightarrow$ H(φ).

D26 *Rabbi 'Eli'ezer* against *Rabbi Yehošu'a*: the final decision is like *Rabbi Yehošu'a*:

$(\varphi =_b$ *Rabbi Yehošu'a* $\wedge$ agst_φ(*Rabbi 'Eli'ezer*, *Rabbi Yehošu'a*)) $\Rightarrow \mathrm{H}(\varphi)$.

D27 *Rabbi 'Eli'ezer* against *Rabbi Gamali'el*: the final decision is like *Rabbi Gamali'el*:

$(\varphi =_b$ *Rabbi Gamali'el* $\wedge$ agst_φ(*Rabbi 'Eli'ezer*, *Rabbi Gamali'el*)) $\Rightarrow \mathrm{H}(\varphi)$.

D28 Wherever *Rabban Šim'on ben Gamli'el* appears in the Mishnah the final decision is like him, except in four named cases:

$(\varphi =_b$ *Rabban Šim'on ben Gamli'el* $\wedge$ agst_φ(*Rabban Šim'on ben Gamli'el*, Q)) $\Rightarrow (\mathrm{H}(\varphi) \vee (\mathrm{exc}_4$(*Rabban Šim'on ben Gamli'el*) $\Rightarrow \varphi))$.

D29 The final decision does not follow the disciples of *Rabbi 'Eli'ezer*, except in eight named cases:

$(\varphi =_b Q \wedge \mathrm{T}(Q,$ *Rabbi 'Eli'ezer*$) \wedge \mathrm{agst}_\varphi(Q, R)) \Rightarrow (\neg\mathrm{H}(\varphi) \vee (\mathrm{exc}_8(Q) \Rightarrow \varphi))$.

D30 Shammai's party against Hillel's party: the *halakha* is like Hillel's party, except in six named cases of Hillel's party and in three named cases of Shammai's party:

$(\varphi =_b Q \wedge \mathrm{T}(Q,$ *Hillel*$) \wedge \mathrm{T}(R,$ *Shammai*$) \wedge \mathrm{agst}_\varphi(Q, R)) \Rightarrow (\mathrm{H}(\varphi) \vee (\mathrm{exc}_6(Q) \Rightarrow \varphi) \vee (\mathrm{exc}_3(R) \Rightarrow \varphi))$.

D31 *Rab* against *Šmu'el*: the final decision is like *Rab* in prohibitions and like *Šmu'el* in laws:

$(\varphi =_b$ *Šmu'el* $\wedge \neg\psi =_b$ *Rab* $\wedge$ agst$_\varphi$(*Rab*, *Šmu'el*) $\wedge$ agst$_{\neg\psi}$(*Rab*, *Šmu'el*)) $\Rightarrow$ (H(φ) $\wedge$ H($\neg\psi$)).

D32 *Rav Ḥisda'* against *Rav Huna'*: the final decision is like *Rav Huna'*:

$(\varphi =_b$ *Rav Huna'* $\wedge$ agst$_\varphi$(*Rav Huna'*, *Rav Ḥisda'*)) $\Rightarrow$ H(φ).

D33 *Rav Šešet* against *Rav Naḥman*: the final decision is like *Rav Šešet* in prohibitions and like *Rav Naḥman* in laws:

$(\varphi =_b$ *Rav Naḥman* $\wedge \neg\psi =_b$ *Rav Šešet* $\wedge$ agst$_\varphi$(*Rav Šešet*, *Rav Naḥman*) $\wedge$ agst$_{\neg\psi}$(*Rav Šešet*, *Rav Naḥman*)) $\Rightarrow$ (H(φ) $\wedge$ H($\neg\psi$)).

D34 *Rav Yehudah* against *Rabbah*: the final decision is like *Rav Yehudah*:

$(\varphi =_b$ *Rav Yehudah* $\wedge$ agst$_\varphi$(*Rav Yehudah*, *Rabbah*)) $\Rightarrow$ H(φ).

D35 *Rabbah* against *Rav Yosef*: the final decision is like *Rabbah*, except in three named cases:

$(\varphi =_b$ *Rabbah* $\wedge$ agst$_\varphi$(*Rabbah*, *Rav Yosef*)) $\Rightarrow$ (H(φ) $\vee$ (exc$_3$(*Rabbah*) $\Rightarrow \varphi$).

D36 *Rav 'Aḥa'* against *Ravina'*: the final decision is like *Ravina'*, except in three named cases:

$(\varphi =_b$ *Ravina'* $\wedge$ agst$_\varphi$(*Ravina'*, *Rav 'Aḥa'*)) $\Rightarrow$ (H(φ) $\vee$ (exc$_3$(*Ravina'*) $\Rightarrow \varphi$).

D37 The final decision is like *Mar bar Rav 'Aši* except where he is in dispute with *Rav 'Aši*:

$(\varphi =_b$ *Mar bar Rav 'Aši* $\wedge \neg(Q \equiv$ *Rav 'Aši*$) \wedge \mathrm{agst}_{\varphi}($*Mar bar Rav 'Aši*$, Q)) \Rightarrow \mathrm{H}(\varphi)$;

$(\varphi =_b$ *Rav 'Aši* $\wedge \mathrm{agst}_{\varphi}($*Rav 'Aši*, *Mar bar Rav 'Aši*$)) \Rightarrow \mathrm{H}(\varphi)$.

This additional axiom is needed because *Rav 'Aši* is his teacher (see D2, D3).

Models for Judaic pragmatic-syllogistics is defined as follows:

Definition 4.9 *A structure* $\mathbf{M} = \langle O = \{\dot{O} : \dot{O} \in Q_2\}, I, \dot{\mathrm{A}}, \dot{\mathrm{H}}, \doteq_b, \dot{\mathrm{T}}, \dot{t}, \dot{*}, \dot{\mathrm{exc}}_n, \dot{\mathrm{agst}} \rangle$ *is a Judaic pragmatic-syllogistic model iff:*

1. O is the set of objects including 28 sages named above.
2. I associates each connective $o \in \{\mathrm{A}, \mathrm{H}, =_b, \mathrm{T}, t, *, \mathrm{exc}_n, \mathrm{agst}\}$ over formulas of P with an appropriate connective $\dot{o} \in \{\dot{\mathrm{A}}, \dot{\mathrm{H}}, \doteq_b, \dot{\mathrm{T}}, \dot{t}, \dot{*}, \dot{\mathrm{exc}}_n, \dot{\mathrm{agst}}\}$.
3. $\dot{\mathrm{A}}$ is a mapping from $\mathbf{F}/\Leftrightarrow$ onto {0, 1} satisfying conditions (12), D7—D12.
4. $\dot{\mathrm{H}}$ is a mapping from $\mathbf{F}/\Leftrightarrow$ onto {0, 1} satisfying conditions (26), (27), (28), D1—D7, D15—D37.
5. $\doteq_b$ is a relation on $(\mathbf{F}/\Leftrightarrow) \times O$ satisfying conditions (9), (10), (11), (16), (23)—(25), D1, D2, D3, D8—D37.
6. $\dot{\mathrm{T}}$ is a quasi (less-than) relation on $O \times O$ satisfying the conditions (13)—(16), D2, D29, D30.
7. $\dot{t}$ is a quasi (less-than) relation on $O \times O$ satisfying the conditions (17)—(19), D3.

8. $\dot{*}$ is a quasi (less-than) ordering relation on **F** such that $[\alpha]\dot{*}[\beta]$ is read as "$[\alpha]$ is less than $[\beta]$," where $[\alpha]\dot{*}[\beta] \in \mathbf{F}/\Leftrightarrow$, it satisfies conditions D4, D5 also.

9. $\dot{\text{e}}\text{x}\dot{\text{c}}_n$ is a mapping from O into $(\mathbf{F}/\Leftrightarrow)^n$ satisfying conditions D28—D30, D35, D36.

10. $\dot{\text{a}}\text{g}\dot{\text{s}}\text{t}$ is a mapping from $\mathbf{F}/\Leftrightarrow$ into $O \times O$ satisfying conditions (7)—(12), (16), D1—D7, D15—D37.

The truth conditions of Boolean combinations of Judaic pragmatic-syllogistic formulas in a Judaic pragmatic-syllogistic model are understood in the way:

Definition 4.10

$\mathsf{M} \models \neg\phi$ iff $\mathsf{M} \not\models \phi$

$\mathsf{M} \models \phi \wedge \psi$ iff $\mathsf{M} \models \phi$ and $\mathsf{M} \models \psi$

$\mathsf{M} \models \phi \vee \psi$ iff $\mathsf{M} \models \phi$ or $\mathsf{M} \models \psi$

$\mathsf{M} \models \phi \Rightarrow \psi$ iff $\mathsf{M} \models \neg\phi$ or $\mathsf{M} \models \psi$

Thus, Judaic pragmatic-syllogistics allows us to take a final decision within disputes of the sages. It defines pragmatic limits for Talmudic debates.

Judaic argumentation theory considers pure formal-logical aspects of three groups of proofs from the pragmatic standpoint: proofs as trees, proofs as parallel processes, proofs as competitive communication. All groups can be regarded from the viewpoint of symbolic logic.

5. Centralized reasoning in Christianity and Platonism

In Christianity there are other attitudes toward debates concerning the subject of religious belief and behavior rules. The specificity of these attitudes can be revealed by means of analyzing the First Council of Nicaea, the council of Christian bishops

convened in Nicaea in Bithynia (present-day İznik in Turkey) by the Roman Emperor Constantine I.

Constantine, summoning the Council, became the first Roman Emperor accepting Christianity. His law issued in the year 321 concerned the right of churches to get real estate and to own it. Through the whole Empire Christian temples have been built up, sometimes for their construction the pagan temples have been used; a number of well-known pagan temples were destroyed by Constantine's command. The Council was the first effort to attain consensus in the Church through an assembly representing all of Christendom. The first ceremonial meeting of the Council occurred on May, 20th 325, and the Council proceeded till August, 25th of the same year. The chair belonged to Eustathius of Antioch and in his absence to Pope Alexander of Alexandria. The Emperor was a honorable chairman. After meetings about the main subject, the first Creed of the Roman Catholic Church was stated at the Council's session on June, 19th.

This Council was summoned by the most powerful person of that time, Constantine I who achieved the individual power over the whole Roman Empire. Constantine the Great began to consider the Church as an integral part of the Empire and himself as the head over all church affairs, not dealing with the doctrine:

> Vos quidem, inquit, in iis quae intra Ecclesiam sunt episcopi estis, ego vero in iis quae extra geruntur episcopus a Deo sum constitutus.
>
> ἀλλ' ὑμεῖς μὲν τῶν εἴσω τῆς ἐκκλησίας, ἐγὼ δὲ τῶν ἐκτὸς ὑπὸ θεοῦ καθεσταμένος ἐπίσκοπος ἂν εἴην.
>
> You are bishops whose jurisdiction is within the Church: I also am a bishop, ordained by God to overlook whatever is external to the Church.

Giving to Christianity the special status within the Empire and supporting the Church, Constantine was actively achieving the consolidation of the Roman Catholic (Greek: καθολικὴ, universal)

Church as the condition of unifying the Empire. He also tried to act as judge in all interchurch disputes. The goal of the First Council of Nicaea consisted, first of all, in the Church unification.

According to Constantine's statement, the doctrine of God was presented at the Council, and all who did not consent to this doctrine were being guided by the devil. The dissent is not admissible; the decision should be accepted unanimously:

> We have received a complete blessing from Divine Providence, namely, we have been relieved from all error and been united in a common confession of one and the same faith. The devil will no longer have any power against us, since all the schemes he in his hatred had devised for our destruction, have been entirely overthrown from their foundations. At the command of God, the splendor of truth has dissolved all the poisons so deadly to unity: dissensions, schisms, commotions, and the like. We all now worship the One by name, and continue to believe that he is the One God. In order to accomplish all of this, at God's summoning I assembled a large number of bishops at the city of Nicaea, and I joined them in investigating the truth, though I am only one of you, who rejoices exceedingly in being your fellow-servant. All points which seemed ambiguous or could possibly lead to dissension have been discussed and accurately examined. May the Divine Majesty forgive the unfortunately huge number of the blasphemies which some were shamelessly uttering against the mighty Savior, our life and hope, as they declared and confessed things contrary to the divinely inspired Scriptures.
>
> More than three hundred bishops, remarkable for their moderation and intellectual keenness, were unanimous in their confirmation of one and the same faith, a faith which has arisen in agreement with the truths of the Law of God. Arius alone had been misled by the devil, and was found to be the only one set on promoting this unholy mischief, first among you, and afterwards among others as well …
>
> This ruling, made by the collective judgment of three hundred bishops, cannot be other than the doctrine of God, especially where the Holy Spirit has illuminated the divine will by placing it upon the minds of so many dignified persons …

(Constantine Augustus, *The letter to the catholic church of the Alexandrians*, see *Bright*).

About the same we read in the Letter of the Council of Nicaea to the Egyptian Church:

> To the great church of the Alexandrians, which is holy by the grace of God, and to our beloved brothers throughout Egypt, Libya, and the Pentapolis. We bishops assembled at Nicaea, constituting the great and holy council, send greetings in the Lord.
> Since, by the grace of God, a great and holy council has been convened at Nicaea, after our most pious sovereign Constantine summoned us out of various cities and provinces for that purpose, we at the sacred council thought it most necessary to write you a letter, in order that you may know what subjects were considered and examined, and what was eventually decided on and decreed. In the first place, the impiety and guilt of Arius and his adherents was examined in the presence of our most pious emperor Constantine.
> We unanimously decided that his impious opinion should be anathematized, with all the blasphemous expressions he has uttered, namely that "the Son of God came to be out of nothing," that "there was a time when he was not," and even that "the Son of God, because he possessed free will, was capable of either both evil and good." They also call him a creature (ktisma) and a work (poiēma) (*Letter of the Council of Nicaea to the Egyptian Church*, see *Bright*).

The creed drawn up at the Council of Nicaea in 325 is significantly different than the creed often referred to as the "Nicene Creed." The latter is actually the Constantinopolitan Creed from the Council of Constantinople in 381. The actual Nicene Creed is as follows:

> Πιστεύομεν εἰς ἕνα θεὸν πατέρα παντοκράτορα, πάντων ὁρατῶν τε καὶ ἀοράτων ποιητήν. Καὶ εἰς ἕνα κύριον Ἰησοῦν Χριστὸν τὸν υἱὸν τοῦ θεοῦ, γεννηθέντα ἐκ τοῦ πατρὸς μονογενῆ, τουτέστιν

ἐκ τῆς οὐσίας τοῦ πατρὸς, θεὸν ἐκ θεοῦ, φῶς ἐκ φωτός, θεὸν ἀληθινὸν ἐκ θεοῦ ἀληθινοῦ, γεννηθέντα, οὐ ποιηθέντα, ὁμοούσιον τῷ πατρὶ, δι᾽ οὗ τὰ πάντα ἐγένετο, τά τε ἐν τῷ οὐρανῷ καὶ τὰ ἐν τῇ γῇ· τὸν δι᾽ ἡμᾶς τοὺς ἀνθρώπους καὶ διὰ τὴν ἡμετέραν σωτηρίαν κατελθόντα καὶ σαρκωθέντα, ἐνανθρωπήσαντα, παθόντα, καὶ ἀναστάντα τῇ τριτῇ ἡμέρᾳ, ἀνελθόντα εἰς οὐρανούς, καὶ ἐρχόμενον κρῖναι ζῶντας καὶ νεκρούς. Καὶ εἰς τὸ ἅγιον πνεῦμα. Τοὺς δὲ λέγοντες· ἦν ποτε ὅτε οὐκ ἦν, καὶ πρὶν γεννηθῆναι οὐκ ἦν, καὶ ὅτι ἐξ οὐκ ὄντων ἐγένετο, ἢ ἐξ ἑτέρας ὑποστάσεως ἢ οὐσίας φάσκοντας εἶναι, ἢ κτιστὸν ἢ τρεπτὸν ἢ ἀλλοιωτὸν τὸν υἱὸν τοῦ θεοῦ, ἀναθεματίζει ἡ καθολικὴ ἐκκλησία (*August Hahn*).

We believe in one God, the Father Almighty, Maker of all things visible and invisible: and in one Lord Jesus Christ, the Son of God, begotten of the Father, Only-begotten, that is, from the essence of the Father; God from God, Light from Light, true God from true God, begotten not made, of the same being as the Father, by whom all things were made, both things in heaven and things on earth; who for us men and for our salvation came down and was made flesh, was made man, suffered, and rose again the third day, ascended into heaven, and will come to judge the living and the dead; and we believe in the Holy Spirit. But those who say, 'Once he did not exist,' and 'He did not exist before he was begotten,' and 'He came to be from nothing,' or those who pretend that the Son of God is 'of another subsistence (*hypostasis*) or being (*ousia*),' or 'created'(*ktistos*), or 'alterable' (*treptos*), or 'changeable' (*alloiōtos*), the catholic church anathematizes.

In the next place, all the Roman Catholic and Orthodox Christians have considered the given creed as the conclusive foundations of theology which cannot be a subject of any doubts or discussions. The divinity of Jesus is affirmed in this creed, which definitively breaks off all communication between Christianity and Judaism. The anti-Judaic orientation of the Nicene Council was

brightly showed in the decision that the date of Easter must be calculated in another way than it has been done by the Jews. Constantine Augustus, negatively disposed towards the Jews, "father-killers," writes as follows:

> At the council we also considered the issue of our holiest day, Easter, and it was determined by common consent that everyone, everywhere should celebrate it on one and the same day … And in the first place, it seemed very unworthy for us to keep this most sacred feast following the custom of the Jews, a people who have soiled their hands in a most terrible outrage, and have thus polluted their souls, and are now deservedly blind … Therefore have nothing in common with that most hostile people, the Jews. We have received another way from the Savior … On what subject are they competent to form a correct judgment, who, after that murder of their Lord lost their senses, and are led not by any rational motive, but by an uncontrollable impulsiveness to wherever their innate fury may drive them? … So first, it was desirable to change the situation so that we have nothing in common with that nation of father-killers who slew their Lord. Second, the order which is observed by all the churches of the western, southern, and northern parts, and by some also in the eastern is quite suitable. Therefore, at the current time, we all thought it was proper that you, intelligent as you are, would also cheerfully accept what is observed with such general unanimity of sentiment in the city of Rome, throughout Italy, Africa, all Egypt, Spain, France, Britain, Libya, the whole of Greece, and the dioceses of Asia, Pontus, and Cilicia (Constantine Augustus, *The letter to the churches*, see *Bright*).

The Nicene Council was the first ecumenical council, i.e. the first conference of ecclesiastical dignitaries and theological experts convened to discuss and settle matters of Church doctrine and practice. This conference built up the tradition of other ecumenical councils. Due to these councils, Christianity became the ecumenical doctrine with the metaphysics of total unity when the dissent is not supposed on a broader scale. After all the ecumenical councils have

given the final decision, what opinion is to be considered as Roman Catholic and what as heresy. Besides henceforth, Christianity has been the official religion of the whole Ecumene. And the Christian doctrine started to play role of political ideology.

In Judaism this specificity of Christianity after Constantine Augustus, its rigid connection with notions of empire, force, power, unequivocal decisions, began to be expressed by the statement that Romans (Christians) are descendants of Esau. This idea is widely reflected in Rashi's commentary on the Torah. In these commentaries he several times stated that Esau was embodied by the Roman Empire or Christianity. For example, in his opinion, Esau's brutal strength and Isaac's words to him: "Yet by your sword you shall live" (*Gen.* 27:40), were verified by the Romans, who destroyed the Second Temple. Furthermore, when *Gen.* 25:23 reads: "Two nations in your womb," Rashi refers to the Roman Caesar Antoninus and *Rabbi Yehudah ha-Naśi'* representing Rome and Israel in their mother's womb:

> **שני גוים בבטנך:** גיים כתיב, אלו אנטונינוס ורבי, שלא פסקו מעל שולחנם
> לא צנון ולא חזרת לא בימות החמה ולא בימות הגשמים:
> **ושני לאמים:** אין לאום אלא מלכות:
> **ממעיך יפרדו:** מן המעים הם נפרדים, זה לרשעו וזה לתומו:
> **מלאם יאמץ:** לא ישוו בגדולה, כשזה קם זה נופל, וכן הוא אומר (יחזקאל
> כו ב) אמלאה החרבה, לא נתמלאה צור אלא מחורבנה של ירושלים:
>
> *Two nations are in your womb:* [The word גּוֹיִם] is written גֵּיִים [which is pronounced] like גֵּאִים (exalted persons). These were Antoninus and Rabbi [Judah the Prince], from whose tables neither radishes nor lettuce were lacking either in the summer or in the winter [*Avodah Zarah* 11a].
> *and two kingdoms:* לְאֻמִּים always means a kingdom [*Avodah Zarah* 2b].
> *will separate from your innards:* From the womb they are separated, this one to his wickedness, and this one to his innocence.
> *will become mightier than the other kingdom:* They will not be equal in greatness; when one rises, the other will fall, and so [Scripture] states [*Ezek.* 26:2]: "I shall become full from the destroyed

> city." Tyre became full [gained power] only from the destruction of Jerusalem [*Megillah* 6a, *Pes.* 42b].

Hence, in Judaism Rome and Christianity are perceived as the incarnation of Esau. According to the Biblical text and its Judaic commentaries, Esau was not inclined to dialogue and sophisticated debate, but he was capable to domination and to establishing large kingdoms. In a sense it turned out in this way. Rashi, living in the 11th century, actually foretold the growth of power and greatness of the European (Christian) people. The European (Christian) civilization more than others has determined the civilized dynamics of mankind. The councils after Nicaea have made Christianity the ecumenical religion. Since the first Council Christianity has lost Judaic (and Semitic on a broader scale) roots, but has become an integral part of the European culture.

The ecumenical Christianity started to take up with Platonism. In due course Platonism became the methodological foundations of Christian theology. The main assumption of Platonism is that out of our world there exists also the world of ideas and it exists not figuratively, but as something completely real. It would seem what is bad in the assumption of real existence of the ideas world? Actually, such an assumption of Platonism puts special frames in the action theory. It justifies setting up supervaluable tasks.

In the Platonic pattern of human activity we cannot be limited to individual agents, concrete people. We have supervaluable, most important tasks, i.e. we have a superpure world of ideas. Therefore we have some ideal, invented agents who can be presented either by virtual collective agents, which ostensibly exist (to say, 'proletariat' for Marxists), or by emperors, leaders, saints who ostensibly possess special sacral abilities to see supervaluable tasks. To collective agents and to the leader's superstatus one usually refers when one wishes to manipulate other people, to inspire that there is the most important task which needs to be obeyed.

Any totalitarian or authoritarian society is being constructed on the basis of Platonic assumptions. Platon became the first-ever ideologist of the totalitarian organization. In a totalitarian or authoritarian state, supervaluable tasks are always spread, the Platonic world of ideas is implicitly preached. In his model of the ideal state Platon was the first who substantiated totalitarianism. Another ideologist of totalitarianism, Shang Yang, the representative of the Chinese culture, substantiated the totalitarian model in hundred years after Platon. By the way, the work by Shang Yang, the *Book of Lord Shang*, was the reference book of Mao Zedong.

Platon's idealism has pagan roots, because it postulates the ideal status of uncountably infinite subjects (gods), though it is considered that all pure ideas (gods) result from the One. As a matter of fact, Platonism conflicts with Judaic-Christian and Islamic values. According to Abrahamian religions, only one God possesses the status of ideal subject. He is the one, a unique. There is no world of ideas-gods absolutely!

In the approach of massive-parallel computation the following assumptions are contained: first, there are only individual agents of activity (collective agents and sacral leaders are not present in fact), second, adequate examples of ideal entities in the Platonic meaning are not more than stabilized for a while complexes of massive-parallel patterns of activity with individual agents.

As we see, the assumption of massive parallelism of processes contradicts to Platonism. There are no ideal entities in the Platonic meaning. There are patterns of activity which have been realized by simultaneously many agents, i.e. there are massive-parallel patterns of activity. Notice that modern logic allows us to mathematically formalize many of such patterns. Moreover, the idea of massive parallelism completely corresponds to democratic values, unlike of Platonism.

Platonism in mathematics showed its inconsistency when it was clarified that in mathematics there are unsolvable problems, tasks for which it is possible to prove effectively that their solution is impossible. It means that mathematical concreteness does not exist initially until the mathematician starts the task solution. Mathematician's activity is a kind of designing the mathematical concreteness from nothing. The mathematician cannot solve any task, let be well formulated, because any activity has the pragmatic limits and mathematician's activity too. This activity is defined not by images, eidoses, intuition, but by the game rules. If in mathematics mathematicians' game rules change, then at least the new mathematics appears. Indeed, there are many mathematics, with the choice axiom and without it, with the foundation axiom and without it, with the Archimedes axiom and without it, with the Church-Turing thesis and without it, etc. In all these mathematics there is an absolutely different concreteness because their game rules are basically updated.

Of course, intuition (e.g. the eidetic/Platonic approach) concerns mathematical knowledge in any way, but it is so far mediated, as if a link between mathematics and religious belief or sexual orientation. The mathematical concreteness is being born if and only if the way of how this concreteness has been designed is shown. An appropriate construction should satisfy thus the game rules of that mathematics in which frames it is shown.

Chapter 4. Anti-Platonic logic and mathematics

1. Platonism in logic and mathematics

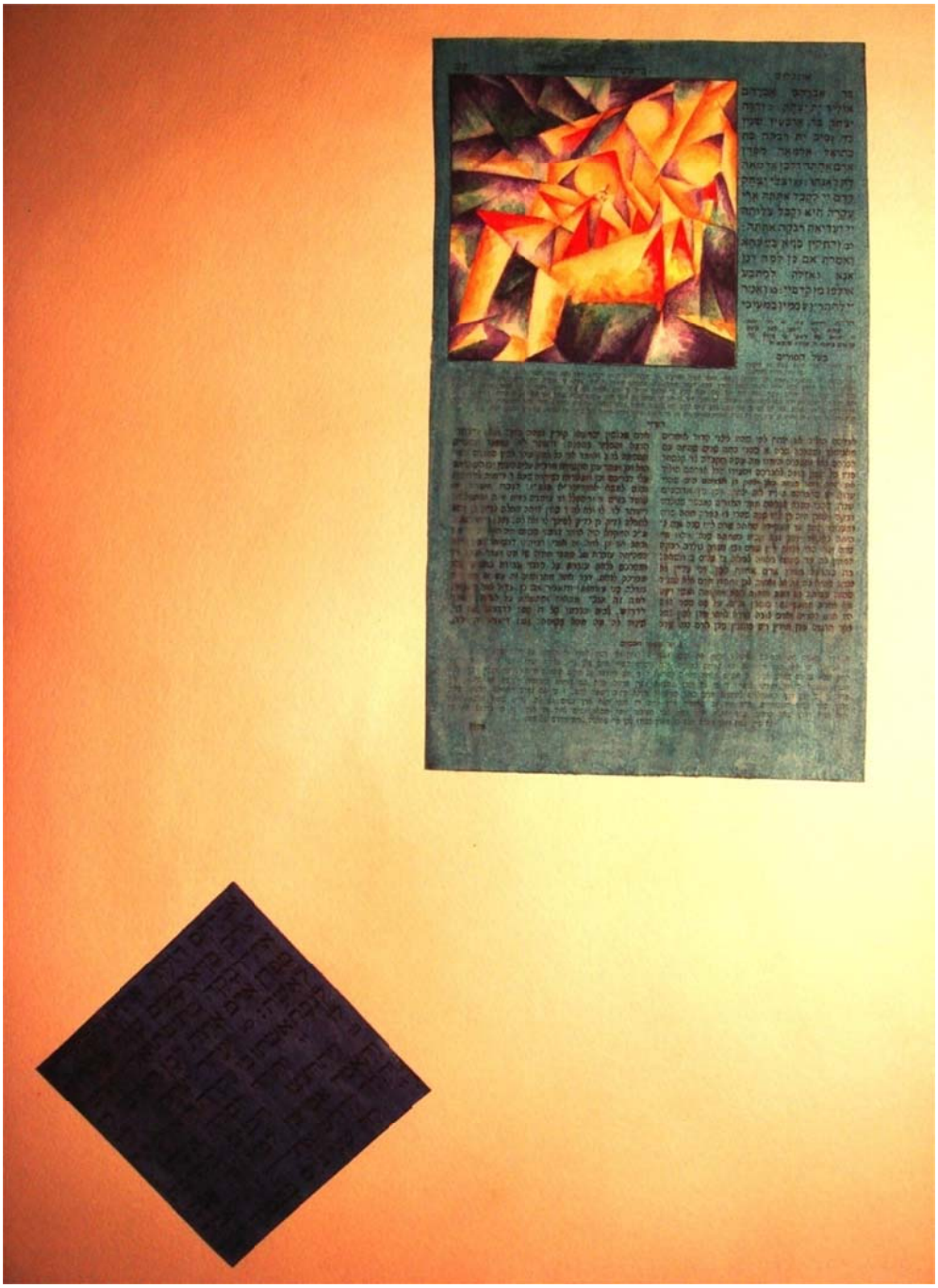
Platonism has made considerable impact on the Western culture and science by assuming ideal entities which should be regarded as the base, the first premises in the theoretical analysis of any phenomena:

> I was afraid that by observing objects with my eyes and trying to comprehend them with each of my other senses I might blind my soul altogether. So I decided that I must have recourse to theories, and use them in trying to discover the truth about things I started off in this way, and in every case I first lay down the theory [principle] which I judge to be soundest, and then whatever seems to agree with it—with regard either to causes or to anything else—I assume to be true, and whatever does not I assume not to be true (*Phaedo* 99d—100a).

The presuppositions of Platonism allowed to generate the axiomatic method in mathematics. According to Platonism, there exist entities with a conclusive absolute status of existence (primary objects of mathematics, e.g. notions of point or line in geometry) and there are propositions with the highest persuasiveness and evidence (axioms). Actually, Platonism is a corner stone of classical mathematics. The Platonic position in mathematics was well stated by Charles Hermite (1822—1901):

> I believe that the numbers and functions of analysis are not the arbitrary product of our spirits: I believe that they exist outside us with the same character of necessity as the objects of objective reality; and we find or discover them and study them as do the physicists, chemists and zoologists.

Many mathematicians have estimated Platonism as an integral part of mathematical thinking:



Picture 8 The birth of Isaac

Picture 9 The birth of Isaac. Fragment

> The value of platonistically inspired mathematical conceptions is that they furnish models of abstract imagination. These stand out by their simplicity and logical strength. They form representations which extrapolate from certain regions of experience and intuition (*Bernays*).

The axiomatic method is aimed to obtain a maximum of information from a minimum of premises. David Hilbert in his program of foundations of mathematics considered the axiomatic method (respectively, Platonic assumptions) as a paramount factor of mathematical thinking. He proposed to convert all the existing mathematics into a formal theory and to prove the consistency of this formal theory (i.e. prove that no proposition can be proved and disproved in it simultaneously). As a privileged part of mathematics, he considered an elementary number theory, which relies only on a "purely intuitive basis of concrete signs." The formal theory obtained by Hilbert should be constructed on the base of axioms of two types: (i) logical axioms which set reasoning rules and (ii) non-logical axioms by which primary object of mathematics are defined.

According to Platonism, mathematical entities, which we obtain from primary terms and axioms, are eternal and unchanging. This means that, on the one hand, they exist in fact and, on the other hand, they are abstract and have supranatural properties instead of spatiotemporal or causal ones. Plato says about as follows:

> The sphere of the intelligible will also have two divisions,— one of mathematics, in which there is no ascent but all is descent; no inquiring into premises, but only drawing of inferences. In this division the mind works with figures and numbers, the images of which are taken not from the shadows, but from the objects, although the truth of them is seen only with the mind's eye; and they are used as hypotheses without being analysed. Whereas in the other division reason uses the hypotheses as stages or steps in the ascent to the idea of good, to which she fastens them, and then again descends, walking firmly in the region of ideas, and of ideas only, in her ascent as well as

> descent, and finally resting in them. 'I partly understand,' he replied; 'you mean that the ideas of science are superior to the hypothetical, metaphorical conceptions of geometry and the other arts or sciences, whichever is to be the name of them; and the latter conceptions you refuse to make subjects of pure intellect, because they have no first principle, although when resting on a first principle, they pass into the higher sphere.' You understand me very well, I said. And now to those four divisions of knowledge you may assign four corresponding faculties—pure intelligence to the highest sphere; active intelligence to the second; to the third, faith; to the fourth, the perception of shadows—and the clearness of the several faculties will be in the same ratio as the truth of the objects to which they are related... (*The Republic* (Jowell translation). Book VII).

The main question of mathematical ontology and epistemology, whether mathematics is discovered or invented, was answered by Plato and Platonists rigorously. They claimed that mathematical entities exist indeed as abstract eternal objects that were discovered, not invented. According to this claim, the mathematical realm is considered as plenitudinous. Such an approach is regarded by Marc Balaguer as full-blooded Platonism. In his words, the pure Platonistic point of view is expressed as follows: "all the mathematical objects (logically possibly) could exist actually do exist" (*Balaguer*). Formally:

$$\forall x\,[(x \text{ is a mathematical object } \&\ x \text{ is logically possible}) \Rightarrow x \text{ exists}]$$

This approach was fully embodied in modern logic and mathematics. Since David Hilbert, logicians and mathematicians have been sure that if a mathematical entity is correctly defined within axiomatic method, then it really exists as eternal object:

> An example of this way of setting up a theory can be found in Hilbert's axiomatization of geometry. If we compare Hilbert's axiom system to Euclid's, ignoring the fact that the Greek geometer fails to include certain [necessary] postulates, we

> notice that Euclid speaks of figures to be constructed, whereas, for Hilbert, system of points, straight lines, and planes exist from the outset. Euclid postulates: One can join two points by a straight line; Hilbert states the axiom: Given any two points, there exists a straight line on which both are situated. "Exists" refers here to existence in the system of straight lines (*Bernays*).

The world of eternal objects that has been assumed in modern logic and mathematics was considered by Gottlob Frege as *das dritte Reich* (the third world) that exists independently on the physical world (the first world) and our subjective impressions, fictions and imaginations (the second world). *Das dritte Reich* contains all interpersonal senses (Sinne):

> Die Gedanken sind weder Dinge der Außenwelt noch Vorstellungen. Ein drittes Reich muß anerkannt werden.
>
> The thoughts neither are things of physical world nor impressions. A third world should be accepted (*Frege 1918/19*).

However, we could prove that modern logic and mathematics may be established without *das dritte Reich*, i.e. without axiomatic method and Platonic presuppositions. In the beginning, let us show, where exactly there are Platonic assumptions in modern logic and mathematics in order to avoid them.

2. Classical logic

Let us take a set consisting of k_1 one-placed predicates, k_2 two-placed predicates, k_n n-placed predicates, P_{11}, P_{12}, …, P_{1k1}, P_{21}, …, P_{2k2}, …, P_{nkn}, i.e. P_{ij} means a j-th i-placed predicate. Let x, y, z, … are individual variables, i.e. their values are individuals (specific names). A proposition of the form $P_{ij}(x_1, x_2, \ldots, x_i)$ is said to be *atomic*. Take now logical connectives ε (' … is …'), & (conjunction), ∨ (disjunction), ⇒, (implication), ¬ (negation). Then *formulas* are defined as follows:

- Each atomic formula is a formula.
- If P_{il}, P_{im} are atomic formulas, then $P_{il}(x_1, x_2, \ldots, x_i) \varepsilon P_{im}(x_1, x_2, \ldots, x_i)$ is a formula (e.g. let H be a predicate to be a human being and M to be mortal. Both predicates are one-placed. Then the formula $H(x) \varepsilon M(x)$ has the meaning: "The human being is mortal").
- If Φ_1, Φ_2 are formulas, then $\Phi_1 \& \Phi_2$, $\Phi_1 \vee \Phi_2$, $\Phi_1 \Rightarrow \Phi_2$, $\neg\Phi_1$, $\neg\Phi_2$ are formulas.

A formula of the form $P_{il}(x_1, x_2, \ldots, x_i) \varepsilon P_{im}(x_1, x_2, \ldots, x_i)$ is called *quasi-atomic*.

Let us take an algebraic system M consisting of k_1 one-placed relations, k_2 two-placed relations, k_n n-placed relations, R_{11}, R_{12}, …, R_{1k1}, R_{21}, …, R_{2k2}, …, R_{nkn}, i.e. R_{ij} means j-th i-placed relations. We assume that these relations are defined on a set A. Then we can assign values on the set A to individual variables $x, y, z, \ldots$ by means of an interpretation I. In other words, $I(x) \in A$. Let Γ be the set of propositions of our formal language. Taking into account that for each predicate P_{ij} there is a relation R_{ij} of algebraic system M, we can consider M as a model of the set Γ, it is denoted by $M \models \Gamma$, if the following conditions are satisfied for all formulas Φ_1, Φ_2 of Γ:

$I(P_{ij}(x_1, x_2, \ldots, x_i)) = R_{ij}(I(x_1), I(x_2), \ldots, I(x_i)) \in A^i$, i.e. $M \models R_{ij}(I(x_1), I(x_2), \ldots, I(x_i))$;

$M \models \neg \Phi_1$ iff it is false that $M \models \Phi_1$;

$M \models \Phi_1 \& \Phi_2$ iff $M \models \Phi_1$ and $M \models \Phi_2$;

$M \models \Phi_1 \vee \Phi_2$ iff $M \models \Phi_1$ or $M \models \Phi_2$;

$M \models \Phi_1 \Rightarrow \Phi_2$ iff from $M \models \Phi_1$ it follows that $M \models \Phi_2$;

$M \models P_{il}(x_1, x_2, \ldots, x_i) \varepsilon P_{im}(x_1, x_2, \ldots, x_i)$ iff $M \models P_{il}(x_1, x_2, \ldots, x_i)$ and $M \models P_{il}(x_1, x_2, \ldots, x_i) \Rightarrow P_{im}(x_1, x_2, \ldots, x_i)$.

A *proof of formula* Φ is a sequence of formulas $\Phi_1, \ldots, \Phi_n$ such that $\Phi_n = \Phi$ and for every i ($i \leq n$) the formula Φ satisfies one of the following conditions:

- Φ_i is an axiom;
- Φ_i is entailed from Φ_j $\{i < j\}$ by one of the inference rules.

If for the formula Φ there is its proof, then Φ is called a *theorem* or *provable formula*. In this case we write $\vdash \Phi$.

A *deduction of formula* Φ from the set of formulas Γ is a sequence of formulas $\Phi_1, \ldots, \Phi_n$ such that $\Phi_n = \Phi$ and for every i ($i \leq n$) the formula Φ satisfies one of the following conditions:

- Φ_i is provable;
- Φ_i belongs to the set of formulas Γ;
- Φ_i is entailed from Φ_j $\{i < j\}$ by one of the inference rules.

If there is a deduction of formula Φ from the set Γ, then the formula Φ is called *deducible* from Γ, and the set of formulas Γ is called the *set of hypotheses*. It is obvious that the provability of formula is equivalent to its deducibility from the empty set of hypotheses. A deducibility of formula Φ from the set Γ is denoted by $\Gamma \vdash \Phi$.

If Γ is a set of propositions, then by $\mathrm{Th}(\Gamma)$ we denote the set of all propositions deducible from Γ, i.e. $\mathrm{Th}(\Gamma) = \{\Phi : \Gamma \vdash \Phi\}$. Instead of $\Gamma \vdash \Phi$ it is possible to write $\Phi \in \mathrm{Th}(\Gamma)$. In particular, if Φ is a theorem and Φ is a proposition, then $\Phi \in \mathrm{Th}(\varnothing)$. The set $\mathrm{Th}(\varnothing)$ consists of axioms and theorems of the first-order logic.

The set Γ of propositions such that $\Gamma = \mathrm{Th}(\Gamma)$ is called the *elementary theory*. Members of Γ are called *theorems of theory* Γ.

If Γ is an elementary theory, $A \subseteq \Gamma$ and $\Gamma = \mathrm{Th}(\Gamma)$, then the set A is called the *set of axioms of theory* Γ or simply the *axiomatics* for Γ, and its members are *axioms*. In particular, Γ is an axiomatics for Γ.

If Γ_1 and Γ_2 are elementary theories and $\Gamma_1 \subseteq \Gamma_2$, then Γ_2 is called the *overtheory* (*extension* of theory Γ_1) and Γ_1 the *subtheory* (*restriction*) of theory Γ_2. Theories Γ_1 and Γ_2 are called *equivalent (deductively equal)* if everyone of them is an extension for another.

Let us get a class of algebraic systems, K, consisting of the same set of relations R_{11}, R_{12}, …, R_{1k1}, R_{21}, …, R_{2k2}, …, R_{nkn}. The proposition Φ is called

- *true on a class* K if Φ is true on each system of a class K;
- *realizable on a class* K if in K there is a system in which Φ is true.

Clearly, valid propositions (propositions from Th(∅)) are true on any class of algebraic systems. Also, if a proposition Φ is true on any class of algebraic systems, then the proposition Φ is valid.

Let Γ be a collection of propositions. A class of all algebraic systems such that on each of them all propositions from Γ are true is denoted by Mod(Γ). Evidently, if $\Gamma_1 \subseteq \Gamma_2$, then $Mod(\Gamma_1) \supseteq Mod(\Gamma_2)$.

Let K be an arbitrary class of algebraic systems. The collection of all propositions that are true on the class K is an elementary theory and it is called the *elementary theory of class* K. The elementary theory of class K is denoted by Th(K). Let us notice that if $K_1 \subseteq K_2$, then $Th(K_1) \supseteq Th(K_2)$. Obviously also that $K \subseteq Mod(Th(K))$. If M is an algebraic system, then Th({M}) is called the *elementary theory of system* M and it is denoted by Th(M). The class K of systems is called axiomatized if K = Mod(Th(K)), i.e. the class K of algebraic systems is axiomatized if and only if there exists an elementary theory Th(K) such that K is *a set of all* models for the set of propositions Th(M).

In the *Critique of Pure Reason*, Kant defines the following four kinds of quasi-atomic propositions:

Analytic proposition: a quasi-atomic proposition $P_{il}(x_1, x_2, \ldots,$

x_i) ε $P_{im}(x_1, x_2, \ldots, x_i)$ whose predicate concept (i.e. $P_{im}(x_1, x_2, \ldots, x_i)$) is a genus for its subject concept (i.e. for $P_{il}(x_1, x_2, \ldots, x_i)$) is analytic, i.e. it is analytic if the formula $P_{il}(x_1, x_2, \ldots, x_i) \Rightarrow P_{im}(x_1, x_2, \ldots, x_i)$ is true in any class K that contains $R_{il}(I(x_1), I(x_2), \ldots, I(x_i))$, $R_{im}(I(x_1), I(x_2), \ldots, I(x_i))$.

Kant's example of analytic propositions/judgements is based on the presupposition that individuals of algebraic models are presented by real things. Now let us take the predicate, H, to be a human being and the predicate, R, to be rational. Then the quasi-atomic formula $H(x)$ ε $R(x)$ is analytic, becouse $H(I(x)) \Rightarrow R(I(x))$ holds everywhere.

Synthetic proposition: a quasi-atomic proposition $P_{il}(x_1, x_2, \ldots, x_i)$ ε $P_{im}(x_1, x_2, \ldots, x_i)$ whose predicate concept is not contained in its subject concept in general case is synthetic. Kant's example is as follows. Let H(x) mean that x is a human being and $F(x)$ mean that x is featherless bipeds. Then $H(I(x)) \Rightarrow F(I(x))$ holds just in some models.

A priori proposition: a quasi-atomic proposition $P_{il}(x_1, x_2, \ldots, x_i)$ ε $P_{im}(x_1, x_2, \ldots, x_i)$ whose justification does not rely upon experience is a priori, i.e. $P_{il}(x_1, x_2, \ldots, x_i)$ ε $P_{im}(x_1, x_2, \ldots, x_i)$ is a priori iff $P_{il}(x_1, x_2, \ldots, x_i) \vdash P_{im}(x_1, x_2, \ldots, x_i)$, namely iff $P_{im}(x_1, x_2, \ldots, x_i)$ is deduceble from $P_{il}(x_1, x_2, \ldots, x_i)$.

A posteriori proposition: a quasi-atomic proposition $P_{il}(x_1, x_2, \ldots, x_i)$ ε $P_{im}(x_1, x_2, \ldots, x_i)$ whose justification does rely upon experience is a posteriori, i.e. $P_{il}(x_1, x_2, \ldots, x_i)$ ε $P_{im}(x_1, x_2, \ldots, x_i)$ is a posteriori iff $P_{im}(x_1, x_2, \ldots, x_i)$ is not deducible from $P_{il}(x_1, x_2, \ldots, x_i)$.

We can extend these definitions up to the case of all formulas.

Analytic proposition: a proposition Φ is analytic iff it is true in any class K. On the basis of implication 'if $\Gamma_1 \subseteq \Gamma_2$, then Mod ($\Gamma_1$) $\supseteq$ Mod(Γ_2)' it is possible to claim that the more propositions are contained in a theory, the less it is analytic.

Synthetic proposition: a proposition Φ is synthetic iff it is not

true in any class K.

A priori proposition: a proposition Φ is a priori iff $\vdash$ Φ. On the basis of implication 'if $K_1 \subseteq K_2$, then $Th(K_1) \supseteq Th(K_2)$' it is possible to claim that the greater the class of algebraic systems K is, the more the theory containing of propositions true on the class K is a priori.

A posteriori proposition: a proposition Φ is a posteriori iff $\Gamma \vdash$ Φ and $\Gamma \neq \varnothing$.

Kant combines his distinction between analytic and synthetic propositions with the distinction between a priori and a posteriori propositions to explain Platonic presuppositions that there are abstract objects, pure sences. As a result of that combination, he obtaines the following kinds of propositions: (i) analytic a priori; (ii) synthetic a priori, (iii) analytic a posteriori; (iv) synthetic a posteriori. Kant claims that the third type is self-contradictory. Indeed, if a formula Φ is analytic (holds in any class K), then it is a priori (is a theorem). Could we say at the same time that if it is a priori, then it is analytic? In Kant's view, the answer is negative. In modern logic it is too, namely there are theorems that do not hold in any class K. Hence, in order to explicate Platonic assumptions in modern logic we should consider the meaning of analytic a priori and synthetic a priori propositions more in details and explain, why there are synthetic a priori propositions.

The family of propositions Th(∅) is analytic, because it holds in any class K. This family is included in any theory Γ: $\forall\Gamma$. ($\Gamma = Th(\Gamma) \Rightarrow Th(\varnothing) \subseteq \Gamma$). It means that this set is a priori. This Th(∅) is called the set of logical axioms. A theory Γ without the propositions of Th(∅) (formally: Γ / Th(∅), where $\Gamma = Th(\Gamma)$) is called the set of non-logical axioms. It is denoted by NonL(Γ). The propositions of NonL(Γ) are not analytic, but they are a priori.

Thus, Kant's distinction between analytic a priori and synthetic a priori propositions could be explicated as follows: (i) analytic a priori propositions are logical axioms of any formal

theory (i.e. belong to the set Th(∅)) and (ii) synthetic a priori propositions are non-logical axioms of an apropriate theory (i.e. they belong to NonL(Γ)).

Now we could find out, where *das dritte Reich* (the world of pure senses) in modern logic has been hidden. In order to ground pure senses (Sinne), Gottlob Frege provides the distinction between sense (Sinn) and reference (*Bedeutung*). He takes into account that propositions $a = a$ and $a = b$ are obviously of differing cognitive value. Propositions of the form $a = a$ are to be labelled analytic a priori, while propositions of the form $a = b$ are synthetic. The first kind of propositions has the same sense, the second kind of propositions expresses two different senses ('*a*' and '*b*'), but have the same reference that makes them true:

> When we found '$a = a$' and '$a = b$' to have different cognitive values, the explanation is that for the purpose of knowledge, the sense of the sentence, viz., the thought expressed by it, is no less relevant than its reference, i.e. its truth value. If now $a = b$, then indeed the reference of '*b*' is the same '*a*,' and thereby the sense expressed in '$a = b$' differs from that of '$a = a$.' In that case the two sentences do not have the same cognitive value. If we understand by 'judgment' the advance from the thought to its truth value, as in the above paper, we can also say that the judgments are different (*Frege 1892*)

In the *Meaning and Necessity* Rudolf Carnap explicates the two kinds of meaning proposed by Frege as follows:

Extension: it is a corresponding class of references (denotations, *Bedeutungen*) for a predicate or proposition. Let Γ be a family of propositions and M be a model for Γ, i.e. $M \in Mod(\Gamma)$. Then two propositions Φ_1, Φ_2 of Γ have the same extension iff $M \models (\Phi_1 \Rightarrow \Phi_2) \,\&\, (\Phi_2 \Rightarrow \Phi_1)$. If H(x) means that *x* is a human being and F(x) means that *x* is featherless bipeds, then there is a model, where $(H(x) \Rightarrow F(x)) \,\&\, (F(x) \Rightarrow H(x))$ is true. In other words, H and F have the same extension in M. For Frege the extension is expressed in propositions of the form $a = b$.

Intension: it is a corresponding family of properties (senses, *Sinne*) for a predicate or proposition. Let Γ be a collection of propositions derivated with the use of predicates P_{11}, P_{12}, …, P_{1k1}, P_{21}, …, P_{2k2}, …, P_{nkn}, and K be a class of algebraic systems consisting of the same set of relations R_{11}, R_{12}, …, R_{1k1}, R_{21}, …, R_{2k2}, …, R_{nkn}. Then two propositions Φ_1, Φ_2 of Γ have the same intension iff $(\Phi_1 \Rightarrow \Phi_2)$ & $(\Phi_2 \Rightarrow \Phi_1)$ is true on K. For instance, if we take the predicate, H, to be a human being and the predicate, R, to be rational, then we obtain the formula $(H(x) \Rightarrow R(x))$ & $(R(x) \Rightarrow H(x))$ is true everywhere. This means that H and R have the same intension. For Frege the intension is expressed in propositions of the form $a = a$.

> Now we shall introduce the terms 'extension' and 'intension' with respect to predicators. If two predicators apply to the same individuals in other words, if they are equivalent it is sometimes said that they are coextensive or that they have the same extension (in one of the various customary uses of this term). The use of 'intension' varies still more than that of 'extension.' Two predicators have the same intension if and only if they are *L*-equivalent (*Carnap*).

3. The analytic a posteriori

Thus, *das dritte Reich* as the Platonic world of pure senses consists of analytic a priori, synthetic a priori and intensions. However, the basic idea of this approach that allows us to build up classical logic assumes that analytic a posteriori does not exist. Is it really so? In the *Naming and Necessity* Saul Kripke states that there exist 'contingent a priori' (instead of this term it is more presicely to say analytic a posteriori) propositions as well. They are constructed by using indexicals like 'I,' 'here,' 'now,' 'actually,' 'you.' The example of analytic a posteriori is Descartes' famous *cogito* argument:

> To say 'I think, therefore I am' expresses a necessary truth only

> if it is taken as an analytic a posteriori principle: 'I know from experience (a posteriori) that I can and do think, and this implies (analytically) that I exist' (*Palmquist*).

According to Platonic presuppositions, which Kant, Frege and Carnap shared, logical relations do not depend upon our expirience (the actual world), they are defined everywhere, i.e. on any class of algebraic systems (since they are analytic a priori). However, as Kripke showes, there are logical relations, namely modal operators ('possible,' 'necessary'), that can not be interpreted on any model. In defining them we can dot the I's and cross the T's only if we appreciate the so-called Kripke models. Modal relations being logical ones are to be labelled contingent a priori (as Kripke says) or analytic a posteriori (as Palmquist says and we are saying).

Kripke proposes to differ necessary truth from a priori knowable truth:

> Philosophers have talked ... [about] various categories of truth, which are sometimes called 'a priori,' 'analytic,' 'necessary' ... these terms are often used as if whether there are things answering to these concepts is an interesting question, but we might as well regard them all as meaning the same thing.
> ... First the notion of a prioricity is a concept of epistemology. I guess the traditional characterization from Kant goes something like: a priori truths are those which can be known independently of any experience.
> ...The second concept which is in question is that of necessity ... what I am concerned with here is a notion which is not a notion of epistemology but of metaphysics ...We ask whether something might have been true, or might have been false. Well, if something is false, it's obviously not necessarily true. If it is true, might it have been otherwise? Is it possible that, in this respect, the world should have been different than the way that it is? ... This in and of itself has nothing to do with anyone's knowledge of anything. It's certainly a philosophical thesis, and not a matter of obvious definitional equivalence, either that everything a priori is necessary or that everything necessary is a priori ... at any rate they are dealing with two

> different domains, two different areas, the epistemological and the metaphysical (*Kripke*).

Necessary truth may be a posteriori. This means that logical relations may be a posteriori, too. From this it follows that we could design logic as theory of necessary truth without Platonic presuppositions, i.e. without axioms. As a result, we could destroy *das dritten Reich*. So, we could hit Goliath' head, i.e. the big head of Platonism.

Stephen Palmquis claims that analytic a posteriori takes place if conceptual (analytic) content satisfies not general, but particular conditions:

> What then is the difference between the 'analytic a posteriori' and Kant's special 'synthetic a priori' class of knowledge? The analytic a posteriori and synthetic a priori are similar classes of knowledge insofar as both are concerned with conditions imposed on the world by the subject (in contrast to the analytic a priori and synthetic a posteriori types, which are concerned with information that can be drawn out of, or deduced from, what we find in experience), but they differ by virtue of the fact that the former imposes *particular* conditions (a posteriori) with conceptual (analytic) content, whereas the latter imposes *general* conditions (a priori) with intuitive (synthetic) content (*Palmquist*).

This understanding of analytic a posteriori is too metaphysic. It is very far from logic and mathematics. In order to formally explicate the notion of analytic a posteriori, instead of primary terms and axioms as basic, initial notions of logic and mathematics let us consider transition and behavior as ground concepts. In this connection we can assume that there are no unchangeable things. Everything changes and nothing remains still (*πάντα χωρεῖ καὶ οὐδὲν μένει*). Entities that are felt eternal by us are an iteration in fact, i.e. a permanent iterance of the same. Eternal entities also move and vary, but in the modification they come back to the initial states.

The eternal existence in logic and mathematics, treated as iteration or recursion, will correspond to the notion 'analytic a posteriori.' On the one hand, iteration or recursion are a posteriori, because they stay in the eternal transition that is detected by experience. On the other hand, they are analytic, because they are eternal and contain a permanent iterance in the modification.

If we admit analytic a posteriori in logic and mathematics, we will already be able to deny notions 'analytic a priori' and 'synthetic a priori.' These notions are exhaling. The eternal in its pure sense disappears. We face just changeable things of two sorts: analytic a posteriori and synthetic a posteriori. The first notion expresses iteration in transitions, the second a permanent modification in transitions.

Logic and mathematics where all propositions (mathematical entities) are either analytic a posteriori or synthetic a posteriori are called anti-Platonic.

Anti-Platonic logic and mathematics (as the novel program of foundations of mathematics without Platonism) satisfy the following basic presuppositions:

- there are no propositions that are true on any class K (i.e. there are no logical axioms);
- there are no non-logical axioms, we draw conclusions from premises having the equal status, all formulas are infered from hypotheses;
- finally, any object has only a relative existence.

Within these assumptions we build up derivations without using axioms, therefore there is no sense in distinguishing logic and theory (i.e. logical and non-logical axioms), derivable and provable formulas, etc. In such a formal theory the conclusion is understood as massive-parallel computing, i.e. the deduction is considered as a transition of automaton. This formal proof theory is reduced to an appropriate cellular automaton which is called a *proof-theoretic cellular automaton*, whose cell states are regarded as well-formed formulas of a logical language. As a result, in deduction we do not obtain

derivation trees and instead of the latter we find out derivation traces, i.e. a linear evolution of each singular premise. Thereby some derivation traces may be circular, i.e. some premises could be derivable from themselves, and hence some derivation traces may be infinite. Propositions from circular derivations are said to be analytic a posteriori. Other propositions are called synthetic a posteriori.

4. Proof-theoretic cellular automata

For any logical language L we can construct a *proof-theoretic cellular automaton* (instead of conventional deductive systems) simulating massive-parallel proofs.

Definition 4.1 *A proof-theoretic cellular automaton is a 4-tuple* $\mathsf{A} = \langle \mathsf{Z}^d, S, N, \delta \rangle$, *where*

- $d \in \mathsf{N}$ is a number of dimensions and the members of Z^d are referred as cells,
- S is a finite or infinite set of elements called the states of an automaton A, the members of Z^d take their values in S, the set S is collected from well-formed formulas of a language L.
- $N \subset \mathsf{Z}^d \setminus \{0\}^d$ is a finite ordered set of n elements, N is said to be a neighborhood,
- $\delta : S^{n+1} \to S$ that is δ is the inference rule of a language L, it plays the role of local transition function of an automaton A.

As we see, an automaton is considered on the endless d-dimensional space of integers, i.e. on Z^d. Discrete time is introduced for $t = 0,1,2,\ldots$ fixing each step of inferring.

For any given $z \in \mathsf{Z}^d$, its neighborhood is determined by $z + N = \{z + \alpha : \alpha \in N\}$. There are two often-used neighborhoods:

- Von Neumann neighborhood $N_{VN} = \{z \in \mathsf{Z}^d : \sum_{k=1}^{d} | z_k | = 1\}$
- Moore neighborhood

$$N_M = \{z \in \mathsf{Z}^d : \max_{k=\overline{1,d}} |z_k| = 1\} = \{-1,0,1\}^d \setminus \{0\}^d$$

For example, if $d = 2$, $N_{VN} = \{(-1,0),\ (1,0),\ (0,-1),\ (0,1)\}$; $N_M = \{(-1,-1),\ (-1,0),\ (-1,1),\ (0,-1),\ (0,1),\ (1,-1),\ (1,0),\ (1,1)\}$.

In the case $d = 1$, von Neumann and Moore neighborhoods coincide. It is easily seen that $|N_{VN}| = 2d$, $|N_M| = 3^d - 1$.

At the moment t, the *configuration* of the whole system (or the *global state*) is given by the mapping $x^t : \mathsf{Z}^d \to S$, and the *evolution* is the sequence $x^0, x^1, x^2, \ldots$ defined as follows: $x^{t+1}(z) = \delta(x^t(z), x^t(z+\alpha_1), \ldots, x^t(z+\alpha_n))$, where $\langle \alpha_1, \ \ldots \ , \ \alpha_n \rangle \in N$. Here x^0 is the initial configuration, and it fully determines the future behavior of the automaton. It is the set of all premises (not axioms).

We assume that δ is an inference rule, i.e. a mapping from the set of premises (their number cannot exceed $n = |N|$) to a conclusion. For any $z \in \mathsf{Z}^d$ the sequence $x^0(z)$, $x^1(z)$, … , $x^t(z)$, is called a *derivation trace from a state* $x^0(z)$. If there exists t such that $x^t(z) = x^l(z)$ for all $l > t$, then a derivation trace is *finite*. It is *circular/cyclic* if there exists l such that $x^t(z) = x^{t+l}(z)$ for all t.

Definition 4.2 *In case all derivation traces of a proof-theoretic cellular automaton* A *are circular, this automaton* A *is said to be reversible.*

Notice that x^{t+1} depends only upon x^t, i.e. the previous configuration. It enables us to build the function $G_{\mathsf{A}} : C_{\mathsf{A}} \to C_{\mathsf{A}}$, where C_{A} is the set of all possible configurations of the cellular automaton A (it is the set of all mappings $\mathsf{Z}^d \to S$, because we can take each element of this set as the initial configuration x^0, though not every element can arise in the evolution of some other configuration). G_{A} is called the *global function* of the automaton.

1. Example of modus ponens *Consider a propositional language* L *that is built in the standard way with the only binary operation of implication* $\supset$. *Let us suppose that well-formed formulas of that language are*

used as the set of states for a proof-theoretic cellular automaton A. *Further, assume that modus ponens is a transition rule of this automaton* A *and it is formulated for any* φ, $\psi \in \mathsf{L}$ *as follows:*

$$x^{t+1}(z) = \begin{cases} \psi, & \text{if } x^t(z) = \varphi \supset \psi \text{ and } \varphi \in (z + N); \\ x^t(z), & \text{otherwise.} \end{cases}$$

The further dynamics will depend on the neighborhood. If we assume the Moor neighborhood in the 2-dimensional space, this dynamics will be exemplified by the evolution of cell states in Fig.4.1—Fig.4.3.

$(p \supset q) \supset r$	$p \supset (p \supset q)$	$p \supset q$	$(p \supset q) \supset (p \supset q)$	$(r \supset p) \supset r$
$(p \supset r) \supset (q \supset r)$	$p \supset q$	p	$p \supset (p \supset q)$	$r \supset p$
$p \supset r$	p	$p \supset (q \supset (p \supset q))$	p	r
$p \supset (q \supset r)$	$p \supset p$	$p \supset q$	$(p \supset r) \supset (q \supset p)$	$p \supset r$
$p \supset q$	$p \supset (q \supset p)$	q	$p \supset r$	p

Figure 4.1 An initial configuration of a proof-theoretic cellular automaton A with the Moor neighborhood in the 2-dimensional space, its states run over formulas set up in a propositional language L with the only binary operation $\supset$, $t = 0$. Notice that p, q, r are propositional variables.

r	$p \supset q$	q	$p \supset q$	r
$q \supset r$	q	p	$p \supset q$	p
r	p	$q \supset (p \supset q)$	p	r
$q \supset r$	p	q	$q \supset p$	r
$p \supset q$	$p \supset (q \supset p)$	q	r	p

Figure 4.2 An evolution of A described in Fig.4.1 at the time step $t = 1$.

r	q	q	q	r
r	q	p	q	p
r	p	q	p	r
r	p	q	p	r
q	p	q	r	p

Figure 4.3 An evolution of A described in Fig. 4.1 at the time step $t = 3$. Its configuration cannot vary further.

This example shows that first we completely avoid axioms and secondly we take premisses from the cell states of the neighborhood according to a transition function. As a result, we do not come across proof trees in our novel approach to deduction taking into account that a cell state has just a linear dynamics (the number of cells and their location do not change). This allows us evidently to simplify deductive systems.

Now we are trying to consider a cellular-automaton presentation of two basic deductive approaches: Hilbert's type and sequent ones.

2. Example of Hilbert's inference rules *Suppose a propositional language* L *contains two basic propositional operations: negation and disjunction. As usual, the set of all formulas of* L *is regarded as the set of states of an appropriate proof-theoretic cellular automata. In that we will use the exclusive disjunction of the following five inference rules converted from Joseph* R. *Shoenfield's deductive system:*

$$x^{t+1}(z) = \begin{cases} \psi \vee \varphi, & \text{if } x^t(z) = \varphi; \\ \varphi, & \text{if } x^t(z) = \varphi \vee \varphi; \\ (\chi \vee \psi) \vee \varphi, & \text{if } x^t(z) = \chi \vee (\psi \vee \varphi); \\ \chi \vee \psi, & \text{if } x^t(z) = \varphi \vee \chi \text{ and } (\neg\varphi \vee \psi) \in (z + N); \\ \chi \vee \psi, & \text{if } x^t(z) = \neg\varphi \vee \psi \text{ and } (\varphi \vee \chi) \in (z + N). \end{cases}$$

3. Example of sequent inference rules *Let us take a sequent propositional language* L*, in which the classical propositional language with negation, conjunction, disjunction and implication is extended by adding the sequent relation* $\circ$*. Recall that a sequent is an expression of the form* $\Gamma_1 \circ \Gamma_2$*, where* $\Gamma_1 = \{\varphi_1, \ldots, \varphi_j\}$*,* $\Gamma_2 = \{\psi_1, \ldots, \psi_i\}$ *are finite sets of well-formed formulas of the standard propositional language, that has the following interpretation:* $\Gamma_1 \circ \Gamma_2$ *is logically valid iff*

$$\bigwedge_j \varphi_j \supset \bigvee_i \psi_i$$

is logically valid. Let S denote the set of all sequents of L, furthermore let us assume that this family S is regarded as the set of states for a proof-theoretic cellular automaton A. The transition rule is an exclusive disjunction of the 14 singular rules (6 structural rules and 8 logical rules):

$$x^{t+1}(z) = \Gamma_1 \circ \Gamma_2, \begin{cases} \textit{if } \Gamma_1 \circ \Gamma_2 \textit{ is a result of applying to } x^t(z) \\ \textit{eather one of structural rules} \\ \textit{or the left (right) introduction of negation} \\ \textit{or the left introduction of conjunction} \\ \textit{or the right introduction of disjunction} \\ \textit{or the right introduction of implication.} \end{cases}$$

$$x^{t+1}(z) = \begin{cases} \Gamma \circ \Gamma', \psi \wedge \chi, & \textit{if } x^t(z) = \Gamma \circ \Gamma', \psi \textit{ and} \\ & (\Gamma \circ \Gamma', \chi) \in (z + N); \\ \Gamma, \psi \vee \chi \circ \Gamma', & \textit{if } x^t(z) = \Gamma, \psi \circ \Gamma' \textit{ and} \\ & (\Gamma, \chi \circ \Gamma') \in (z + N); \\ \psi \supset \chi, \Gamma, \Delta \circ \Gamma', \Delta', & \textit{if } x^t(z) = \Gamma \circ \Gamma', \psi \textit{ and} \\ & (\chi, \Delta \circ \Delta') \in (z + N). \end{cases}$$

4. Example of Brotherston's cyclic proofs *The sequent language used in the previous example we extend by adding predicates N, E, O and appropriate inference rules of Fig.4.4 for them. Further, let us extend also the automaton of Example 3 in the same way by representing inference rules of Fig.4.4 in the cellular-automatic form.*

Now we assume that a cell has an initial state $[\Gamma, N(z)^\circ\Delta, O(z), E(z)]$ and its neighbor cell an initial state $[\Gamma, z = 0^\circ\Delta]$ that is equal to $[\Gamma, z = 0^\circ\Delta, O(z), E(z)]$ for any $t = 4,14,24,\ldots$ and to $[\Gamma, z = 0^\circ\Delta, E(z), O(z)]$ for any $t = 9,19,29,\ldots$. Then we will have the following infinite cycle:

$$[\Gamma, N(z)^\circ\Delta, O(z), E(z)] \to^{(substitution)} [\Gamma, N(y)^\circ\Delta, O(y), E(y)]$$
$$\to [\Gamma, N(y)^\circ\Delta, O(y), O(y+1)] \to [\Gamma, N(y)^\circ\Delta, E(y+1), O(y+1)]$$
$$\to [\Gamma, z = (y+1), N(y)^\circ\Delta, O(z), E(z)] \to^{(case\,N)} [\Gamma, N(z)^\circ\Delta, E(z), O(z)] \to \ldots$$

$$\frac{\Gamma^\circ N(x)}{\Gamma^\circ N(x+1)}, \quad \frac{\Gamma^\circ\Delta}{\Gamma^\circ\Delta, N(0)}, \quad \frac{\Gamma^\circ E(x)}{\Gamma^\circ O(x+1)}, \quad \frac{\Gamma^\circ O(x)}{\Gamma^\circ E(x+1)}, \quad \frac{\Gamma^\circ\Delta}{\Gamma^\circ\Delta, E(0)},$$
$$\frac{N(x)^\circ\Delta}{N(x+1)^\circ\Delta}, \quad \frac{\Gamma^\circ\Delta}{\Gamma, N(0)^\circ\Delta}, \quad \frac{E(x)^\circ\Delta}{O(x+1)^\circ\Delta}, \quad \frac{O(x)^\circ\Delta}{E(x+1)^\circ\Delta}, \quad \frac{\Gamma^\circ\Delta}{\Gamma, E(0)^\circ\Delta},$$
$$\frac{\Gamma, t = 0^\circ\Delta \qquad \Gamma, t = x+1, N(x)^\circ\Delta}{\Gamma, N(t)^\circ\Delta}(Case\,N), where x \notin FV(\Gamma \cup \Delta \cup \{N(t)\}),$$
$$\frac{\Gamma^\circ\Delta}{\Gamma[x]^\circ\Delta[x]}(Substitution).$$

Figure 4.4 Inference rules for predicates N ('being a natural number'), E ('being an even number'), O ('being an odd number'), see *Brotherston*.

As we see, the possibility of cyclic derivation traces depends on configuration of cell states.

Definition 4.3 *Initial states of cyclic derivation traces are called analytic a posteriori propositions. All other states are called synthetic a*

posteriori propositions. For instance, the premise $[\Gamma, N(z)^{\circ}\Delta, O(z), E(z)]$ *of Example 4 is an analytic a posteriori proposition.*

From this it follows immediately that all premisses of a reversible proof-theoretic cellular automaton are analytic *a posteriori* propositions.

Definition 4.4 *A set of all premisses of a cyclic derivation trace is called a logical biota. For instance, the premises* $[\Gamma, N(z)^{\circ}\Delta, O(z), E(z)]$ *and* $[\Gamma, z = 0^{\circ}\Delta]$ *of Example 4 forms a logical biota.*

Each logical biota contains at least one analytic a posteriori proposition.

Proposition 4.1 *Let us assume that a proof-theoretic cellular automaton* A *is not reversible and contains at least one logical biota. Then there is a natural number n such that at the time step n the configuration of logical biota cannot vary further.*

A logical biota is a kind of self-organization in deductions and may be considered as a dissipative system in proof-theoretic cellular automata. Also here it is possible to say that for logical biotas the dissipation (energy dispersion) takes place. It is related by that logical biotas under the influence of an environment can disintegate, too. In other words, their dynamics can stop at the time step *n*.

A vivid example of dissipative systems in the nature is presented by an auto-oscillation, namely by an undamped wave supported from outside in non-linear system with the energy damping. In spite of the fact that auto-oscillations are provided with exterior effects, their properties are determined by parametres of the system. We observe a version of auto-oscillations in logical biotas.

The evolutionary character of behaviour is a major property of the dissipative structure. Meanwhile in the class of evolutionary

processes it is possible to distinguish various types of dissipative systems. The first type includes systems whose evolution moves to a stationary state, the second one contains systems whose evolution goes through a sequence of nonequilibrium stationary states of an open system. Systems of the first type are termed as simple structures, systems of the second type as difficult ones.

Let us notice that an absolute stability in front of exterior effects is achievable only at very great values of entropy and it is equivalent to the thermal death. For cellular automata the state, when the configuration of cellular automata cannot vary further, is an analogue with the thermal death. In other cases the system stays in states which have no ultimate balance. Nevertheless, entropy can have a certain relative maximum. In this situation, the system appears as a dissipative structure having a complex organisation (and it is capable to adaptation, evolution, etc.).

The common feature of self-organized processes is in an increase of the order from the chaos due to the occurrence of inconvertible thermodynamic balance with the environment, which is opposite, however, to the thermal death (a balance of pices of environment that have inetracted in the independent and unstructured way). Self-organized processes of dissipative systems proceed together with processes that lead to a degradation and decay of system as a whole.

In proof-theoretic cellular automata we can consider dissipative and self-organized phenomena as well. The notion of analytic a posteriori allows us to do it. At the same time, taditional tasks concerning proof theory like completeness and independence of axioms lose their sense in massive-parallel proof theory, although it can be readily shown that we can speak about consistency:

Proposition 4.2 *Proof theories given in Examples from 1 to 4 are consistent, i.e. we cannot deduce a contradiction within them.*

5. Unconventional (non-well-founded) logic

We have just constructed the theory of proofs in which there is no place for theorems or provable formulas (a priori propositions). But in this theory, there appears the notion of cyclic proofs (analytic a posteriori propositions). We can go further and construct a formal language in which there is no basic notions of classical semantics, e.g. there are no valid formulas (analytic a priori propositions) and propositions that are true on a class of models (synthetic a priori propositions).

This modern language will be called *unconventional* or *non-well-funded*. Its formulas are defined as follows:

- Each atomic formula is a formula.
- If $\Phi_1 \varepsilon \Phi_2$ or $\Phi_1 \,\&\, \Phi_2$ or $\Phi_1 \vee \Phi_2$ or $\Phi_1 \Rightarrow \Phi_2$ or $\neg\Phi_1$, $\neg\Phi_2$ are formulas, then Φ_1, Φ_2 are formulas.

The above definition assumes that the formula is a syntactic object which does not satisfy the set-theoretic axiom of foundation. It means that the given syntactic objects can already be of an infinite length and comprise cycles. For example, cyclic expressions $(\Phi_1 \Rightarrow (\Phi_2 \Rightarrow (\Phi_1 \Rightarrow (\Phi_2 \Rightarrow (\ldots)))))$ or $(\Phi_1 \,\&\, (\Phi_1 \,\&\, (\Phi_1 \,\&\, (\Phi_1 \,\&\, (\ldots)))))$ are well formed formulas. These expressions can be defined by recursion, too:

The formula of infinite length $(\Phi_1 \Rightarrow (\Phi_2 \Rightarrow (\Phi_1 \Rightarrow (\Phi_2 \Rightarrow (\ldots)))))$ is equivalent to the cyclic definition $\Phi = (\Phi_1 \Rightarrow (\Phi_2 \Rightarrow \Phi))$.

The formula of infinite length $(\Phi_1 \,\&\, (\Phi_1 \,\&\, (\Phi_1 \,\&\, (\Phi_1 \,\&\, (\ldots)))))$ is equivalent to the cyclic definition $\Phi = (\Phi_1 \,\&\, \Phi)$.

Meanings of unconventional-logical connectives ε, &, ∨, ⇒, ¬ are defined not by true tables as this proceeds usually, but by hybrid cellular automata. Let us take now a cellular automaton $\mathsf{A} = \langle \mathsf{Z}^d, S, N, \delta \rangle$, where

- $d \in \mathsf{N}$ is a number of dimensions and the members of Z^d are referred as cells,
- S is a finite or infinite set of elements called the states of an automaton A, the members of Z^d take their values in S, the set S is collected from two truth values 1 and 0 for well-formed formulas of a language L.
- $N \subset \mathsf{Z}^d \setminus \{0\}^d$ is a finite ordered set of n elements, N is said to be a neighborhood,
- $\delta : S^{n+1} \to S$ that is $\delta \in \{\varepsilon, \&, \vee, \Rightarrow, \neg\}$ is the truth valuation for each logical operation of a language L, it plays the role of local transition function of an automaton A.

Definition 5.1 *A truth valuation of conjunction is a transition rule of the automaton* A*, where* $S = \{1, 0\}$*, and it is formulated for any formula* $\Phi_1 \& \Phi_2 \in \mathsf{L}$ *as follows:*

$$x^{t+1}(z) = \begin{cases} 1, & \text{if } x^t(z) = 1 \text{ and } 0 \notin (z + N); \\ 0, & \text{otherwise.} \end{cases}$$

Definition 5.2 *A truth valuation of disjunction is a transition rule of the automaton* A*, where* $S = \{1, 0\}$*, and it is formulated for any formula* $\Phi_1 \vee \Phi_2 \in \mathsf{L}$ *as follows:*

$$x^{t+1}(z) = \begin{cases} 1, & \text{if } x^t(z) = 1 \text{ or } 1 \in (z + N); \\ 0, & \text{otherwise.} \end{cases}$$

Definition 5.3 *A truth valuation of implication (respectively affirmation) is a transition rule of the automaton* A*, where* $S = \{1, 0\}$*, and it is formulated for any formula* $\Phi_1 \Rightarrow \Phi_2 \in \mathsf{L}$ *(respectively* $\Phi_1\ \varepsilon\ \Phi_2 \in \mathsf{L}$*) as follows:*

$$x^{t+1}(z) = \begin{cases} 0, & \text{if } x^t(z) = 0 \text{ and } 0 \notin (z + N); \\ 1, & \text{otherwise.} \end{cases}$$

Definition 5.4 *A truth valuation of negation is a transition rule of the automaton* A*, where* $S = \{1, 0\}$*, and it is formulated for any formula* $\neg\Phi \in \mathsf{L}$ *as follows:*

$$x^{t+1}(z) = \begin{cases} 0, & \text{if } x^t(z) = 1; \\ 1, & \text{otherwise.} \end{cases}$$

Algorithm of truth valuation *Let* Φ *be a formula. If it is atomic, then its truth value is an initial configuration of a cellular automaton with the set of states* $\{1, 0\}$*. If it is not atomic, then we start the evaluation with the most otside connective* k_1*. By using one of definitions 5.1—5.4 that corresponds to* k_1*, we transform an initial configuration of a cellular automaton with the set of states* $\{1, 0\}$*. This transformation is fixed by steps* $t_{k1} = 1, 2, 3, \ldots$ *Then we move to a more inside connective* k_2*. For each step* $t_{k1} = 1, 2, 3, \ldots$ *we transform a configuration at the step* t_{k1} *in accordance with one of definitions 5.1—5.4 that corresponds to the new connective* k_2*. This means that for each step* t_{k1} *we obtain a new cellular automaton for the truth evaluation of* k_2*. Each transformation of that cellular automaton is fixed by new steps* $t_{k2} = 1, 2, 3, \ldots$ *Notice that it is possible that simultaneously we have two inside connectives* k_2 *and* k_2*' of the same range. In this case we do all the same, but in parallel manner. Further, we can move to a more inside connective* k_3*. For each step* $t_{k2} = 1, 2, 3, \ldots$ *we transform a configuration at the step* t_{k2} *in accordance with one of definitions 5.1—5.4 that corresponds to the connective* k_3 *and so on.*

To provide an example, let us evaluate the formula $(\Phi_1 \Rightarrow (\Phi_2 \,\&\, \Phi_1))$ in a cellular automaton A with the Moor neighborhood in the 2-dimensional space:

(I) Initial configuration, $t = 0$

1	0	0
0	1	1
0	1	0

We begin our evaluation with the connective $\Rightarrow$ as most outside. Thereby we are using definition 5.3:

(II) $t = 1$

1	1	1
1	1	1
1	1	0

So, we have only one transformation fixed by step 1. Then we move to the inside connective & and by definition 5.1 we obtain the following data over (II):

(III) $t = 2$

1	1	1
1	0	0
1	0	0

$\Downarrow$

(IV) $t = 3$

0	0	0
0	0	0
0	0	0

The automaton cannot vary further. If we had an additional more inside connective, then we would have two automata (III) and (IV) for further modification in accordance with this more inside connective.

Definition 5.5 *The formula is called analytic a posteriori if its truth valuation cellular automaton is reversible. The most simple case of analytic a posteriori formula is* $\neg\Phi$ *as it follows from definition 5.4 directly.*

As models of unconventional (non-well-founded) formulas we will use coalgebraic systems, e.g. labelled transition systems. The latter are defined as systems $\langle\ S, \rightarrow_S, A\ \rangle$, consisting of a set S of states, a transition relation $\rightarrow_S \subseteq S \times A \times S$, and a set A of labels. As always, $s \rightarrow_S^a s'$ is used to denote $\langle\ s, a, s'\ \rangle \in \rightarrow_S$.

Cellular automata $\mathsf{A} = \langle \mathsf{Z}^d, S, N, \delta \rangle$ may be regarded as a kind of labelled transition systems of the form $\langle S, \rightarrow_S, \{\delta_i\}_i \rangle$, where S is a set of states, $\rightarrow_S \subseteq S \times \{\delta_i\}_i \times S$, and $\{\delta_i\}_i$ is a set of labels for the local transition function δ of an automaton A. The members of $\{\delta_i\}_i$ is obtained as follows. We have i different combinations of neighbor states. This number depends on the number of neighborhood $n = |N|$ and the number of states $k = |S|$, namely $i = k^n$. Hence, $|\{\delta_i\}_i| = i$. Each combination of i has its own label $\delta_{1 \le j \le i}$.

Further let us define

$$\mathrm{B}(X) = \mathrm{P}(A \times X) = \{V : V \subseteq A \times X\}, \text{ for any set } X.$$

Then a labelled transition system $\langle\ S, \rightarrow_S, A\ \rangle$ can be represented as a B-coalgebra $\langle\ S, \alpha_S\ \rangle$ by defining

$$\alpha_S : S \rightarrow \mathrm{B}(S); s \rightarrow \{\langle\ a,\ s'\ \rangle : s \rightarrow_S^a s'\}$$

Notice that the class of all labelled transition systems coincides with the class of all B-coalgebras.

Let us consider now two B-coalgebras $\langle S, \alpha_S \rangle$ and $\langle T, \alpha_T \rangle$. A B-bisimulation between S and T is said to be a relation $R \subseteq S \times T$ satisfying, for all $\langle s, t \rangle \in R$, the following conditions:

1. for all s' in S, if $s \rightarrow_S^a s'$, then there is t' in T with $t \rightarrow_T^a t'$ and $\langle s', t' \rangle \in R$,

2. for all t' in T, if $t \rightarrow_T^a t'$, then there is s' in S with $s \rightarrow_S^a s'$ and $\langle s', t' \rangle \in R$

Instead of the induction proof principle used in algebras, the coinduction proof principle is involved into coalgebras: for every bisimulation R on S, $R \subseteq \Delta_S$ (where $\Delta_S = \{\langle s, s \rangle : s \in S\}$. Equivalently, for all s and s' in S, if s and s' are bisimilar in S, then $s = s'$.

Let M be a B-coalgebra $\langle S, \alpha_S \rangle$. Let us take the set consisting of k_1 one-placed relations, k_2 two-placed relations, k_n n-placed relations, R_{11}, R_{12}, …, R_{1k1}, R_{21}, …, R_{2k2}, …, R_{nkn}, i.e. R_{ij} means j-th i-placed relations. Then an i-placed relation in M is defined as a B-coalgebra over S^i, namely $\alpha_S : S^i \rightarrow B(S^i)$.

To individual variables $x, y, z, \ldots$ we can assign meanings in the set S by an interpretation I. In other words, $I(x) \in S$. To predicates P_{ij} we can assign relations R_{ij} understood as B-coalgebras over S^i.

Let Γ be a set of propositions of our formal language. Taking into account that for every predicate P_{ij} there exists a relation R_{ij} of B-coalgebra M, we can consider M as a model of Γ (it is denoted by $M \models \Gamma$) if the following conditions are satisfied for any formulas Φ_1, Φ_2 of Γ:

$I(P_{ij}(x_1, x_2, \ldots, x_i)) = R_{ij}(I(x_1), I(x_2), \ldots, I(x_i))$ is a B-coalgebra over S^i, i.e. $M \models R_{ij}(I(x_1), I(x_2), \ldots, I(x_i))$;

$M \models \neg \Phi_1$ iff the truth valuation of $\neg \Phi_1$ is bisimilar with M;

$M \models \Phi_1 \& \Phi_2$ iff the truth valuation of $\Phi_1 \& \Phi_2$ is bisimilar with M;

$M \models \Phi_1 \vee \Phi_2$ iff the truth valuation of $\Phi_1 \vee \Phi_2$ is bisimilar with M;

$M \models \Phi_1 \Rightarrow \Phi_2$ iff the truth valuation of $\Phi_1 \Rightarrow \Phi_2$ is bisimilar with M;

$M \models \Phi_1 \varepsilon \Phi_2$ iff iff the truth valuation of $\Phi_1 \varepsilon \Phi_2$ is bisimilar with M.

Let us assume that there is a class of coalgebras consisting of the same set of relations R_{11}, R_{12}, …, R_{1k1}, R_{21}, …, R_{2k2}, …, R_{nkn}. Then there is no proposition Φ that is true on any system of the class K. However, there exists a proposition Φ that is rearizable on the class K, i.e. K contains a system where Φ is true.

Let Γ be a family of propositions. The class of all coalgebraic systems, where all propositions of Γ are true, is denoted by $\mathrm{Mod}(\Gamma)$.

Proposition 5.1 If $\Gamma_1 \subseteq \Gamma_2$, then $\mathrm{Mod}(\Gamma_1) \subseteq \mathrm{Mod}(\Gamma_2)$.

Let K be a class of coalgebraic systems. The family of all propositions that are true on the class K is an elementary theory and is called an elementary theory of the class K. It is denoted by Th(K).

Proposition 5.2 If $K_1 \subseteq K_2$, then $\mathrm{Th}(K_1) \subseteq \mathrm{Th}(K_2)$.

Propositions 5.1—5.2 demonstrate some features of anti-Platonic logic and mathematics: the more there are propositions,

the more there are models and also the more there are models, the more there are propositions.

6. The analytic a posteriori and unconventional computing

Modern logic was finally formed in the beginning of the 20th century. At that time the formal language of logic has been created and since then its main properties have been studied. Then modern logic has had no key modifications. Since then and till now it has been constructed as axiomatic theory. There is, say, many-valued logic. However, the way of its designing does not differ from two-valued logic as a matter of principle. We have there an axiomatic system in which axioms differ, but the formal language is almost the same.

The main advantage of logic is that on its basis the computer architecture has been constructed (for example, circuits are modelled by logical relations), and also programming languages have been developed. Since then the computer equipment has not varied essentially from the logical point of view. For instance, due to computers' new materials memory, speed are to increase, but the logic remained almost the same.

In the last decades one has carried out many researches in direction of how it is possible to design logic which would describe dynamics of any natural process as computation. The given field of research is called unconventional computing. According to this approach any physical, chemical or biological process is an unconventional computer. In unconventional computers, the architecture ("iron") and the software coincide, i.e. during computations the architecture of the computer varies, too.

Conventional computers are based on logic that was obtained more than a century ago. Now the problem is set up, if unconventional computers (chemical, biological, etc.) are possible. For designing them we should know, what logic they need, whether

this logic is conventional or not. Many (if not all) experts claim that logic of unconventional computing (e.g. logic of circuits) should differ from conventional one. Some new logic such as glider logic was proposed. However, universal logical methods for unconventional computing were not obtained still.

If one does the trick to build up universal logic of unconventional computers (and sooner or later it will be possible), then rational robots-humanoids round us will become a part of everyday's reality. Every thing in our world is an unconventional computer, any physical, chemical or biological system. And the human being is not an exception to the rule here. If we understand logic of these systems, it will make an enormous break-in in computer equipments and in all exact sciences. We will already live in another world. Our reality will vary cardinally.

One of the tasks of logic for unconventional computing consists in the logical formalization of self-organized phenomena. These phenomena can be understood as logical biotas, i.e. cyclic deductions.

As we see, logic will soon become a science that rocks the world. Logic will privatise life for good and all. All exact sciences including classical ontology will disappear. Please ponder what the project of logic for unconventional computing means in fact. It is a project of absorbing all exact sciences. The necessity for operating theoretical terms of physics, chemistry, biology will disappear. There will be no foggy notions such as "matter," "energy," "chemical reaction," "morphology," "physiology." Instead of them there will be notions of computations describing processes in physical, chemical and biological systems. There will be a reduction of all natural-science knowledge to logic!

Logic has now grown up from children's panties of semiotics. In a few time, it will absorb all other sciences. Then we will speak not only about being, but also about any natural processes purely from the logical point of view. Logic will become a unified general science. As concerns engineering, we will face the

most grandious revolutionary changes in the human civilisation that have never been still!

Picture 10 The field of potatoes

7. The proof-theoretic cellular automaton for Belousov-Zhabotinsky reaction

Unconventional computing assumes that massive-parallel computations are observed everywhere in natural systems. There are different approaches to nature-inspired computing: reaction-diffusion computing (*Adamatzky, De Lacy Costello, Asai*), chemical computing (*Berry & Boudol*), biological computing (*Adamatzky 2007, Schumann & Adamatzky 2011*), etc. In all those computational models parallel inferring and concurrency are assumed as key

notions. In the paper (*Khrennikov & Schumann 2009*) a hypothesis was put forward that the paradigm of parallel and concurrent computation caused by rejecting the set-theoretic axiom of foundation can be widely applied in modern physics. In this section we are analyzing simulating Belousov-Zhabotinsky reaction within the framework of our theory of massive-parallel proofs.

Belousov-Zhabotinsky reaction is the best known example of auto-wave phenomenon in chemistry, i.e. the most famous example of self-organizing system there. We could show that the Belousov-Zhabotinsky reaction can be understood as a logical biota in massively parallel proof theory. In other words, we need a proof-theoretic cellular automaton with circular proofs for formalizing the Belousov-Zhabotinsky reaction containing feedback relations. The mechanism of this reaction (namely cerium(III) $\leftrightarrow$ cerium(IV) catalyzed reaction) is very complicated: its recent model contains 80 elementary steps and 26 variable species concentrations. Let us consider a simplification of Belousov-Zhabotinsky reaction assuming that the set of states consists just of the following reactants: Ce^{3+}, $HBrO_2$, BrO_3^-, H^+, Ce^{4+}, H_2O, $BrCH\ (COOH)_2$, Br^-, $HCOOH$, CO_2, $HOBr$, Br_2, $CH_2(COOH)_2$ which interact according to inference rules (reactions) (1)—(7). In this reaction we observe sudden oscillations in color from yellow to colorless, allowing the oscillations to be observed visually. In spatially nonhomogeneous systems (such as a simple petri dish), the oscillations propagate as spiral wave fronts. The oscillations last about one minute and are repeated over a long period of time. The color changes are caused by alternating oxidation-reductions in which cerium changes its oxidation state from cerium(III) to cerium(IV) and vice versa: $Ce^{3+} \to Ce^{4+} \to Ce^{3+} \to \ldots$

When Br^- has been significantly lowered, the reaction pictured by inference rule (1) causes an exponential increase in bromous acid ($HBrO_2$) and the oxidized form of the metal ion catalyst and indicator, cerium(IV). Bromous acid is subsequently converted to bromate (BrO_3^-) and $HOBr$ (the step (3)). Meanwhile,

the step (2) reduces the cerium(IV) to cerium(III) and simultaneously increase bromide (Br^-) concentration. Once the bromide concentration is high enough, it reacts with bromate (BrO_3^-) and $HOBr$ in (4) and (6) to form Br_2, further Br_2 reacts with $CH_2(COOH)_2$ to form $BrCH(COOH)_2$ and the process begins again. Thus, parallel processes in (1)—(7) have several cycles which are performed synchronously.

The proof-theoretic simulation of Belousov-Zhabotinsky reaction can be defined as follows:

Definition 7.1 *Consider a propositional language* L *with the only binary operation* $\oplus$, *it is built in the standard way over the set of variables* $S = \{Ce^{3+}, HBrO_2, BrO_3^-, H^+, Ce^{4+}, H_2O, BrCH(COOH)_2, Br^-, HCOOH, CO_2, HOBr, Br_2, CH_2(COOH)_2\}$. *Let* S *be the set of states of proof-theoretic cellular automaton* A. *The inference rule of the automaton is presented by the conjunction of singular inference rules (1)—(7):*

$$(1) \wedge (2) \wedge (3) \wedge (4) \wedge (5) \wedge (6) \wedge (7).$$

The operation $\oplus$ has the following meaning: $A \oplus B$ defines a probability distribution of events A and B in neighbor cells participated in a reaction caused the appearance of $A \oplus B$. Then A simulates the Belousov-Zhabotinsky reaction.

Definition 7.2 *Let* $p, s_i, s_{i+1} \in \{Ce^{3+}, HBrO_2, BrO_3^-, H^+, Ce^{4+}, H_2O, BrCH(COOH)_2, Br^-, HCOOH, CO_2, HOBr, Br_2, CH_2(COOH)_2\}$. *A state* p *is called a premise for deducing* s_{i+1} *from* s_i *by the inference rule* $(1) \wedge (2) \wedge \ldots \wedge (7)$ *iff*

- p is s_i or
- in a neighbor cell we find out an expression of the form $p \oplus A, B \oplus C$, where A, B, C are propositional metavariables, i.e. they run over either the empty set or the set of states closed under the operation $\oplus$. Thus, we assume that each premise should occur in a

separate cell. This means that if we find out an expression $p_i \oplus p_j, B \oplus C$ or $p_i \oplus A, p_j \oplus C$ in a neighbor cell and both p_i and p_j are needed for deducing, whereas p_i, p_j do not occur in other neighbor cells, then p_i, p_j could not be considered as premises.

$$x^{t+1}(z) = \begin{cases} Ce^{4+} \oplus HBrO_2 \oplus H_2O, if\ x^t(z) \in \{Ce^{3+}\}\ and \\ \ premises\ HBrO_2, BrO_3^-, H^+ \in (z+N); \\ \\ x^t(z), otherwise. \end{cases} \quad (1)$$

$$x^{t+1}(z) = \begin{cases} Br^- \oplus Ce^{3+} \oplus HCOOH \oplus CO_2 \oplus H^+, if\ x^t(z) \in \{Ce^{4+}\} \\ and\ \ premises\ BrCH(COOH)_2, H_2O \in (z+N); \\ \\ x^t(z), otherwise. \end{cases} \quad (2)$$

$$x^{t+1}(z) = \begin{cases} HOBr \oplus BrO_3^- \oplus H^+, & if\ x^t(z) \in \{HBrO_2\}; \\ \\ x^t(z), & otherwise. \end{cases} \quad (3)$$

$$x^{t+1}(z) = \begin{cases} HOBr \oplus HBrO_2, & if\ x^t(z) \in \{BrO_3^-\}\ and \\ & \ premises\ Br^-, H^+ \in (z+N); \\ \\ x^t(z), & otherwise. \end{cases} \quad (4)$$

$$x^{t+1}(z) = \begin{cases} HOBr, & if\ x^t(z) \in \{Br^-\}\ and \\ & \ premises\ HBrO_2, H^+ \in (z+N); \\ \\ x^t(z), & otherwise. \end{cases} \quad (5)$$

$$x^{t+1}(z)=\begin{cases} Br_2 \oplus H_2O, & \text{if } x^t(z)\in\{HOBr\} \text{ and} \\ & \text{premises } Br^-, H^+ \in (z+N); \\ x^t(z), & \text{otherwise.} \end{cases} \quad (6)$$

$$x^{t+1}(z)=\begin{cases} Br^- \oplus H^+ \oplus BrCH(COOH)_2, \text{if } x^t(z)\in\{Br_2\} \\ \text{and premises } CH_2(COOH)_2 \in (z+N); \\ x^t(z), \text{otherwise.} \end{cases} \quad (7)$$

Example of Belousov-Zhabotinsky's cyclic proofs *We can simplify the automaton defined above assuming that* $\oplus$ *is a metatheoretic operation with the following operational semantics:*

$$\frac{A\oplus B}{A} \qquad \frac{A\oplus B}{B},$$

where A and B are metavariables defined on S. The informal meaning of that operation is that we can ignore one of both variables coupled by $\oplus$. In the cellular automaton A this metaoperation will be used as follows:

$$x^{t+1}(z)=\begin{cases} X,Y, & \text{if } x^t(z)=A\oplus B \text{ and according to rules } (1)-(7), \\ & X \text{ changes from } A \text{ and } Y \text{ changes from } B; \\ X, & \text{if } x^t(z)=A\oplus B \text{ and according to rules } (1)-(7), \\ & X \text{ changes from } A \text{ and } B \text{ does not change}; \\ Y, & \text{if } x^t(z)=A\oplus B \text{ and according to rules } (1)-(7), \\ & Y \text{ changes from } B \text{ and } A \text{ does not change}; \\ A\oplus B, & \text{if } x^t(z)=A\oplus B \text{ and rules } (1)-(7) \\ & \text{cannot be applied to } A \text{ or } B. \end{cases} \quad (8)$$

Let us suppose now that X,Y run over the set of states closed under the operation $\oplus$.

$$x^{t+1}(z)=\begin{cases} X,Y, & \text{if } (i)\ x^t(z)=X,Y \text{ and } (ii) \text{ both } X \text{ and } Y \\ & \text{are simultaneously usable } (\text{not usable}) \\ & \text{as premises in at least two different rules} \\ & \text{of } (1)-(7) \text{ (see definition } 7.2); \\ X, & \text{if } (i)\ x^t(z)=X,Y \text{ and } (ii) \text{ only } X \text{ is usable} \\ & \text{as a premise in at least one rule of } (1)-(7); \\ Y, & \text{if } (i)\ x^t(z)=X,Y \text{ and } (ii) \text{ only } Y \text{ is usable} \\ & \text{as a premise in at least one rule of } (1)-(7). \end{cases} \quad (9)$$

$$\text{idempotency}: A ::= A, A. \quad (10)$$

$$\text{commutativity}: A, B ::= B, A. \quad (11)$$

Hence, we cannot ignore one of both variables coupled by $\oplus$ and should accept both them if in the neighborhood there are reactants that catenate both variables and change them. This rule is the simplest interpretation of $A \oplus B$ in definition 7.1. We have three cases: (i) both variables are catenated with reactants from the neighborhood, in this case we mean that the probability distribution of events A and B is the same and equal to 0.5 and, as a result, we cannot choose one of them and accept both; (ii) only A is catenated with reactants from the neighborhood, then the probability distribution of event A is equal to 1.0 and that of B to 0.0; (iii) only B is catenated with reactants from the neighborhood, then the probability distribution of event B is equal to 1.0 and that of A to 0.0. Thus, $A \oplus B$ is a function that associates either exactly one value with its arguments (i.e. either A or B) or simultaneously both values (i.e. A and B).

This simplified version of the automaton A is exemplified in Fig.7.1.

Evidently, reducing the complicated dynamics of Belousov-Zhabotinsky reaction to conventional logical proofs is a task that cannot be solved in easy way differently from simulating within massive-parallel proofs.

(I) Initial configuration, $t = 0$

$HBrO_2$	BrO_3^-	H^+
$BrCH(COOH)_2$	Ce^{3+}	H_2O
Br^-	$CH_2(COOH)_2$	BrO_3^-

$\Downarrow$

(II) $t = 1$

$HOBr \oplus BrO_3^- \oplus H^+$	BrO_3^-	H^+
$BrCH(COOH)_2$	$Ce^{4+} \oplus HBrO_2 \oplus H_2O$	H_2O
Br^-	$CH_2(COOH)_2$	BrO_3^-

$\Downarrow$

(III) $t = 2$

$HOBr \oplus BrO_3^- \oplus H^+$	BrO_3^-	H^+
$BrCH(COOH)_2$	$Br^- \oplus Ce^{3+} \oplus HCOOH \oplus CO_2 \oplus H^+$, $HOBr \oplus BrO_3^- \oplus H^+$	H_2O
Br^-	$CH_2(COOH)_2$	BrO_3^-

$\Downarrow$

(IV) $t = 3$

$HOBr \oplus BrO_3^- \oplus H^+$	$HOBr \oplus HBrO_2$	H^+
$BrCH(COOH)_2$	$Br^- \oplus Ce^{3+} \oplus HCOOH \oplus CO_2 \oplus H^+$, $Br_2 \oplus H_2O$, $HOBr \oplus HBrO_2$	H_2O
Br^-	$CH_2(COOH)_2$	BrO_3^-

$\Downarrow$

(V) $t = 4$

$HOBr \oplus BrO_3^- \oplus H^+$	$Br_2 \oplus H_2O$, $HOBr \oplus BrO_3^- \oplus H^+$	H^+
$BrCH(COOH)_2$	$Ce^{4+} \oplus HBrO_2 \oplus H_2O$, $Br_2 \oplus H_2O$, $HOBr \oplus BrO_3^- \oplus H^+$, $HOBr$, $Br^- \oplus H^+ \oplus BrCH(COOH)_2$	H_2O
Br^-	$CH_2(COOH)_2$	BrO_3^-

$\Downarrow$

(VI) $t = 5$

$Br_2 \oplus H_2O$, $HOBr \oplus HBrO_2$	$Br_2 \oplus H_2O$, $HOBr \oplus HBrO_2$	H^+
$BrCH(COOH)_2$	$Br_2 \oplus H_2O$, $Br^- \oplus Ce^{3+} \oplus HCOOH \oplus CO_2 \oplus H^+$, $HOBr \oplus BrO_3^- \oplus H^+$, $HOBr \oplus HBrO_2$, $Br^- \oplus H^+ \oplus BrCH(COOH)_2$	H_2O
Br^-	$CH_2(COOH)_2$	BrO_3^-

$\Downarrow$

…

Figure 7.1 The evolution of a reversible proof-theoretic cellular automaton A with the Moor neighborhood in the 2-dimensional space for the Belousov-Zhabotinsky reaction. This automaton simulates the circular feedback $Ce^{3+} \to Ce^{4+} \to Ce^{3+} \to \ldots$ (more precisely temporal oscillations in a well-stirred solution): Ce^{3+} is colorless and Ce^{4+} is yellow. The initial configuration of A-cells described in (I) occurs in the same form at the further steps and the cycle repeats several times. For entailing (I) $\to$ (II) we have just used inference rule (1) (row 2, column 2) and inference rule (3)

(row 1, column 1), for entailing (II) → (III) inference rules (2), (3) and (8) (row 2, column 2), for entailing (III) → (IV) inference rule (4) (row 1, column 2) and inference rules (4), (6), (8) and (9) (row 2, column 2), for entailing (IV) → (V) inference rules (3) and (6) (row 1, column 2) inference rules (1), (3), (5), (6), (7), (8) and (9) (row 2, column 2), for entailing (V) → (VI) inference rules (4), (6) (row 1, column 1), inference rules (4), (6), (10) (row 1, column 2), inference rules (2), (3), (4), (6), (7), (9), (10), (11) (row 2, column 2).

8. The proof-theoretic cellular automaton for dynamics of *Plasmodium* of *Physarum polycephalum*

The dynamics of plasmodium of *Physarum polycephalum* could be regarded as another simple example of the natural proof-theoretic automata. The point is that when the plasmodium is cultivated on a nutrient-rich substrate (agar gel containing crushed oat flakes) it exhibits uniform circular growth similar to the excitation waves in the excitable Belousov-Zhabotinsky medium (Fig. 8.1). If the growth substrate lacks nutrients, e.g. the plasmodium is cultivated on a non-nutrient and repellent containing gel, a wet filter paper or even glass surface localizations emerge and branching patterns become clearly visible (Fig. 8.2, 8.3).

The plasmodium continues its spreading, reconfiguration and development as long as there are enough nutrients. When the supply of nutrients is over, the plasmodium either switches to fructification state (if level of illumination is high enough), when sporangia are produced, or forms sclerotium (encapsulates itself in hard membrane), if in darkness.

The pseudopodium propagates in a manner analogous to the formation of wave-fragments in sub-excitable Belousov-Zhabotinsky systems. Starting in the initial conditions the plasmodium exhibits foraging behavior, searching for sources of nutrients (Fig.8.1). When such sources are located and taken over,

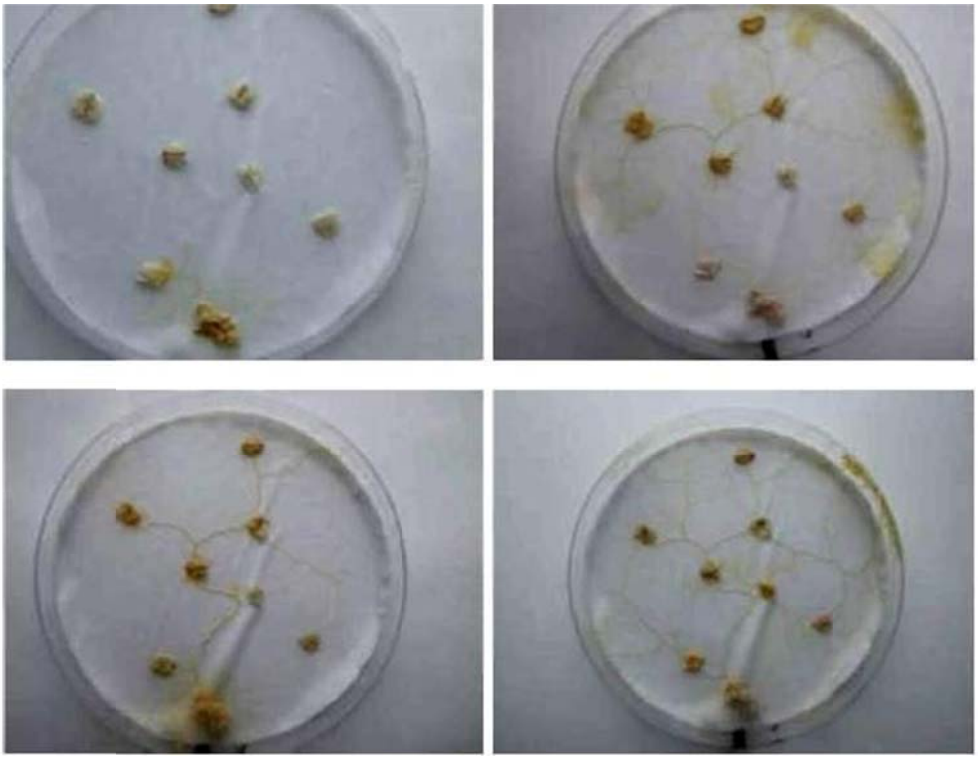


Figure 8.1 An example of computational process in *Physarum* machine. Photographs are taken with time lapse circa 24 hours. *Courtesy of Andy Adamatzky*.

the plasmodium forms characteristic veins of protoplasm, which contracts periodically. Belousov-Zhabotinsky reaction and plasmodium are light-sensitive, which gives us the means to program them. *Physarum* exhibits articulated negative phototaxis, Belousov-Zhabotinsky reaction is inhibited by light. Therefore by using masks of illumination one can control the dynamics of localizations in these media. Light-sensitivity of Plasmodium has been already explored in the design of robotics controllers (*Adamatzky 2007*, *Adamatzky & Jones 2009*).

Experiments with *Physarum polycephalum* were carried out by Prof. Adamatzky as follows. The plasmodia of *Physarum polycephalum*

were cultured on wet paper towels, fed with oat flakes, and moistened regularly. We subcultured the plasmodium every 5—7 days.

Experiments were performed in standard Petri dishes, 9 cm in diameter. Depending on particular experiments we used 2% agar gel or moisten filter paper, nutrient-poor substrates, and 2% oatmeal agar, nutrient-rich substrate (Sigma-Aldrich). All experiments were conducted in a room with diffusive light of 3—5 cd/m, 22^{o}C temperature. In each experiment an oat flake colonized by the plasmodium was placed on a substrate in a Petri dish, and few intact oat flakes distributed on the substrate. The intact oat flakes acted as source of nutrients, attractants for the plasmodium. Petri dishes with plasmodium were scanned on a standard HP scanner. The only editing done to scanned images is color enhancement: increase of saturation and contrast.

Repellents were implemented with illumination domains using blue electroluminescent sheets, see details in *Adamatzky 2010*. Masks were prepared from black plastic, namely the triangle was cut in the plastic, when this mask was placed on top of the electro-luminescent sheet, the light was passing only through the cuts (*Adamatzky 2010*).

Results of experiments may be described in terms of proof-theoretic cellular automata. Let us assume that its set of states consists of the entities from the following sets (Fig.8.2).

- The set of *neutral zones*, $\{Z_1, Z_2,\ldots\}$, where nothing goes.
- The set of *growing pseudopodia*, $\{P_1, P_2,\ldots\}$, localized in *active zones* (see an example on Fig.8.2a). On a nutrient-rich substrate plasmodium propagates as a typical circular, target wave, while on the nutrient-poor substrates localized wave-fragments are formed.
- The set of *attractants* $\{A_1, A_2,\ldots\}$, they are sources of nutrients, on which the plasmodium feeds. It is still subject of discussion how exactly plasmodium feels presence of attracts, indeed diffusion of some kind is involved. Based on

previous experiments by Prof. Adamatzky we can assume that if the whole experimental area is about $8-10 \approx cm$ in diameter then the plasmodium can locate and colonize nearby sources of nutrients.

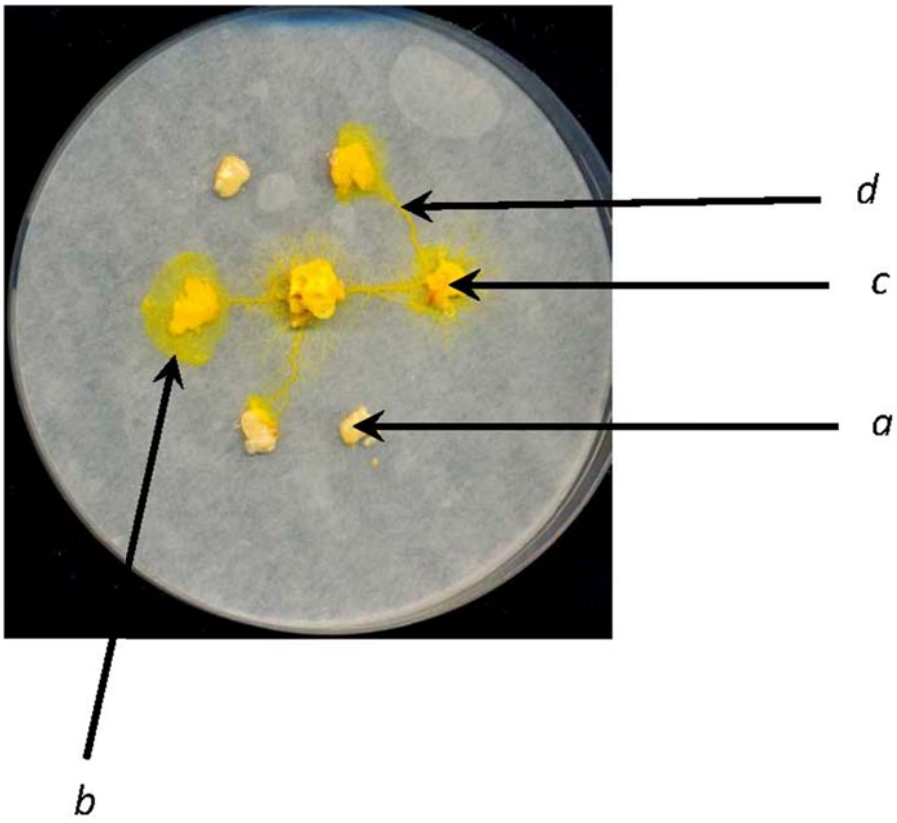


Figure 8.2 Basic components of *Physarum* and its environment: (a) oat flake, (b) propagating pseudopodium, plasmodium's wave-fragment, (c) oat flake colonized by plasmodium, (d) protoplasmic tube. *Courtesy of Andy Adamatzky*.

- The set of *repellents* $\{R_1, R_2, \ldots\}$. Plasmodium of *Physarum* avoids light. Thus, domains of high illumination are repellents such that each repellent R is characterized by its

position and intensity of illumination, or force of repelling.

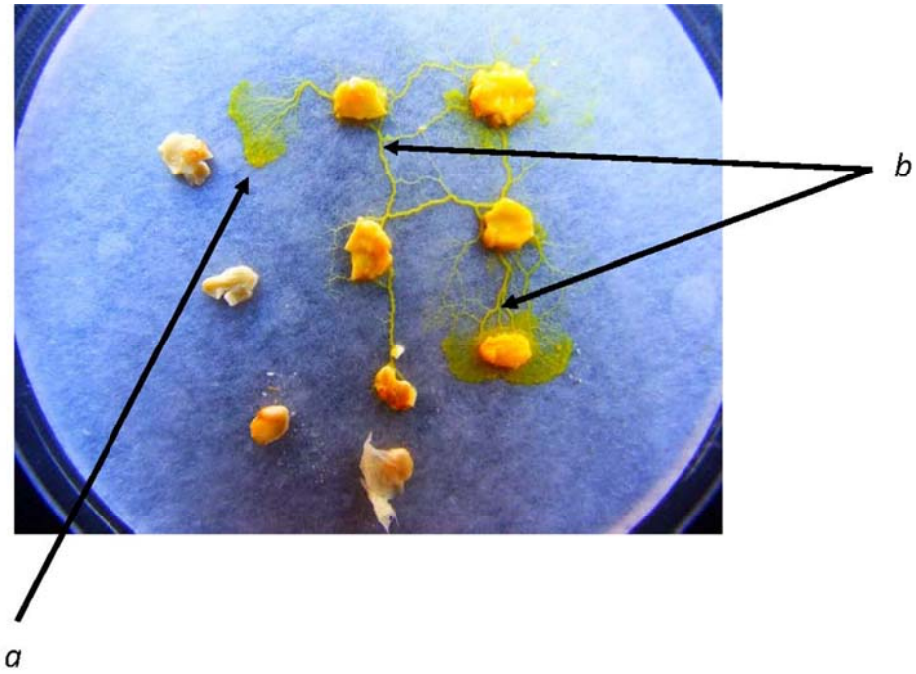


Figure 8.3 Snapshot of experimental dish with propagating plasmodium, new activated propagating zone (a) and sites of branching pseudopodia, junctions of protoplasmic tubes (b) are shown by arrows. *Courtesy of Andy Adamatzky*.

- The set of *protoplasmic tubes* $\{C_1, C_2, \ldots\}$. Typically plasmodium spans sources of nutrients with protoplasmic tubes/veins (Fig.8.2). The plasmodium builds a planar graph, where nodes are sources of nutrients, e.g. oat flakes, and edges are protoplasmic tubes.

Hence, the set of states in the proof-theoretic cellular automaton for dynamics of Plasmodium of *Physarum polycephalum* is equal to $\{Z_1, Z_2, \ldots\} \cup \{P_1, P_2, \ldots\} \cup \{A_1, A_2, \ldots\} \cup \{R_1, R_2, \ldots\} \cup \{C_1, C_2, \ldots\}$.

The proof-theoretic simulation of *Physarum polycephalum* is defined as follows:

Definition 8.11 *Consider a propositional language* L *with the only binary operation* $\oplus$*, it is built in the standard way over the set of variables* $S = \{Z_1, Z_2, \ldots, P_1, P_2, \ldots, A_1, A_2, \ldots, R_1, R_2, \ldots, C_1, C_2, \ldots\}$*. Let* S *be the set of states of proof-theoretic cellular automaton* A*. The inference rule of the automaton is presented by the conjunction of singular inference rules (1)—(7):*

$$(1) \wedge (2) \wedge (3) \wedge (4) \wedge (5) \wedge (6) \wedge (7).$$

The operation $\oplus$ has the following meaning: $A \oplus B$ defines a probability distribution of events A and B in cells. The operation $\oplus$ is idempotent, commutative and associative. Then A simulates the dynamics of Plasmodium of *Physarum polycephalum*.

Definition 8.2 *Let* $p, s_i, s_{i+1} \in \{Z_1, Z_2, \ldots, P_1, P_2, \ldots, A_1, A_2, \ldots, R_1, R_2, \ldots, C_1, C_2, \ldots\}$*. A state* p *is called a premise for deducing* s_{i+1} *from* s_i *by inference rules (1)—(7) iff*

- p is s_i or
- in a neighbor cell we find out an expression of the form $p \oplus X$, where X is a propositional metavariable, i.e. it runs over either the empty set or the set of states closed under the operation $\oplus$. Thus, we assume that each premise should occur in a separate cell. This means that if we find out an expression $p_i \oplus p_j \oplus X$ in a neighbor cell and both p_i and p_j are needed for deducing, whereas p_i, p_j do not occur in other neighbor cells, then p_i, p_j could not be considered as premises. This restriction is just for rule (7).

$$x^{t+1}(z) = \begin{cases} X \text{ if } x^t(z) = X \oplus A_i \text{ and } X = Y \oplus P_j \\ \\ x^t(z), \text{otherwise.} \end{cases} \quad (1)$$

$$x^{t+1}(z) = \begin{cases} X \oplus P_j \text{ if } x^t(z) = X \oplus C_n \oplus A_i \\ \\ x^t(z), \text{otherwise.} \end{cases} \quad (2)$$

$$x^{t+1}(z)=\begin{cases} X\oplus P_j \ \textit{if}\ x^t(z)=X\oplus P_j\oplus A_i \\ \\ x^t(z), \textit{otherwise.}\end{cases} \tag{3}$$

$$x^{t+1}(z)=\begin{cases} X\oplus P_i\vee X\oplus P_j \ \textit{if}\ x^t(z)=X\oplus P_i\oplus P_j \\ \\ x^t(z), \textit{otherwise.}\end{cases} \tag{4}$$

$$x^{t+1}(z)=\begin{cases} X\oplus C_i\vee X\oplus C_j \ \textit{if}\ x^t(z)=X\oplus C_i\oplus C_j \\ \\ x^t(z), \textit{otherwise.}\end{cases} \tag{5}$$

$$x^{t+1}(z)=\begin{cases} X\oplus C_i, & \textit{if}\ x^t(z)=X \ \textit{and} \\ & \textit{premises}\ R_i\notin(z+N), \\ & \textit{premises}\ C_j, A_i\in(z+N); \\ \\ x^t(z), & \textit{otherwise.}\end{cases} \tag{6}$$

$$x^{t+1}(z)=\begin{cases} X\oplus C_i, & \textit{if}\ x^t(z)=X \ \textit{and} \\ & \textit{premises}\ R_i\notin(z+N), \\ & \textit{premises}\ P_j, A_i\in(z+N); \\ \\ x^t(z), & \textit{otherwise.}\end{cases} \tag{7}$$

(I) Initial configuration, $t=0$

A_{11}	Z_{21}	A_{31}
R_{12}	A_{22}	Z_{32}
R_{13}	Z_{23}	P_{33}

$\Downarrow$

(II) $t=1$

A_{11}	Z_{21}	A_{31}
R_{12}	A_{22}	$Z_{32}\oplus C_{32}$
R_{13}	Z_{23}	P_{33}

$\Downarrow$

(III) $t = 2$

A_{11}	Z_{21}	$A_{31} \oplus C_{31}$
R_{12}	A_{22}	$Z_{32} \oplus C_{32}$
R_{13}	Z_{23}	$P_{33} \oplus C_{33}$

$\Downarrow$

(IV) $t = 3$

A_{11}	Z_{21}	P_{31}
R_{12}	A_{22}	$Z_{32} \oplus C_{32}$
R_{13}	Z_{23}	$P_{33} \oplus C_{33}$

Figure 8.4 The evolution of a proof-theoretic cellular automaton A with the Moor neighborhood in the 2-dimensional space for *Physarum polycephalum*.

9. Unconventional computing as novel paradigm in natural sciences

In natural sciences there are many theoretical terms which as it seems to us allow to understand reality better, but actually they replace true reality by images made up. For example, in understanding the Belousov-Zhabotinsky reaction the following theoretical terms of chemistry are used: matter, chemical element, chemical reaction, autowave, valence etc. In understanding the dynamics of Plasmodium of *Physarum polycephalum* the following theoretical terms of biology are assumed: contiguous living system, cell, protoplasm, physiology, morphology etc.

If we share positions of unconventional computing and agree that any natural process is a computation and nature as a whole is a computer, theoretical terms of physics, chemistry and biology will lose a sense. We can eliminate them from the language of scientific theories.

Theoretical terms cannot be verified, i.e. they are not reduced to observation terms. However, within the limits of unconventional computing we can substitute logical terms simulating the process of

computation for theoretical terms. Logical terms do not require already verification. They form only the structure and algorithmization of observation terms.

Let us suppose that we created the universal language L_U of *unconventional computing*, i.e. the universal logical language that allows us to consider any natural process as computation process. Then the language L_U, first, eliminates all theoretical terms of natural sciences, replacing them by logical terms of massively parallel proof theory and, second, instead of huge number of scientific theories of physics, chemistry, biology we obtain an opportunity to have the universal logical theory that simulates the Universe.

Let us show, how we could eliminate theoretical terms in the language L_U. Take n theoretical terms: $T_1, T_2, \ldots, T_n$, linked to observation terms by correspondence rules of an appropriate theory and m observation terms, i.e. their references are found out immediately: $O_1, O_2, \ldots, O_m$. The complete theoretical statement is a union consisting of T - and O-terms:

$$\langle T_1, T_2, \ldots, T_n; O_1, O_2, \ldots, O_m \rangle.$$

This formula can be replaced by another, identical to the first in the way that all theoretical terms are replaced by logical terms $U_1, U_2, \ldots, U_k$ which describe the process of computation of an appropriate automata with features $\langle O_1, O_2, \ldots, O_m \rangle$:

$$\langle U_1, U_2, \ldots, U_k; O_1, O_2, \ldots, O_m \rangle.$$

In such a statement, empirical values of theoretical terms are positioned, i.e. they obtain references. These references are expressed by observation terms. Such a representation of theoretical expressions has a number of advantages, e.g. sophisticated theoretical terms of natural sciences could find the classes of references. For example, it is obvious that the statements

describing Earth's rotation have the set {the Earth} as a class of references. However, it is not clearly, which set of physical bodies realises the statements describing the concept 'electron.'

In a proposition of language L_U the sophisticated expression setting theoretical terms is transformed into an expression in which there are only observation terms as well as terms of anti-Platonic (a posteriori) logic and mathematics, therefore the concept 'electron' may be eliminated from theory expressions.

In our simulation of Belousov-Zhabotinsky reaction as observation terms the following states of an appropriate cellular automaton were used: $\{Ce^{3+}, HBrO_2, BrO_3^-, H^+, Ce^{4+}, H_2O, BrCH(COOH)_2, Br^-, HCOOH, CO_2, HOBr, Br_2, CH_2(COOH)_2\}$, and in the simulation of the dynamics of Plasmodium of *Physarum polycephalum* the following states: $\{Z_1, Z_2,\ldots,P_1, P_2,\ldots,A_1, A_2,\ldots,R_1,R_2, \ldots,C_1,C_2,\ldots\}$.

The theoretical terms needed for understanding the Belousov-Zhabotinsky reaction were replaced by inference rules of definition 7.1, and theoretical terms needed for understanding the dynamics of Plasmodium of *Physarum polycephalum* by inference rules of definition 8.1.

10. Kabbalistic-Leibnizian automata

It is a miracle that ideas of unconventional computing and analytic a posteriori computation have their roots in Judaism, namely in Kabbalah, i.e. they may be regarded as a part of revelation. In Judaism there is a claim that only the Lord exists. He has created the world from nothing. He has control over anything. He stands above all. In a hymn of the Jewish liturgy there is the following statement about Him:

מָלַךְ אֲשֶׁר עוֹלָם אָדוֹן	נִבְרָא יְצִיר כָּל בְּטָרֵם
כּ֫ל בְּחֶפְצוֹ נַעֲשָׂה לְעֵת	נִקְרָא שְׁמוֹ מֶלֶךְ אֲזַי

הַכֹּ֥ל כְּכִלּוֹת וְאַחֲרֵי נוֹרָא יִמְלֹ֖ךְ לְבַדּוֹ

ה֥וּא וְהוּא הָיָה וְהוּא בְּתִפְאָרָה יִהְיֶה וְהוּא

Eternal master, who reigned supreme, Before all of creation was drawn;
When it was finished according to his will, Then "King" his name was proclaimed to be
When this our world shall be no more, In majesty he still shall reign,
And he was, and he is, And he will be in glory.

In Kabbalah, the esoteric teaching of Judaism, the illusiveness of the world is proved by that the world does not exist in itself. The world is a combination of the characters of sacred (Hebrew) language, in which the Holy Scripture is written. According to the *Sefer Yetzirah*, the best known Kabbalistic source, the entire Universe was created from the set of 22 Hebrew letters, each of which is also a number. God's holy words have created all:

> בשלשים ושתים נתיבות פליאות חכמה חקק י"ה יהו"ה צבאות את עולמו בשלשה ספרים בספר ספר וספור עשרים ושתים אותיות יסוד, שלש אמות ושבע כפולות וי"ב פשוטות
>
> In two and thirty most occult and wonderful paths of wisdom did JAH the Lord God of the armies of Israel. He created this universe by the three Sepharim, Number, Writing, and Speech. Ten are the numbers, as are the Sephiroth, and twenty-two the letters, these are the Foundation of all things. Of these letters, three are mothers, seven are double, and twelve are simple (*Sefer Yetzirah*).

Each character of the Hebrew alphabet is given an assigned number, beginning with one for א, two for ב, and so on. The tenth character, י, is numerically equivalent to 10, and successive characters equal 20, 30, 40, and so on. The letter ק equals 100, ר 200, ש 300 and the last letter, ת, equals 400. The final forms (sofit) of the Hebrew letters are counted as a continuation of the numerical sequence for the alphabet. These sofit letters are assigned from 500 to 900, respectively.

letter	*numeric value*	*letter*	*numeric value*	*letter*	*numeric value*
א	1	**י**	10	**ק**	100
ב	2	**כ**	20	**ר**	200
ג	3	**ל**	30	**ש**	300
ד	4	**מ**	40	**ת**	400
ה	5	**נ**	50	**ך**	500
ו	6	**ס**	60	**ם**	600
ז	7	**ע**	70	**ן**	700
ח	8	**פ**	80	**ף**	800
ט	9	**צ**	90	**ץ**	900

Kabbalah reconstructs the dynamics of natural phenomena from the two points of view: on the one hand, Hebrew phrases of the Torah can be represented as numeric algoriths for describing natural phenomena, on the other hand, mathematical calculations over phenomena can be read as Hebrew phrases.

The roots of Kabbalah have grown from Gematria [גימטריא], the cryptogramme giving instead of an initial word its numerical value or replacing one word by another word whose letters correspond to the letters of the first word by a special combination of characters. These two forms of Gematria occuring in the Talmud and Midrash are as follows:

1) *Numeric values of words*. Two basic subcases are:

i) A number, hinting in the Holy Scripture at a subject. For example, in the Torah (*Gen.* 14:14) it is stated that Abraham, waging the war against the four Eastern kings, armed and took 318 of his trained servants; meanwhile there exists a Midrash according to that Abraham took only one servant, Eliezer. Gematria solves this contradiction by refering that the sum of numerical values of letters of the name Eliezer is equal 318 (א=1; ל=30; י=10; ע=70; ז=7; ר=200):

> **שמונה עשר ושלש מאות:** רבותינו אמרו אליעזר לבדו היה, והוא מנין גימטריא של שמו:

three hundred and eighteen: Our Sages said (*Gen. Rabbah* 43:2, *Ned.* 32a): It was Eliezer alone, and it [the number 318] is the numerical value of his name.

ii) A word of the Holy Scripture hinting at a number, person or subject. For instance, a word can be accepted in its ordinary numerical value, for example the word הכסף, money, which King Ahasuerus promised to return to Haman, presaged העץ, the gallows-tree on which he was hung up, because both words = 165 (*Esther* 3:11). In this form Gematria is a simple arithmetic equation, for example צמח=מנחם=138. Further, one of the members of equality or both members can be added up with a unit or two, an 'external number' for those words whose numerical value is regarded. For example: both תורה and יראת are equal 611, by adding the number 2, (an 'external number' for both words), we sum up 613 (*Numb. Rabbah* 18:21).

2. *Cryptography*. All or some letters of a word can be replaced also by appropriate letters in accordance with the cryptographic alphabet. One of the methods of cryptography is *Atbash* [אתבש] that exchanges each letter in a word or a phrase by opposite letters. Opposite letters are determined by substituting the first letter of the Hebrew alphabet [א] with the last letter [ת], the second letter [ב] with the next to last [ש], etc. The result can be interpreted as a secret message or calculated by the standard Gematria methods. For example, Prophet Jeremiah (51:1) calls Babylon the name לב קמי for unclear reasons. By Atbash, לֵב קָמָי is a cryptography, having the value כַּשְׂדִים (Chaldea). Another instance of Atbash is when the country שֵׁשַׁךְ is understood as בָּבֶל (*Jeremiah* 25:26 and 51:41).

Many Kabbalists had attempts to recreate some samples of the Divine speech creating the Universe by using different Gematria methods. They supposed that the Divine words may be generated by a purely mechanical process and might have no meaning in human language. In fact, as we see, the idea of automaton, a self-operating machine, flashed upon Kabbalists for

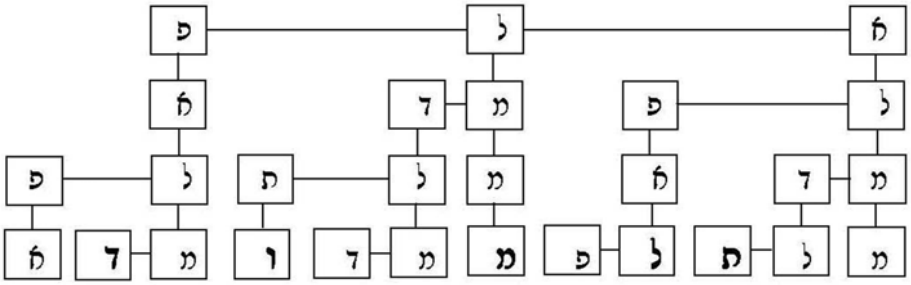
the first time (for more details see *Schumann (ed.) 2010, Schumann (ed.) 2011*).

As an example of Kabbalistic automata let us consider one of the automata simulating the Divine speech that has been proposed by Eleazar ben Judah ben Kalonymus of Worms (Rokeach), the Talmudist and Kabbalist, born probably at Mayence about 1176 and died at Worms in 1238. He was a descendant of the great Kalonymus family of Mayence.

In the following example of Kabbalistic automaton, we can see, how the letters of word 'Talmud' emerge from *'alef* (the first letter of the Hebrew alphabet):

Example.
Halting pattern: *t*, *l*, *m*, *w*, *d* (Talmud)
Input string: *'a l f*
Generation1: *lf md'a*
Generation2: *md'a m lt lf*
Generation3: *m l**t** l**f** **m** md **w** m**d**'a*

All five letters of the word 'Talmud' appear after three generations. The letter *'alef* was first spelt out horizontally and then branches into a tree:



The idea of Kabbalistic automata in which operations over characters of the Hebrew alphabet should simulate physical and biological processes, suggested Leibniz an idea to work at a

characteristica universalis or *universal characteristic*, built on an alphabet of human thought in which each fundamental concept would be represented by a unique character:

> It is obvious that if we could find characters or signs suited for expressing all our thoughts as clearly and as exactly as arithmetic expresses numbers or geometry expresses lines, we could do in all matters insofar as they are subject to reasoning all that we can do in arithmetic and geometry. For all investigations which depend on reasoning would be carried out by transposing these characters and by a species of calculus (*Leibniz 1677*).

In universal characteristic complex thoughts would be represented by combining characters for atomic thoughts. Characters of the Hebrew alphabet, which in Kabbalists' intention are atoms of any information, were prototypes of characters in Leibniz' meaning. For Leibniz such atoms should be presented by the system of coding of all elementary concepts by using prime numbers due to the uniqueness of prime factorization (also this idea was used in Gödel numbering).

Universal characteristic should have become an automaton simulating all physical processes, an automaton who knows the "Answer to the Ultimate Question of Life, the Universe, and Everything." This automaton should have become a universal checker which can check and verify any thought mechanically:

> The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate [calculemus], without further ado, to see who is right (*Leibniz 1685*).

Due to such possibilities, universal characteristic may be regarded as sacral language, i.e. a unique language in which it is possible to think unmistakably, language that objectively reflects any physical and biological process, language which already

coincides with reality. It is obvious that sacred language in Kabbalistic meaning was a prototype of universal language in Leibnizian meaning. The latter should have become a universal language for communications of all people:

> And although learnt men have long since thought of some kind of language or universal characteristic by which all concepts and things can be put into beautiful order, and with whose help different nations might communicate their thoughts and each read in his own language what another has written in his, yet no one has attempted a language or characteristic which includes at once both the arts of discovery and judgement, that is, one whose signs and characters serve the same purpose that arithmetical signs serve for numbers, and algebraic signs for quantities taken abstractly. Yet it does seem that since God has bestowed these two sciences on mankind, he has sought to notify us that a far greater secret lies hidden in our understanding, of which these are but the shadows (*Leibniz 1966*).

Leibniz proposed some examples of how it is possible to design universal characteristic. In these examples Leibniz presented 'character' as a certain number, assigned to a subject or a predicate as eigenvalue in the way that a derivated number corresponds to a combination (superposition) of numbers of initial terms. This combination can be presented, for example, by multiplication of numbers which are 'characters' of initial terms (atoms of universal characteristic). For instance, the expression 'rational animal,' consisting of two terms: 'rational' with character a, and 'animal' with character r, obtains a new character ar, i.e. 'man' ('man' = 'rational animal'). In particular, if $a = 3$, $r = 2$, then character of 'man' = 6.

In categorical propositions, relations between characters are defined by means of the following rules:

1) The universal affirmative proposition is true if and only if the character of subject is divided by the character of predicate without remainder of division, e.g. in the proposition 'man is a

rational animal' we have: 'man' (S) / 'animal' (P) = $ar/r = a$.

2) The particular affirmative proposition is true if and only if the character of subject is divided by the character of predicate without remainder of division or the character of predicate is divided by the character of subject without remainder of division.

3) The universal negative proposition is true if and only if the character of subject cannot be divided by the character of predicate without remainder of division and also the character of predicate cannot be divided by the character of subject without remainder of division.

4) The particular negative proposition is true if and only if the character of subject cannot be divided by the character of predicate without remainder of division.

In case categorical propositions do not satisfy the above-mentioned rules, they are considered false. The favour of similar numerical interpretation of atomic propositions consists in that by means of simple operating over numbers it is possible to prove all syllogistic laws. At the same time, the evident assumption is as follows: characters $H = ABC$ and $J = BC$ for combinations of characters of simpler terms A, B, C are divided by the same number of term X without remainder of division if $X < H$ and $X < J$. For example, let the number 'man' be H and the number of 'animal' be A in the proposition 'man is animal.' Then fraction H/A is reduced to prime numbers r/v such that r/v is an integer, i.e. $v = 1$. From this assumption it follows

(1) 'Every H is A' iff $vH = rA$ and $v = 1$.

(2) 'Some A are H' (or 'Some H are A') iff $rA = vH$ and ($r = 1$ or $v = 1$).

(3) 'No H is A' (or 'No A is H') iff $vH = rA$ and ($r > 1$ and $v > 1$).

(4) 'Some A are not H' iff $rA = vH$ and $r > 1$.

The given interpretation satisfies the square of opposition.

Indeed, in (2) and (3) (resp. in (1) and (4)) the universal negative proposition and the particular affirmative proposition (resp. the universal affirmative proposition and the particular negative proposition) contradict each other (if one of them is true, another is false and vice versa). Further, from (1) we can infer (2) (resp. from (3) it follows that (4)). And so on.

Leibniz' intention to construct universal characteristic was extremely grandiose. In his days people trusted in unlimited powers of reason. It seemed to them, a bit later and the whole world will be completely investigated.

An automaton (abstract machine) which would simulate any physical and biological process, is said to be a *Kabbalistic-Leibnizian automaton*:

Definition 10.1 *Any simulation of chemical, physical and biological process by means of proof-theoretic cellular automata is called a Kabbalistic-Leibnizian automaton.*

The main assumptions of Kabbalistic-Leibnizian automata are as follows:

• There is only information. Any chemical, physical or biological process is an unconventional computer. Thus, any chemical, physical or biological process can be simulated by an abstract machine.

• Any chemical, physical or biological process can be considered as transition in massive-parallel proofs. Therefore any chemical, physical or biological phenomenon may be completely reduced to logical relations, e.g. to logical inference rules.

• Any theoretical terms of chemistry, physics, biology can be eliminated if we replace them by logical inference rules.

As a theoretical basis of Kabbalistic-Leibnizian automata we can consider the theory of massive-parallel proofs constructed above, where inference rules play the role of cellular-automatic local transition functions. Some exemples of Kabbalistic-Leibnizian

automata are presented by the proof-theoretic cellular automaton for Belousov-Zhabotinsky reaction and the proof-theoretic cellular automaton for dynamics of *Plasmodium* of *Physarum polycephalum*.

Both examples show that for designing Kabbalistic-Leibnizian automata it is necessary to solve Leibniz' problem to find out characters (atoms of any information), i.e. to find out the minimum set of observation terms, sufficient for an elimination of any theoretical terms. This problem is similar to the problem of looking for Higgs boson. Its solution will allow to create the universal language of natural sciences, to construct a supercomputer "Earth" to come up with the 'Answer to The Ultimate Question of Life, the Universe, and Everything.'

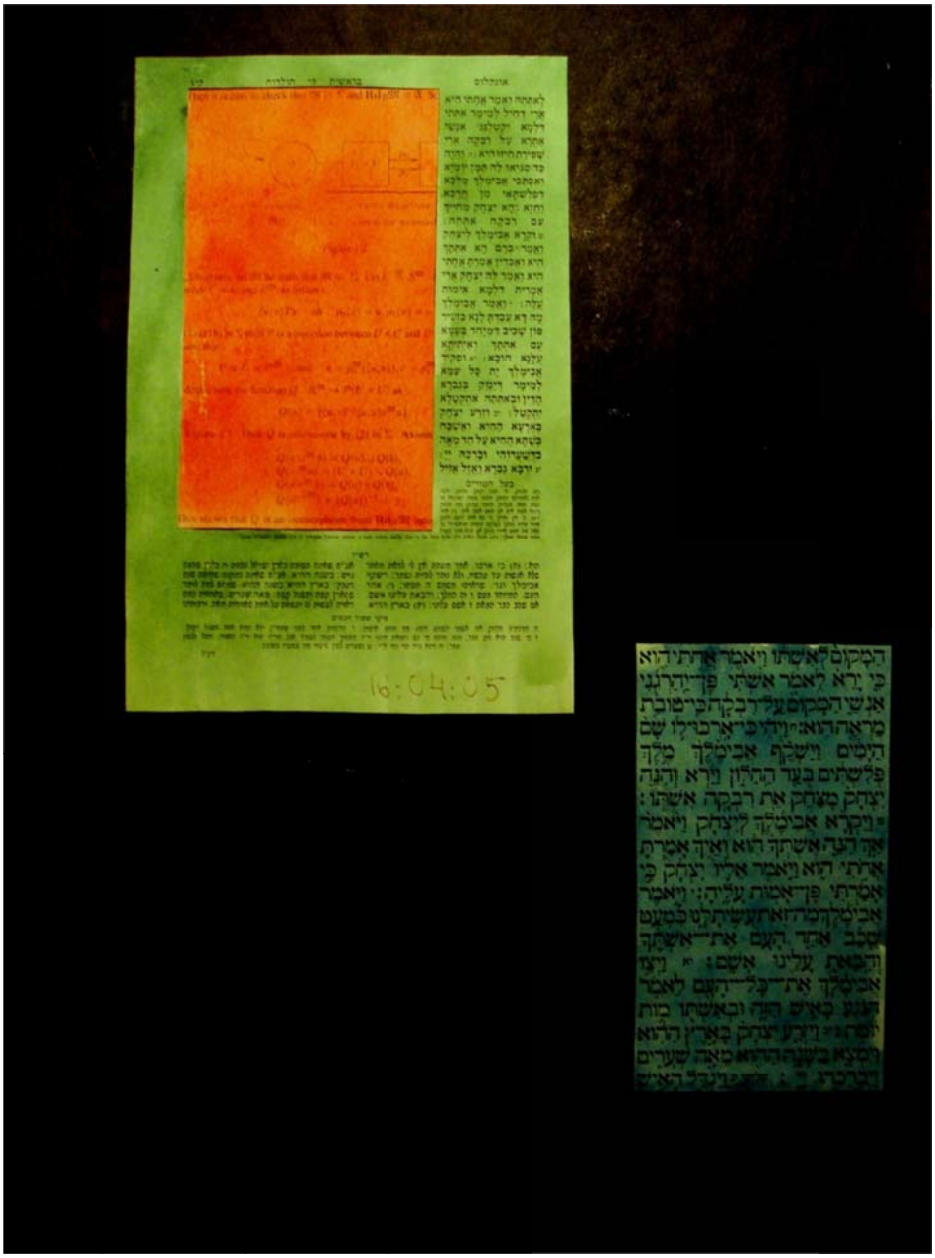
Chapter 5. Massive-parallelism in halakhic reasoning

1. Judaic semantics

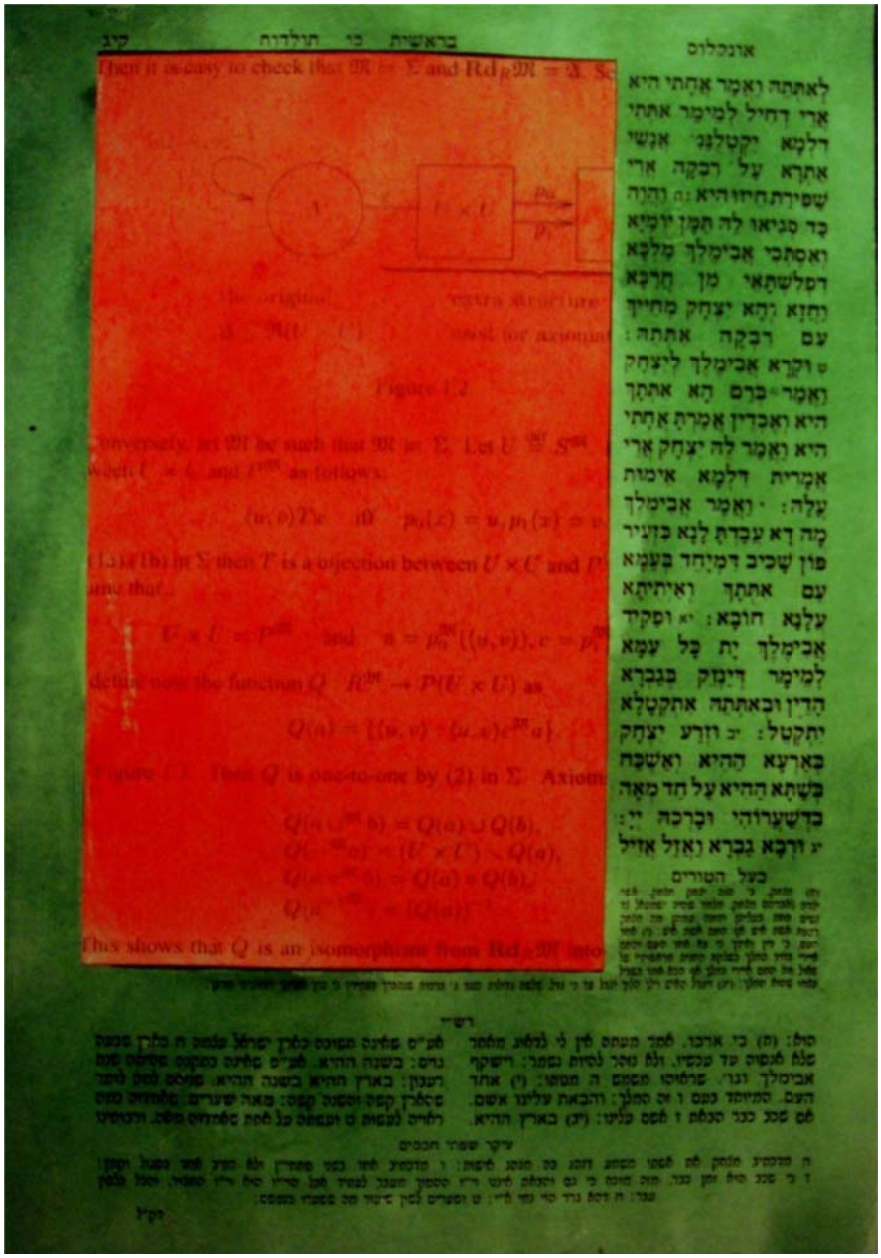
Platonic assumption of ideal objects has been adapted in Christianity. However, it has appeared unacceptable for Judaism. It may be explained by that in Platonism, the existence of many ideal entities is supposed. On the contrary, in Judaism such an attribute is assigned only to the Lord. At the same time, God is unknowable: His ideality cannot be compared with something else, therefore ideal entities cannot be used in reasoning at all. Ideality is transcendental and unutterable. Hence, Platonism supposing a plurality and discursiveness of ideal is estimated as paganism from a Judaic point of view. Only thanks to Kabbalah, some Platonic ideas have been adapted in Judaism. However, in Judaism of the Talmud we cannot find out a Platonic influence.

In order to formalize Judaic reasoning we should avoid Platonism. Just within *proof-theoretic cellular automata* it is easy to formalize the Talmudic reasoning, whose basic rules are set by Rabbi Ishmael. It speaks of two circumstances: (1) the wise men of the Talmud denied the importance of Platonism for legal (halakhic) reasoning, (2) they supposed logic in which there are no axioms (no first premises in sense of the Aristotelian syllogistics) and conclusions are drawn in a massive-parallel way.

Let us build up a proof-theoretic cellular automaton for Judaic reasoning. First of all, we might assume that $\mathsf{A}=\{p,q,r,\ldots\}$ is the set of all halakhic propositions of the Torah closed under logical connectives: conjunction ($\wedge$), disjunction ($\vee$), implication ($\Rightarrow$). We should describe the semantics for A. We know that according to the Judaic tradition, halakhic propositions cannot be evaluated as 'true' or 'false,' but they are estimated as 'particular' and 'general.' Among the thirteen rules of Rabbi Ishmael there are the following rules that concern the semantics of halakhic propositions:



Picture 11 The birth of algebra



Picture 12 The birth of algebra. Fragment

I4. *Kelal u-fraṭ ('general and particular' or 'particular after general')*

I5. *Fraṭ u-kelal ('particular and general' or 'general after particular').*

I6. *Kelal u-fraṭ u-kelal ('general and particular and general' or 'general after particular after general').*

I7. *Kelal ha-ẓarikh li-fraṭ u-fraṭ ha-ẓarikh li-kelal ('the general which requires elucidation by the particular, and the particular which requires elucidation by the general').*

Now let us try to explicate the set-theoretic and semantic presuppositions laid down in I4—I7. In considering these Talmudic inference rules there might be two basic approaches:

- the first one, in case we assume that there are denotations/individuals (this approach satisfies the human everyday evidence that there are some individuals),
- the second one, in case there are no individuals.

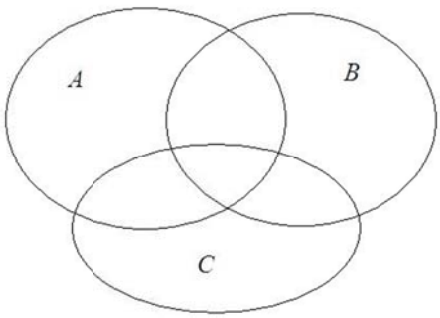


Figure 1.1 The intersection of A, B, C is a class of denotations for the Talmudic notion 'first-born,' where A is a class of denotations for 'first child,' B for 'males,' C for 'openeth the womb.' For example, $\neg C \cap \neg A \cap \neg B$ is a group of kids after Caesarian operation who are not males and not first-born.

Within the first approach, due to rules I4—I7 we can define the inclusion relation $\subset$ over the set of halakhic rules: if all individuals of A also belong to B and some individuals of B do not belong to A, then $A \subset B$ and A is called the particular and B is called the general. Then rule I7 introduces the intersection relation $\cap$. Indeed, we read: "All the firstling..." This may mean "all the firstling of any sex" (general). However, further we have two restrictions: males and openeth the womb (two particulars). It is possible to follow another way, too: all the openeth the womb (general) with two restrictions: first-born and males (two particulars). Hence, the meaning of the rule is that we combine two ways: from 'first-born' to 'males' and 'openeth the womb' and from 'openeth the womb' to 'males' and 'first-born.' In modern logic, this meaning may be explained as follows: we define an intersection between three notions: 'first child' $\cap$ 'males' $\cap$ 'openeth the womb' (see Fig.1.1) to define 'first-born.'

Another example of I7 can be provided by the passage

> And whatsoever man there be of the children of Israel, or of the strangers that sojourn among you, which hunteth and catcheth any beast or fowl that may be eaten; he shall even pour out the blood thereof, and cover it with dust (*Lev.* 17:13),

where the word *'and cover'* might be taken as a general, since there are various ways of covering a thing. Further, the word '*dust'* is a particular, and according to rule I4 the interpretation of this law would be presented by the idea that the blood must be covered with dust and with nothing else. However, the Hebrew expression *we-kisahu* ('and cover') admits of different meanings including *to cover* as well as *to hide*. Therefore *we-kisahu* does not include all cases of 'and cover with dust.' It is a general that requires the particular, i.e. an intersection relation.

One of the basic presuppositions of modern logic and classical set theory is that there are individuals that play role of denotations. However, most probably in Rabbi Ishmael's rules and

in Talmudic legal (halakhic) reasoning in general we cannot find out such a presupposition that there are individuals. The point is that the technical language used by Rabbi Ishmael does not concern everyday situations where the existence of individuals is obvious, but it concerns technical terms of law and ways of inferring Judaic laws from the Pentateuch. So, we could ask whether there are individuals in Judaic technical terms of law. Thus, another possible way to consider I4—I7 is that we assume that there are no individuals in their everyday meaning. In this case, we cannot use intersection relations and, as a result, while rules I4—I7 cover conventional inclusion relations $\subset$ again, when one group is considered as a subset of another group, rule I7 covers unconventional, self-referent (circular, cyclic) inclusion relations $\subset^{SR}$. This understanding corresponds to Rabbi Ishmael's definition of I7 as a circular inclusion.

For a formal definition of the inclusion relation on the assumption of non-existence of individuals, we should refer to non-well-founded set theory (*Aczel 1988*). Let us postulate $\dot{A}$, the set of groups of situations, instead of groups of individuals. Situations are uncertain and do not exist out of their group. Their meanings are just contextual and absolutely depends on their groups. Let us consider the ordered system $\langle \dot{A}; \subset \rangle$, where $\dot{A}$ is the set of groups (halakhic rules) and $\subset$ is the inclusion relation between halakhic rules.

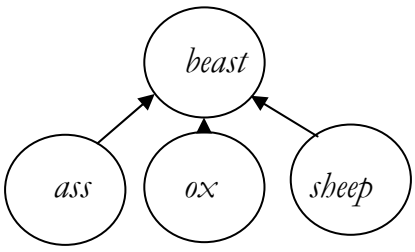


Figure 1.2 *Ex.* 22:10.

Halakhic groups of situations may be pictured using upward growing trees of graphs, see Figure 1.2, where the root is a general rule. Each *graph* consists of a set of nodes and a set of edges. Every edge is an ordered pair of nodes (in Figure 1.2, we have a pair $\langle ass, beast \rangle$ as an example, more precisely this example is as follows: $\langle$ '*deliver unto his neighbour an ass*,' '*deliver unto his neighbour any beast*' $\rangle$). If $\langle ass, beast \rangle$ is an edge then we will say that *beast* is a child of *ass*. A *path* is a finite or infinite sequence of nodes $a, b, c, d, \ldots \in$ Ȧ linked by edges $\langle a,b \rangle, \langle b,c \rangle, \langle c,d \rangle,$ A pointed graph is a graph together with a distinguished node called its point. This graph is said to be accessible if for every its node a there is a path from the node a to the point d. If this path is always unique then the pointed graph is a tree and the point d is the root of the tree (i.e., it is the most general halakhic rule).

We will use accessible pointed graphs as our pictures of halakhic rules of the Torah. In the diagrams the point will always be located at the top. The graphs will be said to be *decorated in the Torah* if there exist appropriate pictures of halakhic rules. Now we are differing two kinds of graphs: well-founded and non-well-founded ones. A graph (tree) is called well-founded if it has no infinite path.

Taking into account Rabbi Ishmael's rule I7, we can suppose the existence of pictures of halakhic rules with infinite (e.g., cyclic) paths. For accepting these paths we need to postulate the following axiom formulated first by Aczel (*1988*) and restricted to accepting on the set Ȧ:

Anti-Foundation Axiom. *Every graph has a unique decoration. In the Torah there exist non-well-founded graphs picturing genus-species relations.*

According to this axiom, a cyclic graph is decorated in the Torah by the following path: $\langle$ '*first-born*,' '*males*' $\rangle$, $\langle$ '*males*,' '*openeth the womb*' $\rangle$, $\langle$ '*openeth the womb*,' '*first-born*' $\rangle$ (*Deut.* 15:19, *Ex.* 13:2). No doubts, this graph is not well-founded because of containing a

cyclic path. Hence, Anti-Foundation Axiom allows us to use non-well-founded trees of halakhic rules in decorated graphs.

Thus, let us assume that the ordered system $\langle \dot{\mathsf{A}}; \subset \rangle$ is pictured by graphs with supposing Anti-Foundation Axiom. Then this system contains meanings of atomic propositions $V(\mathsf{A})$ from the set A of all halakhic propositions of the Torah. Now we should define logical connectives on the base of I4—I7.

Let v be an evaluation function mapping from $V(\mathsf{A})$ into $\dot{\mathsf{A}}$. The atomic proposition $A \in V(\mathsf{A})$ is called '*particular*' if $v(A) \in \dot{\mathsf{A}}$ is a node that has a child and it is called '*general*' if $v(A)$ is a node that is a child of another node. Two atomic propositions A, B are both '*particular*' (respectively, '*general*') if $v(A)$, $v(B)$ belong to different trees (they are incompatible under $\subset$) and $v(A)$, $v(B)$ have children (respectively, they are children of other nodes).

We might notice that logical connectives are understood in I4—I7 in a unique way, namely as different concatenations. First of all, the connective "… and (in Hebrew: *we* or *u*)…" (it is said to be Judaic conjunction, $\wedge$) means a (historical) sequence: $A \wedge B$ means B is later than A. Judaic conjunction is defined by the following matrix:

	A	B	$A \wedge B$
A, B are both particulars	$v(A)$	$v(B)$	$\langle v(A), v(B) \rangle$
$v(A) \subset v(B)$	$v(A)$	$v(B)$	$v(B)$
$v(B) \subset v(A)$	$v(A)$	$v(B)$	$v(B)$
A, B are both generals	$v(A)$	$v(B)$	$\langle v(A), v(B) \rangle$

According to this meaning, Judaic conjunction is not commutative (the equality $A \wedge B = B \wedge A$ is not valid), also it is not idempotent (the equality $A \wedge A = A$ does not hold, because the operation $A \wedge A$ cannot be defined), but it is associative (the equality $A \wedge (B \wedge C) = (A \wedge B) \wedge C$ holds).

• An example of $A \wedge B$ if A, B are both particulars: "wash your feet (A) and rest yourselves under the tree (B)" (*Gen.* 18:4). In Rashi's commentary on this verse, "wash your feet" is a particular referring to the following general:

> thou shalt not make unto thee any graven image, or any likeness of any thing that is in heaven above, or that is in the earth beneath, or that is in the water under the earth (*Ex.* 20:4).

According to Rashi, these people welcomed by Abraham might be Arabs who worship fine sand on their feet. Therefore they needed to wash their feet to become ritually clean. On the other hand, "rest yourselves under the tree" is a particular too. It refers to the verse: "as ye are, so shall the stranger be before the Lord" (*Numb.* 15:15).

• An example of $A \wedge B$ if $v(A) \subset v(B)$: "any bullock and sheep, wherein is blemish (A), any defect (B)" (*Deut.* 17:1).

• An example of $A \wedge B$ if $v(B) \subset v(A)$: "animals (A), of the cattle, of the flock (B)" (*Lev.* 1:2).

• An example of $A \wedge B$ if A, B are both generals: "neither shalt thou desire thy neighbour's wife (A), neither shalt thou covet thy neighbour's house (B)" (*Deut.* 5:21).

The connective "… or (in Hebrew: *'o*)…" (it is called Judaic disjunction, $\vee$) is defined as follows:

	A	B	$A \vee B$
A, B are both particulars	$v(A)$	$v(B)$	$v(A \vee B)$ such that $v(A), v(B) \subset v(A \vee B)$
$v(A) \subset v(B)$	$v(A)$	$v(B)$	$v(B)$
$v(B) \subset v(A)$	$v(A)$	$v(B)$	$v(B)$
A, B are both generals	$v(A)$	$v(B)$	$v(A \vee B)$ such that $v(A), v(B) \subset v(A \vee B)$

Judaic disjunction is non-commutative, non-idempotent, and associative (as well as Judaic conjunction).

• An example of $A \vee B$ if A, B are both particulars:

> behold, the money, which we found in our sacks' mouths, we brought again unto thee out of the land of Canaan: how then should we steal out of thy lord's house silver (A) or gold (B)? (*Gen*. 44:8).

• An example of $A \vee B$ if $v(A) \subset v(B)$:

> or if a soul swear, pronouncing with his lips to do evil, or to do good (A), whatsoever it be that a man shall pronounce with an oath (B) (*Lev*. 5:4).

• An example of $A \vee B$ if $v(B) \subset v(A)$: "or if a soul touch any unclean thing, whether it be a carcase of an unclean beast" (*Lev*. 5:2).

• An example of $A \vee B$ if A, B are both generals: "and if he smite out his manservant's tooth (A), or his maidservant's tooth (B)" (*Ex*. 21:27).

The connective "if…, then… (in Hebrew: *ki*, *'im*)" (it is said to be Judaic implication, $\Rightarrow$) is defined as follows:

	A	B	$A \Rightarrow B$
A, B are both particulars	$v(A)$	$v(B)$	$v(B)$
$v(A) \subset v(B)$	$v(A)$	$v(B)$	$v(B)$
$v(B) \subset v(A)$	$v(A)$	$v(B)$	$v(A)$
A, B are both generals	$v(A)$	$v(B)$	$v(A \wedge B) = \langle v(A), v(B) \rangle$

Judaic implication is not commutative, idempotent and associative.

• An example of $A \Rightarrow B$ if A, B are both particulars: "if Cain shall be avenged sevenfold (A), truly Lamech seventy and sevenfold (B)" (*Gen.* 4:24). We could add that if Lamech shall be avenged seventy and sevenfold, Cain sevenfold.

• An example of $A \Rightarrow B$ if $v(A) \subset v(B)$:

> and when a stranger shall sojourn with thee, and will keep the passover to the Lord (A), let all his males be circumcised, and then let him come near and keep it; and he shall be as one that is born in the land (B) (*Ex.* 12:48).

• An example of $A \Rightarrow B$ if $v(B) \subset v(A)$:

> and said, If thou wilt diligently hearken to the voice of the Lord thy God, and wilt do that which is right in his sight, and wilt give ear to his commandments, and keep all his statutes (A), I will put none of these diseases upon thee, which I have brought upon the Egyptians: for I am the Lord that healeth thee (B) (*Ex.* 15:26).

• An example of $A \Rightarrow B$ if A, B are both generals:

> now therefore, if ye will obey my voice indeed, and keep my covenant (A), then ye shall be a peculiar treasure unto me above all people: for all the earth is mine (B) (*Ex.* 19:5).

Hence, *Judaic logic* is built up in the language $\langle \mathsf{A}; \wedge, \vee, \Rightarrow \rangle$ whose semantics is associated with the ordered system $\langle \dot{\mathsf{A}}; \subset, \wedge, \vee, \Rightarrow \rangle$, where the connectives $\wedge, \vee, \Rightarrow$ are defined in meaning-matrices above.

2. *Qal wa-ḥomer*

Deductions in such logic are constructed by using different inference rules. The first one we consider is called *qal wa-ḥomer*

(I1)—the first of the thirteen rules of Rabbi Ishmael. This inference rule considerably differs from all conventional inference rules involved in a deduction procedure of classical and non-classical logics because of a massive parallelism supposed in it.

The direct meaning of the word *qal* is 'light in weight.' From a logical point of view, *qal* is regarded as *minor*, i.e. as being less important, less significant. The direct meaning of *ḥomer* is 'heaviness.' It is major, i.e. more important, more significant. Hence, *qal wa-ḥomer* is meant an inference from minor to major, and vice versa, from major to minor. For example, the Sabbath day is in some respects regarded as minor of a common holiday, festival (*yom ṭov*). Therefore, if a certain kind of work is permitted on Sabbath we can infer that such a work is the more permissible on festival; and vice versa, if a certain work is forbidden on festival it has to be forbidden on Sabbath also.

> all these things they [the Rabbis] prescribed [as culpable] on a festival, how much more [are they culpable] on Sabbath [כל אלו ביו"ט אמרו ק"ו בשבת]. The festival, differs from the Sabbath only in respect of the preparation of food alone (*Mishnah Bezah 5:2*).

By I1, we take two particulars, *A* and *B*, belonging to the same general and having different degrees of importance (e.g. both festivals and Sabbath are holiday). If they stand to each other in the relation of major and minor indeed, we can draw a conclusion. Then one of both, to say *A*, is called '*teaching*' [מלמד] expressing the subject to a certain law, which by I1 is to be transferred to *B* also. Thereby *B* is termed '*learning*' [למד]. An inference itself is called '*judgment*' [דין]. The peculiar law found in the teaching is said to be judged from [נדון]. The result of the inference presents the inferred law [הבא מין הדין].

In the Talmud it is claimed that it is sufficient to derive from an inference a result that is equivalent to the law from which it is made, i.e. the law transferred to the major must never surpass in

severity the original law in the minor, from which the inference was drown. This way of thinking is said to be the *dayo* principle. Also, there are two other restrictions in applying the inference from minor to major: first, rule I1 is not to be applied in the penal law; second, no inferences by I1 must be made from traditional laws (halakhah) to establish a new law.

Rule I1 has been one of the most actively used inference rules in Judaism for not less than two thousands. As a result, there has been a plenty of sophistical cases of applying I1, e.g.:

> אומרין צדוקיין קובלין אנו עליכם פרושים מה אם שורי וחמורי שאיני חייב בהן מצוות הריני חייב בנזקן עבדי ואמתי שאני חייב בהן מצוות אינו דין שאהא חייב בנזקן
>
> The Sadducees say: we complain against you, o ye Pharisees, in that you say, my ox or ass which has done injury is liable, yet my manservant or maidservant who has done injury is not liable. Now if in the case of my ox or my ass for which i am not responsible if they do not fulfil religious duties, yet I am responsible for their damage, in the case of my manservant or maidservant for whom i am responsible to see that they fulfil religious duties, how much more so that I should be responsible for their damage? (*Mishnah Yadaim 4:7*)

The Pharisees answered that we are responsible for animals, which have no free will, but not for slaves, who have knowledge and deliberation, so that rule I1 is unaplied here, because these two kinds of responsibility belong to different genera.

The Biblical example of I1:

> And the Lord said unto Moses, If her father had but spit in her face, should she not be ashamed seven days? Let her be shut out from the camp seven days, and after that let her be received in again (*Num.* 12:14).

It is an inference made from minor to major, namely, from a father's disfavor to the Lord's disfavor. Miriam had been punished with leprosy and she must have been shut out of the camp seven days in consequence of that disfavor.

Let us consider a Talmudic example of I1:

אמרת קל וחומר ומה שומר שכר שפטור משבורה ומתה חייב בגניבה ואבידה שואל שחייב בשבורה ומתה אינו דין שחייב בגניבה ואבידה וזה הוא קל וחומר שאין עליו תשובה מאי אין עליו תשובה וכ"ת איכא למיפרך מה לשומר שכר שכן משלם תשלומי כפל בטוען טענת לסטים מזויין אפ"ה קרנא דשואל עדיפא איבעית אימא קסבר לסטים מזויין גזלן

You can reason a *minori*: if a paid bailee, who is not responsible for injury and death, is nevertheless liable for theft and loss; then a borrower, who is liable for the former, is surely liable for the latter too! And this is an a *minori* argument which cannot be refuted. Why state that it 'cannot be refuted?'—For should you object, It may be refuted, thus: as for a paid bailee, [he is responsible for theft and loss] because he must make restitution of twice the principal [if discovered] in a [false] plea of [loss through] an armed robber, [I would reply,] yet notwithstanding, the fact that the borrower is responsible for the principal is a greater severity. Alternatively, he maintains that an armed robber is a gazlan. (*Baba Mezia* 95a).

The point is that in *Exodus* 22, there is a statement that if a man borrows an animal or a thing, and the animal dies or the object is destroyed, the borrower is responsible for the loss. But there is no information, whether the borrower is also responsible when the borrowed animal or thing is stolen. Nevertheless, we can apply inference rule I1 by using the depositary as minor and the borrower as major. Then the conclusion is that the borrower, who is responsible for damage and death, is liable also to restore the thing stolen from him:

$$\begin{array}{lcccc} & & \textit{destroyed} & & \textit{stolen} \\ \textit{depositary}: & \left(\begin{array}{cc} \textit{pay} & 0\,\% \\ \textit{pay} & 100\,\% \end{array}\right) & \leq & \left(\begin{array}{cc} \textit{pay} & 100\,\% \\ \textit{pay} & x\,\% \end{array}\right) \\ \textit{borrower}: & & \geq & \\ \hline & & x = 100\,\% & & \end{array}$$

Using our non-well-founded inclusion relation $\subset^{SR}$ we could represent this reasoning in the following way: (i) by the *dayo*

principle, '*the borrower for the stolen things*' $\subset^{SR}$ '*the borrower for the destroyed things*,' (ii) '*the borrower for the stolen things*' $\subset^{SR}$ '*the depository for the stolen things*.'

The formal meaning of I1 is as follows:

Definition 2.1 *Let* $A_1, A_2, \ldots, A_n, B_1, B_2, \ldots, B_{m+1}, C \in \mathsf{A}$ *are particulars such that* $A_1, A_2, \ldots, A_n \subset A$ *and* $B_1, B_2, \ldots, B_m \subset B$ *and* $A, B \subset C$. *Assume that* $n > m$. *Then we could define/deduce* B_{m+1} *satisfying the following conditions: (i)* $B_{m+1} \subset^{SR} B_m$ *(the dayo principle), (ii)* $B_{m+1} \subset^{SR} A_{m+1}$. *This inference rule is called qal wa-ḥomer.*

Notice that the rule defined above considerably differs from Aristotle's deductive syllogisms. Indeed, let '*man*' $\subset$ '*mortal*' and '*Socrates*' $\subset$ '*man*.' Then according to syllogism, we deduce '*Socrates*' $\subset$ '*mortal*.' However, we cannot obtain this conclusion directly from those premises by I1. The point is that while by syllogism we move from the general to the particular, by I1 we move from the particular to the particular. Thus, Aristotle's deductive syllogisms might be called vertical inferring, I1 horizontal inferring. The goal of I1 is to add branches of trees. As we see, I1 provides us with an algorithm for massively-parallel proofs. The Torah can be presented as the set $\langle \dot{\mathsf{A}}; \subset \rangle$ of different trees, some of them have joint points and some of them not. According to I1, we trace so that all branches with the joint root should have the same number of subbranches. Hence, it could be presented as a massive-parallel automaton, in particular as a cellular automaton:

Definition 2.2 *A qal wa-ḥomer cellular automaton is defined as follows: it is a 4-tuple* $\langle \mathbf{Z}^d, S, N, q \rangle$, *where*

• $\mathbf{Z}$ *is the set of all integers,* $d \in \mathbf{N}$ *is a number of dimensions and the members of* $\mathbf{Z}^d$ *are referred as cells,*

• *S is a finite set of elements called the states of an automaton, the members of $\mathbf{Z}^d$ take their values in S, the set S is collected from the set A of all halakhic propositions of the Pentateuch.*

• *$N \subset \mathbf{Z}^d \setminus \{0\}^d$ is a finite set of n elements, N is said to be a radius of q, this set consists of propositions applicable for q (their meanings are particulars, whose generals have the same root),*

• *$q : S^{n+1} \to S$ is a deduction step, where q is the inference rule called qal wa-ḥomer.*

In this way, Judaic reasoning may be formalized only by using non-well-founded mathematics and massive-parallel computing. Taking into account this fact we can claim that Judaism contains some philosophical roots of massive-parallel ideas of modern computing.

3. *Gezerah šawah* and *heqeš*

In the Talmudic terminology, *gezerah šawah* (I2) whose literal meaning is a similar section or decree denotes an *analogy* based on identical or similar words occurring in two different passages of the Scripture. In an appropriate principle opening the way of applying rule I2 one maintains as follows:

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the indefinite is to be explained by the definite.

In other words, if an expression in one passage of the Holy Writ is used ambiguously, its meaning may be explicated by another passage, where the same expression is clearly defined. For instance, in the law concerning the Paschal lamb there in nothing regarding whether it is to be brought also on the Sabbath day, but we read that the Passover should be kept 'at his appointed season' (*Num.* 9:2). The same expression occurs in *Num.* 23:2, where the law laid

Picture 13 Woman

down in relation to the daily offering says that it was to be brought 'at his appointed season,' and we know that it was to be brought also on the Sabbath day (*Num.* 28:10). According to I2, the

expression 'at his appointed season' means that the offering must take place at the appointed time under all circumstances, even on Sabbath and occuring the same expression 'at his appointed season' in regard to the Paschal lamb shows that this offering takes place even on the Sabbath day, too.

There are two limitations of applying I2: first, no one is permitted to reason by I2 himself, without an appropriate reference to using I2 by tradition; second, in order to infer by I2, the identical expression occurring in two different laws must at least in one of them be 'empty,' namely pleonastic and not engaged for another deduction.

A variety of I2 is presented by the Talmudic inference rule called *heqeš* [היקש]. This Hebrew word means a comparison based on the close connection of two subjects in one and the same passage of the Torah. According to *heqeš*, if two particulars are connected in the law by a common general, the same provisions in regard to one of them are under certain circumstances applicable also to the other. For instance, women are free from the performance of all periodical rites and religious duties kept by male Israelites. However, we deduce by *heqeš* that it is incumbent on women to obey prohibitory commandments, because there is a passage in the Torah:

> Speak unto the children of Israel, when a man or woman shall commit any sin that men commit, to do a trespass against the Lord, and that person be guilty (*Num.* 5:6),

where women and men belong to one general in regard to a trespass against the law.

Another example of *heqeš* appears to prove that a wife is taken in matrimony by means of a written contract of marriage. There is a statement:

> And when she is departed out of his house, she may go and be another man's wife (*Deut.* 24:2)

according to that the divorse is made by means of a bill of divorcement. In one verse it is talked about departing out of the house and becoming another man's wife. Hence, this becoming a wife is effected by means of a written document also.

Symbolically, I2 is expressed by means of non-well-founded inclusion relation $\subset^{SR}$ in case of *gezerah šawah* (as well as it was in formalizing *qal wa-ḥomer*) or by means of standard inclusion relation $\subset$ in case of *heqeš*:

Definition 3.1 *Let* $A, B, C, X \in \mathsf{A}$ *are particulars such that* $A, B \subset C$. *Assume* $A \subset^{SR} X$ *(or* $A \subset X$*), where* X *is termed by the expression occurring in the passage of* A *and this expression occurs in the passage of* B *as well. Then we could define/deduce* $B \subset^{SR} X$ *(or* $B \subset X$*). This inference rule is called gezerah šawah (heqeš).*

Definition 3.2 *A gezerah šawah/heqeš cellular automaton is defined as follows: it is a 4-tuple* $\langle \mathbf{Z}^d, S, N, q \rangle$, *where*

- $\mathbf{Z}$ *is the set of all integers,* $d \in \mathbf{N}$ *is a number of dimensions and the members of* $\mathbf{Z}^d$ *are referred as cells,*
- S *is a finite set of elements called the states of an automaton, the members of* $\mathbf{Z}^d$ *take their values in* S, *the set* S *is collected from the set* A *of all halakhic propositions of the Pentateuch.*
- $N \subset \mathbf{Z}^d \setminus \{0\}^d$ *is a finite set of* n *elements,* N *is said to be a radius of* q, *this set consists of propositions applicable for* q *(their meanings are particulars, whose generals have the same root),*
- $q : S^{n+1} \to S$ *is a deduction step, where* q *is the inference rule called gezerah šawah / heqeš.*

4. *Binyan 'av*

The rule *binyan 'av* (I3) is to point out in the special case some characteristic peculiarities used for the generalisation of a special law, when any other case having the same peculiarities is

considered as an analogous case, subject to the same law. A similar legislative approach is found out in the Latin tradition:

> Quando lex specialis, ratio autem generalis, generaliter lex est intelligenda.
>
> When the law is special, but its reason general, the law is to be understood generally.

Rule I3 allows us to obtain genera. For instance, in *Exodus* 21:26—27 we read:

> And if a man smite the eye of his servant, or the eye of his maid, that it perish; he shall let him go free for his eye's sake. And if he smite out his manservant's tooth, or his maidservant's tooth; he shall let him go free for his tooth's sake.

We see that two particulars are contained here: one regarding servant's eye and one regarding his tooth. The least (nearest) genus for eye and tooth is that they are essential parts of the human body and the loss of them cannot be restored. Thus, by I3 we can draw the general from these two particulars that the mutilation of any member of the slave's body causes the manumission of that servant.

Another example: "No man shall take the nether or the upper millstone to pledge: for he taketh a man's life to pledge" (*Deut.* 24:6). The law laid down by this verse is particular. It prohibits very specified things, the nether and the upper millstones, to be taken as pledges. We can generalize this law by using the nearest genus for things mentioned in the passage. As a result, the conclusion is being drown that everything which is used for preparing food is forbidden to be taken as pledge.

One more example: "Thou shalt not plow with an ox and an ass together" (*Deut.* 22:10). Rabbis made two generalizations from this verse: first, the yoking together of any two other animals of different species is forbidden; second, for any purpose it is prohibited to yoke different animals together, not only in plowing.

According to rule I3, there ever exist genera and if we find out some particulars in the Scripture without the general, then we deduce this general as the nearest genus for considered particulars:

Definition 4.1 *Let* $A, B, X, Y \in \mathsf{A}$. *Then there exists* X *such that* $A \subset X$, $B \subset X$ *and there is no* Y *such that* $Y \subset X$, $A \subset Y$, $B \subset Y$. *This inference rule is called binyan 'av.*

Definition 4.2 *A binyan 'av cellular automaton is defined as follows: it is a 4-tuple* $\langle \mathbf{Z}^d, S, N, q \rangle$, *where*

• $\mathbf{Z}$ *is the set of all integers,* $d \in \mathbf{N}$ *is a number of dimensions and the members of* $\mathbf{Z}^d$ *are referred as cells,*

• S *is a finite set of elements called the states of an automaton, the members of* $\mathbf{Z}^d$ *take their values in* S, *the set* S *is collected from the set* A *of all halakhic propositions of the Pentateuch.*

• $N \subset \mathbf{Z}^d \setminus \{0\}^d$ *is a finite set of* n *elements,* N *is said to be a radius of* q, *this set consists of propositions applicable for* q *(their meanings are particulars, whose generals have the same root),*

• $q : S^{n+1} \to S$ *is a deduction step, where* q *is the inference rule called binyan 'av.*

5. *Kelalim u-fraṭim*

Inference rules I4—I11 of chapter 1 concern the Judaic treatement of genus-species relations called *kelalim u-fraṭim* in the Talmudic frazeology. Let us consider their formal meaning more in details.

I4. *Kelal u-fraṭ ('general and particular' or 'particular after general').* For instance, this rule is applied in the following law in *Deut.* 22:11: "Thou shalt not wear a garment of divers sorts [שעטנז], as of woollen and linen together." In the verse, the general שעטנז, meaning a mixture of different sorts, is followed by the particulars 'woollen and linen together,' then we accept only particulars:

Definition 5.1 *Let* $A, B \in \mathsf{A}$. *Suppose we have a sequence* $\langle A, B \rangle$ *such that* $B \subset A$. *Then we deduce* B. *This inference rule is called kelal u-fraṭ.*

I5. *Fraṭ u-kelal ('particular and general' or 'general after particular').* For example, we read in the Holy Scripture:

> If a man deliver unto his neighbour an ass, or an ox, or a sheep, or any beast, to keep; and it die, or be hurt, or driven away, no man seeing it (*Ex.* 22:10).

As we see, the particulars 'ass,' 'ox,' 'sheep' are followed by the general 'any beast,' then the particulars being regarded merely are illustrative examples of that general:

Definition 5.2 *Let* $A, B \in \mathsf{A}$. *Suppose we have a sequence* $\langle A, B \rangle$ *such that* $A \subset B$. *Then we deduce* B. *This inference rule is called fraṭ u-kelal.*

I6. *Kelal u-fraṭ u-kelal ('general and particular and general' or 'general after particular after general').* For example:

> For all manner of trespass [general], whether it be for ox, for ass, for sheep, for raiment [particulars], or for any manner of lost thing [general], which another challengeth to be his, the cause of both parties shall come before the judges; and whom the judges shall condemn, he shall pay double unto his neighbour (*Ex.* 22:9).

This law states that in all cases when a person is guilty of having embezzled property, he will pay the double amount of the embezzlement. In the meantime, the payment of the double amount is to be made only for such embezzled property which resembles the particular ('ox, ass, sheep, raiment'), then we generalize the particulars up to the case that it is a movable property and it is an object of intrinsic value. Therefore the fine of double payment for the embezzled property is not used in cases of

real estate which is not movable or documents which have no intrinsic value.

Definition 5.3 *Let* $A, B, C \in \mathsf{A}$. *Suppose we have a sequence* $\langle A, B, C \rangle$ *such that* $B \subset A$ *and* $B \subset C$. *Then we deduce* C. *This inference rule is called kelal u-fraṭ u-kelal.*

I7. *Kelal ha-ẓarikh li-fraṭ u-fraṭ ha-ẓarikh li-kelal ('the general which requires elucidation by the particular, and the particular which requires elucidation by the general').* This rule introduces the non-well-founded inclusion relation $\subset^{SR}$, when the general requires the supplement of the particular and the particular needs the supplement of the general, therefore the general cannot be treated as a standard general and the particular cannot be regarded as a standard particular.

I8. *Davar še-hayah bi-khelal we-yaẓa' min ha-kelal lelammed lo' le-lammed 'al 'aẓmo yaẓa' 'ella' le-lammed 'al ha-kelal kullo yaẓa' ('the particular implied in the general and excepted from it for pedagogic purposes elucidates the general as well as the particular').* For instance, the witchcraft is regarded as a capital crime: "Thou shalt not suffer a witch to live" (*Ex.* 22:18). However, nothing is said about the way of the capital punishment in this law. We know that this way is shown for having a familiar spirit and being a wizard as particulars of witchcraft:

> A man also or woman that hath a familiar spirit, or that is a wizard, shall surely be put to death: they shall stone them with stones: their blood shall be upon them (*Lev.* 20:27).

This means that by I8 we infer that stoning is for any kind of witchcraft.

Definition 5.4 *Let* $A, B, C \in \mathsf{A}$. *Suppose that* $B \subset^{SR} A$ *and* $A \subset C$. *Then we deduce* $B \subset^{SR} C$. *This inference rule is called davar še-hayah bi-khelal we-yaẓa' min ha-kelal lelammed lo' le-lammed 'al 'aẓmo yaẓa' 'ella' le-lammed 'al ha-kelal kullo yaẓa'.*

I9. *Davar še-hayah bi-khelal we-yaẓa' liṭ'on ṭo'an 'aḥer še-hu' khe'inyano yaẓa' lehaqel we-lo' lehaḥmir ('a particular that was implied in a general principle and was later singled out to discuss another point similar to the general principle was singled out in order to decrease rather than to increase the rigidity of its application').* This rule may be illustrated by the passage in *Ex.* 35:3: "Ye shall kindle no fire throughout your habitations upon the Sabbath day." Here kindling fire is included in the general prohibition of doing any labor on the Sabbath day. This case is expressly mentioned. Then by I9, we conclude that it was said for the purpose of alleviating this particular from the rigor of the general in respect to prohibition of doing any labor on Sabbath. Therefore kindling fire on the Sabbath day is to transgress only a prohibitory law, but it is not subject of the severe punishment for other kinds of labor.

Definition 5.5 *Let* $A, B, C \in \mathsf{A}$. *Suppose that* $A \subset B$ *and* $C \subset^{SR} A$ *(similar context).Then we deduce that* A *is more alleviating than* B. *This inference rule is called davar še-hayah bi-khelal we-yaẓa' liṭ'on ṭo'an 'aḥer še-hu' khe'inyano yaẓa' lehaqel we-lo' lehaḥmir.*

I10. *Davar še-hayah bi-khelal we-yaẓa' liṭ'on ṭo'an 'aḥer še-lo' khe'inyano yaẓa' lehaqel we-lehaḥmir ('a particular that was part of a general principle and was later singled out to discuss another point not similar to the general principle was singled out in order to be more lenient as well as to be more stringent').* For example, let us consider the passages of *Exodus* 21:28—32:

> If an ox gore a man or a woman, that they die: then the ox shall be surely stoned, and his flesh shall not be eaten; but the owner

> of the ox shall be quit. But if the ox were wont to push with his horn in time past, and it hath been testified to his owner, and he hath not kept him in, but that he hath killed a man or a woman; the ox shall be stoned, and his owner also shall be put to death. If there be laid on him a sum of money, then he shall give for the ransom of his life whatsoever is laid upon him. Whether he have gored a son, or have gored a daughter, according to this judgment shall it be done unto him. If the ox shall push a manservant or a maidservant; he shall give unto their master thirty shekels of silver, and the ox shall be stoned.

The law concerns cases when a man or woman was killed by an ox, whose savage nature was known to the proprietor of the ox. Then the proprietor has to pay to the bereaved family an indemnification laid upon him by the court. It is the general. Further, the law adds that if a male or female slave was killed by an ox, its proprietor has to pay to the master of the slave an indemnification of thirty shekels. It is the particular. That separate mention is for the purpose of alleviating (if the actual value of the killed slave is more than thirty shekels) as well as aggravating (if it is less).

Definition 5.6 *Let A, B, $C \in \mathsf{A}$. Suppose that $A \subset B$ and $A \subset C$ (new context). Then we deduce that A is more alleviating as well as more aggravating than B. This inference rule is called davar še-hayah bi-khelal we-yaẓa' liṭ'on ṭo'an 'aḥer še-lo' khe'inyano yaẓa' lehaqel we-lehaḥmir.*

I11. *Davar še-hayah bi-khelal we-yaẓa' lidon ba-davar ḥadaš 'i 'attah yakhol lehaḥaziro li-khelalo 'ad še-yaḥazirennu ha-katuv be-piruš ('a particular that was part of a general principle and was singled out to be considered in a new matter, you cannot return it to its general principle unless Scripture does so explicitly').* For example:

> And the priest that maketh him clean shall present the man that is to be made clean, and those things, before the Lord, at the door of the tabernacle of the congregation: And the priest shall

take one he lamb, and offer him for a trespass offering, and the log of oil, and wave them for a wave offering before the Lord: And he shall slay the lamb in the place where he shall kill the sin offering and the burnt offering, in the holy place: for as the sin offering is the priest's, so is the trespass offering: it is most holy: And the priest shall take some of the blood of the trespass offering, and the priest shall put it upon the tip of the right ear of him that is to be cleansed, and upon the thumb of his right hand, and upon the great toe of his right foot: And the priest shall take some of the log of oil, and pour it into the palm of his own left hand: And the priest shall dip his right finger in the oil that is in his left hand, and shall sprinkle of the oil with his finger seven times before the Lord (*Lev*. 14:11—16).

The trespass offering [אשם] may be sacrificed for making clean from leper. In the general the blood of trespass-offerings was sprinkled only on the altar. However, the offering of the healed leper is an exception to the rule, because some of its blood was put upon the person to be made clean. This peculiar way of sprinkling is the new provision by which this sacrifice is excepted from the general law of trespass-offerings. We obtain an absolutely new case that cannot be returned to the general. This reasoning in the Talmud:

לכדתניא דבר שהיה בכלל ויצא לידון בדבר החדש אי אתה רשאי להחזירו לכללו עד שיחזירנו הכתוב לכללו בפירוש כיצד (ויקרא יד) ושחט את הכבש במקום אשר ישחט את החטאת ואת העולה במקום הקדש כי כחטאת האשם הוא וגו' שאין ת"ל כחטאת האשם מה ת"ל כחטאת האשם לפי שיצא אשם מצורע לידון בדבר החדש בבוהן יד ובוהן רגל ואזן ימנית יכול לא יהא טעון מתן דמים ואימורין לגבי מזבח תלמוד לומר כחטאת האשם הוא מה חטאת טעונה מתן דמים ואימורין לגבי מזבח אף אשם מצורע טעון מתן דמים ואימורין לגבי מזבח אם כן נכתוב בהאי ולא נכתוב בהאי הניחא אי אבל כללו גמר סבירא לן יצא לידון בדבר החדש איהו הוא דלא גמר מכללו מיניה <לצפון> שפיר אלא אי סבירא לן דלא הוא גמר מכללו ולא כללו גמר מיניה האי לגופיה איצטריך כיון דאהדריה אהדריה

Surely it is required for what was taught: If anything was included in a general proposition, and was then singled out for

a new law, you cannot restore it to [the terms of] its general proposition, unless the Writ explicitly restores it to [the terms of] its general proposition. How so? [Scripture saith,] And he shall kill the he-lamb in the place where they kill the sin-offering and the guilt-offering, in the place of the sanctuary; for as the sin-offering so is the guilt-offering: it is the priest's; it is most holy. Now, 'as the sin-offering so is the guilt-offering' need not be said. Why then is 'as the sin-offering so is the guilt-offering' said? Because a leper's guilt-offering was singled out and made subject to a new law, viz., that in respect of the thumb of the hand, the big toe of the foot, and the right ear, you might think that it does not require the presentation of [its] blood and emurim at the altar; therefore Scripture says, 'as the sin-offering so is the guilt-offering:' as the sin-offering requires the presentation of [its] blood and emurim at the altar, so does a leper's guilt-offering require the presentation of blood and emurim at the altar?—If so, let it be written in the latter [passage] and not in the former. Now, that is well if we hold that when anything is made the subject of a new law, it cannot be learnt from its general law, but its general law can be learnt from it: then it is correct. But if we hold that neither can it be learnt from the general proposition, nor can the general proposition be learnt from it, then this [law] is required for its own purpose?—Since [Scripture] restored it, it restored it (*Zevachim* 49a—49b).

Definition 5.7 *Let* $A, B, C, X \in \mathsf{A}$. *Suppose that* $A \subset B$ *and* $A \subset C$ *(new context). Then we deduce that* $X \not\subset A$ *for all* X *such that* $X \subset B$. *This inference rule is called davar še-hayah bi-khelal we-yaẓa' lidon ba-davar ḥadaš 'i 'attah yakhol lehaḥaziro li-khelalo 'ad še-yaḥazirennu ha-katuv be-piruš.*

On the base of definitions 5.1—5.7 we could define now a *kelalim u-fraṭim* cellular automaton:

Definition 5.8 *A kelalim u-fraṭim cellular automaton is defined as follows: it is a 4-tuple* $\langle \mathbf{Z}^d, S, N, q \rangle$, *where*

• $\mathbf{Z}$ *is the set of all integers,* $d \in \mathbf{N}$ *is a number of dimensions and the members of* $\mathbf{Z}^d$ *are referred as cells,*

• S *is a finite set of elements called the states of an automaton, the members of* $\mathbf{Z}^d$ *take their values in* S*, the set* S *is collected from the set* A *of all halakhic propositions of the Pentateuch.*

• $N \subset \mathbf{Z}^d \setminus \{0\}^d$ *is a finite set of* n *elements,* N *is said to be a radius of* q*, this set consists of propositions applicable for* q *(their meanings are particulars, whose generals have the same root),*

• $q : S^{n+1} \to S$ *is a deduction step, where* q *is one of the inference rules set up in definitions from 5.1 to 5.7.*

Chapter 6. Talmudic hermeneutics and Christianity

1. Aramaic primacy

For many centuries Christian theologians have been convinced that the spoken language of Jews in Palestine was Greek, therefore, on the one hand, the Jewish community of the first century needed the translation of the Bible into Greek which was called the *Septuagint* and, on the other hand, the Evangelists must have written their texts in Greek (otherwise common Jews could not have learnt anything about Christianity). In the Roman Catholic circle this idea was refused only in Renaissance. Now there are no doubts that the spoken language of Jews was Syriac (Aramaic), one spoke in Greek only in large cities of the Syrian province, for example in Antioch.

By the Jewish tradition, the work on the *Septuagint* text was completed on Tebet, 10th (in the Jewish calendar, this day occurs exactly in a week after the last day of Hanukkah). Already for more than 2500 this day of the Jewish history has been memorable—since the army of the Babylonian king Nebuchadnezzar besieged Jerusalem. The siege was completed by the capture of the city and its total destroying. The prophets of that time decided that the day of the beginning of the siege, 10 Tebet, will be henceforth a day of pray and fast for all subsequent generations of Jews. Such a coincidence of dates is not casual, namely that the *Septuagint* appearance was at date of mourning. Actually it turns out that the day of finishing the *Septuagint* is labelled by the fast for the Jews.

The *Septuagint* translation in the Jewish circle was at once condemned. This translation has never been used practically, as well as Titus Flavius Josephus' texts. All these sources were included into the Greek literature, but their Hebrew roots were completely lost. It can easily be explained. The Bible text was never perceived by the Jews as historical chronicles whereas due to the Greek translation this sacred text became one of the historical chronicles

of the Greek scientists, such as Herodotus. Hence, according to Judaism, this translation of the Holy Scripture into the Greek language is its false interpretation. The Greek language is a powerful tool of the absolutely other culture, other world, which was hostile to all Jewish at that time and aimed to adopt and assimilate all the Syriac-speaking people.

While the Greek translation of the Scriptures was rejected at once, in the Jewish world there have been a lot of translations of the Holy Scripture into the Syriac language (i.e. Aramaic) for a long time. These translations are called *Targumim*. Some of these translations have been conserving up to now and belong to the corpus which is a necessary part for Judaic studies. The translation by Onkelos, the Syrian by origin, who accepted Judaism, has been considered by rabbis of all generations as the most authoritative translation of the Hebrew Bible into the Syriac language.

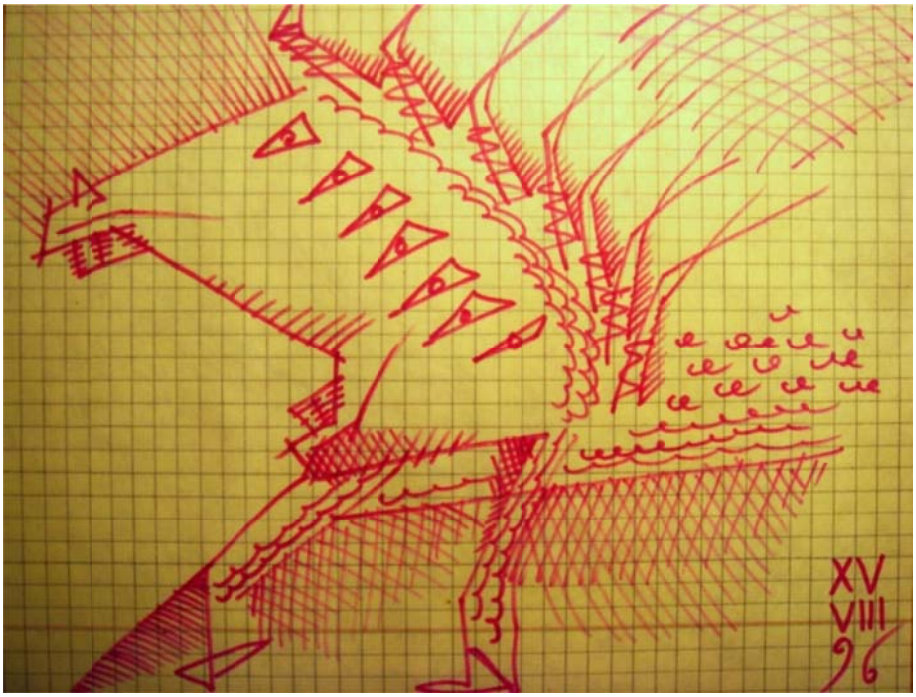
The Jewish culture was a part of the Syrian world. The spoken language of Jews was Syriac, called at that time Aramaic. Under the testimonies of some Talmudic authorities, Syriac always was the spoken language of Jews (already since Moses), Hebrew was used only as pray language. According to the Judaic tradition, all the Syriac-speaking people ('Aramaeans') descend from the forefather Abraham, which can be learnt from the etymology of his name: *Ab Aram* ('the father of Aramaeans'). As Titus Flavius Josephus states, the Syrian and Jewish communities in diaspora have traditionally supported each other. In a word, in the cultural opposition between Syrians and Greeks, Jews always supported Syrians.

Thus, now that is the actual question what the primary language of the New Testament is, either Greek or Syriac.

Some Syriac versions of the New Testament known recently are as follows:

1. The *Diatessaron* (*διά τεσσάρων*—'interval of a fourth') is the most prominent Gospel harmony created by Tatian in 160—175 in Syriac,

2.The Old Syriac versions, different fragments such as Curetonian Syriac, **Syr^{c}**, and Synaitic Syriac, **Syr^{s}**, (these texts are very close to the vocabulary of the Hebrew *Targumim*, Judaic canonic translations of the Bible into Aramaic).



Picture 14 The Lamb of Apocalypse

3.The *Peshitta*, **Syr^{p}**, contains 22 books of the New Testament, lacking the shorter General epistles (2—3 *John*, 2 *Peter*, *Jude*, as well as *John* 7:53—8:11), was finally edited in the beginning of the 5th century (this text was accepted by all Syrian communities and it is close to the lexicon of the Hebrew *Targumim* also).

4.The Harclensis version, **Syr^{h}**, when on behalf of Philoxenus of Mabbug Polycarp made a

translation of the Bible from Greek into Syriac, its revision was fulfilled a century later in the early 7th century by Thomas of Harqel, dismissed Monophysite bishop of Mabbug in Syria, and Paul of Tella lived as exiles in a Coptic monastery near Alexandria. They followed the behalf of Athanasios I, the Monophysite patriarch of Antioch. This version has an evident Greek influence.

5.The Palestinian Syriac version, **Syr^{pal},** written in the 6th to 7th centuries in the Palestinian dialect close to the dialect of Jews of Galilee (it has an evident Greek influence too).

The version **Syr^{c}** was first talked after 1842 when the British museum bought a number of the Syriac manuscripts from the the Nitrian Desert in Egypt. Among the manuscripts William Cureton examined the fragments of the earliest unknown Syriac version of the Gospels. The version **Syr^{s}** was discovered by Agnes Smith Lewis in the Monastery of St. Catherine in 1892 at Mount Sinai.

The texts **Syr^{c}** and **Syr^{s}** woke a notable interest of experts because of the significant difference from the Greek text. For instance, in **Syr^{s}** we read: "Jacob begat Joseph. Joseph the husband of Mary begat Jesus, who is called Christ" (*Matthew* 1:16), in Syriac:

ܝܥܩܘܒ ܝܘܠܕ ܠܝܘܣܦ ܝܘܣܦ ܒܥܠ ܡܪܝܡ ܝܘܠܕ ܠܝܫܘܥ ܗܘ ܕܡܬܩܪܐ ܡܫܝܚܐ

The Western scholars studied the texts of **Syr^{p}** for the first time due to its print copies prepared by the humanist Johann Albrecht Widmanstadt (1506—1559), who in 1533 became secretary of the pope, first Pope Clement VII, then Pope Paul III. The second edition of **Syr^{p}** was published in 1569 due to the diligence of Emmanuel Tremellius, Professor of Biblical Hebrew at the Heidelberg University from 1561 to 1577. Notice that this text was printed by the Hebrew letters.

The text of **Syr^{h}**, unlike **Syr^{p}**, uses many derivations from Greek (in **Syr^{p}** it is impossible to find out them practically), and even in cases when it is more convenient to use primordially Syriac words. For instance, in **Syr^{h}** Semitic names are often expressed by a transliteration from Greek: for example, the name *Ἰάκωβος* [יעקב] has the following form in accusative of Syriac: ܝܰܥܩܘܒ݂ܳܐ. In **Syr^{h}** it is possible also to find out a scrupulous imitation of the Greek word order. There are no doubts that it was the real translation from Greek.

In the vocabulary of **Syr^{pal}** there are many Greek derivations also. For example, if in **Syr^{p}** the name *Ἰησούς* is expressed as the Semitic ܝܫܘܥ (*yšw'*), then in **Syr^{pal}** is used the Greek ܝܣܘܣ (*ysws*). Or another vivid example: in *Matthew* 16:18 we see the name transliteration *Πέτρος*, instead of the Semitic *Kêphā'*, though the word-play meant here by the Evangelist is lost.

Thus, we can claim that it is obvious that **Syr^{h}** and **Syr^{pal}** were translations from Greek into Syriac. But with the same confidence it is impossible to state this about the earlier versions of the Syriac New Testament, about **Syr^{c}**, **Syr^{s}**, **Syr^{p}**. For example, in **Syr^{c}**, **Syr^{s}**, **Syr^{p}** all the quotes from the Hebrew Bible completely correspond to the statements of Judaic *Targumim*, even in cases when they essentially differ from the statements of citations in the Greek New Testament. On the other hand, in **Syr^{h}** and **Syr^{pal}** all the quotes from the Hebrew Bible are interlinear translations from Greek and consequently they strictly correspond to the statements of these citations in the Greek New Testament.

If we are assured that the Syriac Bible is a translation from Greek in fact, it would be much more logical if the earlier versions of the Syriac New Testament contained a lot of Greek derivations and in length of time the initial translations were more Syriac. However on the contrary, the earlier text (it is a question of **Syr^{c}**, **Syr^{s}**, **Syr^{p}**), the more diverse from the Greek texts it is.

There are many indirect testimonies in favour of that the Syriac New Testament in days of the early Christianity was the most authoritative source, e.g.:

1. The Syriac influence is traced in the best known apocryphal Gospel, the *Gospel of Thomas* which was written in the beginning of the second century (it contains many Aramaic derivations);

2. There are authoritative opinions that the Ethiopian text of the New Testament was made from the Syriac original;

3. The Armenian Bible translation, one of the earliest translations of the Gospels, most likely, was fulfilled from Syriac, i.e. Aramaic. So, according to chronicles, authors of translation were Saint Mesrop Mashtots (361 or 362—440) and Catholicus Sahak (390—439). Under the testimony of Moses of Chorene, the nephew and student of St. Mesrop, Sahak translated the Bible from the Syriac language. It is for the indirect argument confirming that the Syriac text of the Gospels in the end of the 4th century AD was considered at least as not less authoritative source, than the Greek text.

Also, let us notice that the Syriac Old Testament has its roots in the Hebrew *Targumim*. For the first time, this link was observed by J. Perles in his thesis *Meletemata Peshitthoniana* (Breslau, 1859). He put forward idea that the Old Testament of **Syr^{p}** is an edited version of *targum* and could have been created only in the Judaic circle. As arguments, Perles specified those places of **Syr^{p}** where the meaning can be understood only in the context of halakhic and haggadic sources. For example, in *Ezra* 9:4 the Hebrew expression 'until the evening sacrifice' is translated as 'at the ninth o'clock,' which corresponds to Judaic exegesis of the given verse.

In **Syr^{p}** there are many essential differences from the Greek New Testament, which could not appear in case of translating from Greek. Let us consider two vivid examples.

1. The first one (*Col.* 3:11) is in *King James Version* as follows:

> Where there is neither Greek nor Jew, circumcision nor uncircumcision, Barbarian, Scythian, bond nor free: but Christ is all, and in all.

In the Syriac language of **Syr**^{P} it looks so:

ܡܢ ܟܝܢܐ ܘܠܐ ܡܢ ܫܘܠܝܐ ܫܘܝܐ ܡܢ ܟܝܢܐ
ܘܠܐ ܡܢ ܥܡܐ ܒܪܒܪܝܐ
ܐܦ ܟܣܝܐ ܥܒܕܐ ܠ ܡܢ ܚܪܝܬܐ ܐܦ

In my translation:

> where there is neither Jew nor Aramaean [A. Sch.: Gentile], legal decision (ܫܘܝܐ) nor circumcision, Greek nor Barbarian, slave nor free: but the Messiah is all, and in all.

The matter is that in the Talmudic tradition the term 'Aramaeandom' [ארמיותא] expresses heathendom, therefore the word ܟܝܢܐ can be translated both as Aramaean and Gentile.

2. The second example (*James* 2:26) in *King James Version*: "For as the body without the spirit is dead, so faith without works is dead also." The same place in Aramaic:

ܗܟ ܦܓܪܐ ܕܠܝܬ ܡܢ ܪܘܚܐ ܚܝ ܗܘ ܐܦ ܗܝܡܢܘܬܐ
ܡܢ ܕܠܝܬ ܦܥܠܐ

In my translation: "For as the body without the spirit is dead, so faith without service ['bd'] is dead." The word 'service' (Hebrew: עבודה, Syriac: עבדא, ܥܒܕܐ) has a special semantics in Judaic tradition: it means fulfillment of all commandments of the Pentateuch and in days of Jesus had also the meaning 'service in the Temple.' Therefore another treatment of this verse can be as follows:

> For as the body without the spirit is dead, so faith without fulfillment of all commandments of the Pentateuch [or: without service in the Temple] is dead.

In **Syr**^{P} we almost do not find out Greek derivations. At the same time in the Greek text of the New Testament there are many

Semitic derivations from Hebrew and especially from Syriac. For example, instead of the word 'people' the New Testament uses the expression 'sons of man,' instead of simple 'angry' (for example, *Mark* 3:17) sometimes uses the strange for the Greek language expression 'sons of thunder' (*υιοι βροντης*, בני רעם), instead of 'contemporaries' (for example, *Luke* 16:8) 'children of this world' (*οι υιοι του αιωνος τουτου*, בני העולם הזה) and so on. The use of the Greek expressions *ρημα* and *λογος*, designating 'word,' in the New Testament have Hebrew semantics obviously. Since only in Syriac and Hebrew 'word' (Hebrew: דבר, Syriac: ܡܠܬܐ) has also meanings 'order,' 'event,' 'thing.'

Besides the huge number of Semitic vocabulary derivations, in the Greek text of the New Testament there are also syntactic influences from Syriac, the majority of syntactic constructions are Aramaic in fact. For example, in the Syriac language, as well as in Hebrew, the verb occurs at the first place, but it is impossible in the Greek standard sentence. For instance, we read in *Luke* 1:46—1:47 the following passages: "And Mary said, My soul doth magnify the Lord, And my spirit hath rejoiced in God my Saviour." The Greek language does not use coordinate conjunction as a rule. Nevertheless, in the Greek text of the New Testament we basically find out coordinate conjunction 'and' (Greek: *και*, Hebrew: ו), which is absolutely unconventional in the Greek language.

The simplest explanation of Semitic derivations in the Greek New Testament is that the Evangelists did not write the Gospels in Greek that was not used in Galilee at all.

The overwhelming majority of early Christians were Syriac-speaking. The point is that the first Christian communities were grounded in the Syrian province, in cities of Syria and Phoenicia, such as Antioch, Damascus, Latakia, Tripoli, Sidon, Edessa. Serious changes in the confession language have occurred only after Constantine the Great, since the first Ecumenical Council in Nicaea 325. Since then Christianity in the Roman Empire has become the state religion and the language of confession of the majority people

have been Greek or Latin henceforth. Mass persecutions of heretics started. In order to illustrate the common heat of intolerance of those years, it is possible to mention the killing of Hypatia (the head of the Platonist school at Alexandria).

> They [A.Sch.: Christians] took her to the church called Caesareum, where they completely stripped her, and then murdered her by scraping her skin off with tiles and bits of shell. After tearing her body in pieces, they took her mangled limbs to a place called Cinaron, and there burnt them (Socrates Scholasticus, *Ecclesiastical History*).

Since then there has been the real opposition of the Greek and Syrian Christianity. Especially this opposition became aggravated during the epoch of Justinian the Great. The bloody events of those years are described by John of Ephesus:

> In many parts of Antioch, in the whole Arabia and Palestine, in many southern and northern cities and in wilderness of anchorites and even up to boundaries of Persia and in other cities of the East the population is expelled from residences and disseminated; those who were captured, put in shackles, and taken into dungeons, and subjected to all punishments and crucifying… The simultaneous and prompt storm of witch-hunt severely and uncontrollably burst out over all earths, all cities and all villages. The properties of all churches and monasteries, cities and villages were plundered with greed and cruelty that belonged not only to church, monastic and monacal, but also to civil, women and children.

The history had once already known the opposition between the Western and Semitic cultures that was reflected in the motto: *Carthago delenda est* ('Carthago should be destroyed'). Carthago then was defeated by Rome. The history repeated again. The Greek Christianity has completely superseded the Syrian adepts. Many of them have been forced to run to the East, to Persia and even India. The Christian anti-Semitism that had been impossible within the limits of Syrian world, started now.

2. The hermeneutic rules *qal wa-ḥomer* and *gezerah šawah* in the New Testament

Authors of the New Testament obtained quite good Judaic education, therefore it is no wonder that they referred mainly to Judaic hermeneutic rules in their argumentation. They simply did not know other rules. In his sermon, Jesus often used the rule *qal wa-ḥomer*. Let us consider some examples.

Jesus was engaged in the healing of people and when Pharisees objected why it could have happened on Saturday, Jesus replied by applying the rule *qal wa-ḥomer*, where as minor of the rule he reminded that circumcision can occur on Saturday too:

ܐܢ ܒܫܒܬܐ ܡܬܡܠ ܒܪ ܐܢܫܐ ܕܢܬܬܠ ܢܡܘܣܐ ܠܐ ܢܬܒܛܠ ܠܝ ܡܬܪܓܙܢ ܐܢܬܘܢ ܥܠܝ ܕܐܣܝܬ ܒܪ ܐܢܫܐ ܟܠܗ ܒܫܒܬܐ

> And if a child is circumcised on the sabbath day, that the law of Moses may not be violated, do ye murmur at me, because I have made a man entirely sound on the sabbath day? (*John* 7:23).

The expression ܐܣܝܬ from the verse can be translated as 'get well' in a broad sense, not only as recovery. Madding somebody entirely sound on Sabbath is major of *qal wa-ḥomer*. All expression needs to be understood in such a manner that the meaning of expression to made a man entirely sound on the Sabbath day should be understood from the position of minor (as if to circumcise a child on Sabbath). Thus, from the viewpoint of *qal wa-ḥomer* it is possible to assume that in the verse it was a question of medical operation. The matter is that minor and major in *qal wa-ḥomer* should belong to the common genus. For circumcision on Saturday and madding a man entirely sound on the Sabbath day the medical operation made on Saturday is the nearest genus.

Another argument of *qal wa-ḥomer* supporting the thesis to make a man well on Sabbath uses as minor to lift a sheep out if it fall into a pit on Sabbath:

ܘܐܡܪ ܠܗܘܢ ܡܢ ܒܪܢܫ ܐܝܬ ܒܟܘܢ ܕܐܝܬ ܠܗ ܐܡܐ ܚܕܐ ܘܢܦܠ ܒܫܒܬܐ ܠܒܝܪܐ
ܘܠܐ ܡܚܙܩ ܒܗ ܘܢܩܝܡܝܗ ❖ ܐܦ ܟܡܐ ܛܒ ܐܢܫܐ ܡܢ ܥܠܐ ܗܐ ܠܟ ܕܫܪܝܪ
ܠܡܥܒܕ ܒܫܒܬܐ ❖

And he said to them, What man among you is there, who, if he have a sheep, and it fall into a pit on the day of the sabbath, doth not lay hold of it and lift it out? Now, how much better is a human being than a sheep! Wherefore it is lawful to do what is good, on the Sabbath (*Matthew* 12:11—12).

From this verse it is possible to draw the conclusion that the meaning of that expression to do what is good is a salvage of a person from a dangerous situation, since for lifting a sheep out (minor) and doing what is good (major) the salvage from an obvious damage on Saturday is the nearest genus.

The subject of possibility of violating Saturday appears often in Jesus' sermons. The case when Pharisees reproached Jesus that he and his pupils began to harvest to eat once on Saturday is the most interesting, because Jesus as minor referred to the King David who broke the same law because of hunger, too:

ܒܗܘ ܙܒܢܐ ܐܙܠ ܝܫܘܥ ܒܫܒܬܐ ܒܝܢ ܙܪܥܐ ܘܬܠܡܝܕܘܗܝ ܐܬܟܦܢ
ܘܫܪܘ ܠܡܩܛܦ ܫܝܢܐ ܘܡܐܟܠ ❖ ܘܦܪܝܫܐ ܟܕ ܚܙܘ ܐܡܪܘ ܠܗ ܗܐ ܬܠܡܝܕܟ
ܥܒܕܝܢ ܕܠܐ ܫܪܝ ܠܡܥܒܕ ܒܫܒܬܐ ❖ ܐܡܪ ܠܗܘܢ ܠܐ ܩܪܝܬܘܢ ܗܢܐ ܕܥܒܕ
ܕܘܝܕ ❖ ܡܬܬܝ
ܐܦ ܠܒܝܬ ܡܕܪܫܐ ܕܫܡܘܐܝܠ ܥܠ ܫܪܪ ܒܫܒܬܐ ❖ ܗܘ ܥܠ ܒܝܬ ܐܠܗܐ ܘܠܚܡ
ܕܪܘܡܝܐ ❖ ܐܟܠ ܐܠܐ ܟܗܢܐ ܒܠܚܘܕ ❖ ܠܗ ܠܐ ܫܪܝ ܠܡܐܟܠ ܐܠܐ ܠܟܗܢܐ
ܒܠܚܘܕ ❖ ܐܦ ܒܫܒܬܐ ܟܗܢܐ ܒܗܝܟܠܐ ❖ ܒܫܒܬܐ ❖ ܠܐ ܩܪܝܬܘܢ ܒܢܡܘܣܐ
ܕܒܫܒܬܐ ܟܗܢܐ ܒܗܝܟܠܐ ❖ ܚܛܝܢ ❖ ܐܡܪ ܠܗܘܢ ܐܡܪ ܐܢܐ ܠܟܘܢ ❖ ܐܠܐ
ܕܗܐ ❖ ܐܠܐ ܐܡܪ ܐܢܐ ❖ ܠܟܘܢ ❖ ܕܗܐ ❖ ܕܝܢ ❖ ܐܝܬ ❖ ܠܟܘܢ ❖ ❖ ❖
ܕܐܝܬ ❖ ❖ ❖

> on the sabbath. But he said to them, Have ye not read what David did when he was hungry, and they that were with him? How he went to the house of God, and ate the bread of the Lord's table; which it was not lawful for him to eat, nor for them that were with him, but solely for the priests? Or have ye not read in the law, how that in the temple the priests profane the sabbath, and are without fault? But I say to you, that a greater than the temple is here. And if ye had known what that is, I desire mercy, and not sacrifice; ye would not have criminated them who are without fault. For the Son of man, is lord of the sabbath (*Matthew* 12:1—8).

These verses are really remarkable, since the case with the King David is considered as minor and at the same time it is meant that the case with Jesus and his pupils is major. Actually, by drawing such *qal wa-ḥomer* Jesus openly declares that he is the Messiah, namely that he possesses the status which is not smaller, than that of the King David.

The fact that Jesus used the rule *qal wa-ḥomer* so often is not surprising. It was one of the most widespread logical rules in the Judaic circle in days of Jesus. It is affirmed as the most significant hermeneutic and logical rule of Judaism by the contemporary of Jesus, Hillel the Elder (the late first century B.C.E. to the early first century C.E.). He grounded an influential school of Judaic exegesis. In those days there were only two such influential schools: the first one belonging to Hillel and the second one belonging to Shammai. It was considered that Hillel was a master at teaching the 'spirit of the Torah' and Shammai the 'letter of the Torah.'

Between the sermon of Jesus and Hillel there is a lot of common. For example, Hillel was known for his love for people so that there was a saying in his day, "Let a man be always humble and patient like Hillel, and not passionate like Shammai" (*Shab.* 30b). The best known statement of Hillel the Elder about love of neighbor is as follows:

> דעלך סני לחברך לא תעביד זו היא כל התורה כולה ואידך פירושה הוא זיל גמור

What is hateful to you, do not do to your neighbor: that is the whole Torah; all the rest of it is commentary; go and learn (*Shabbat* 31a).

It is well known that love of neighbor as the Torah foundation was preached as well by Jesus. Everybody should love other people and love God, since God loves human beings and all alive. In order to prove that God loves people, Jesus applies the same rule *qal wa-ḥomer*:

ܚܙܘ ܠܛܝܪܐ ܕܫܡܝܐ ܕܠܐ ܙܪܥܝܢ ܘܠܐ ܚܨܕܝܢ ܘܠܐ ܟܢܫܝܢ ܠܒܝܬ ܡܟܣܐ ܐܠܐ ܐܒܘܟܘܢ ܕܒܫܡܝܐ ܡܬܝܢ ܠܗܘܢ ܠܐ ܐܢܬܘܢ ܝܬܝܪ ܡܢܗܘܢ ܐܢܬܘܢ

Look at the birds of heaven; which sow not, and reap not, and gather not into storehouses, but your Father who is in heaven feedeth them. Are not ye more important than they? (*Matthew* 6:26).

ܣܒܪܘ ܠܥܘܪܒܐ ܕܠܐ ܙܪܥܝܢ ܘܠܐ ܚܨܕܝܢ ܕܠܝܬ ܠܗܘܢ ܡܟܣܐ ܘܐܘܨܪܐ ܠܐ ܐܝܬ ܠܗܘܢ ܐܠܐ ܐܠܗܐ ܡܙܝܢ ܠܗܘܢ ܠܥܘܪܒܐ ܟܡܐ ܕܝܩܪ ܐܢܬܘܢ ܡܢ ܥܘܦܐ

Consider the ravens, which sow not, nor reap, nor have cellars and storehouses; yet God provideth them food. How much more important now are ye, than the birds! (*Luke* 12:24).

In both verses minor is that God provideth the birds of heaven [ܥܘܦܐ ܕܫܡܝܐ] food. Major is that the Father feedeth men. By the rule *qal wa-ḥomer* this means that the Lord cares about people not less, than about birds. In other words, God loves us. However, the Lord can punish people in the same measure as He punishes plants, which is proved again by means of *qal wa-ḥomer*:

ܘܐܢ ܥܣܒܐ ܕܒܪܐ ܗܟܢܐ ܐܠܗܐ ܡܫܪܐ ܕܗܘܐ ܠܝܘܡܐ ܘܠܡܚܪ ܢܬܪܡܐ ܠܬܢܘܪܐ ܠܐ ܝܬ ܠܗ ܠܡܠܒܫܟܘܢ ܐܦ ܝܬ ܠܗ ܠܟܘܢ ܙܥܘܪܝ ܗܝܡܢܘܬܐ

And if God so clothe the grass of the field, which exists today, and tomorrow falls into the oven, will he not much more [clothe] you, ye small in faith? (*Matthew* 6:30).

Picture 15 Woman

ܗܠܐ ܗܝܕܐ ܕܒ ܝܬܐ ܟܠܝܠ ,ܡܫܘܢ ܟܕܐ ܗܝܕܐ ܒ ܐ
ܟܐܦܐ ,ܝܬ ܐ ܠܐ ܝܗ ܟܐ ܒܛ ܟܐ

And if God so clothe the herb, which today exists in the field, and tomorrow falls into the oven, how much more you, ye little in faith? (*Luke* 12:28).

Jesus considered himself as the head of an autonomic community and by means of the rule *qal wa-ḥomer* he aims to prove the importance of community (church) which is not less significant, than Jesus himself.

ܡܝܢ ܐ ܡܝ ܗ ܐ ܕܬ ܐ ܡ ܗ ܕ ܐ ܡ ܗ ܕ ܐ ܕ ܐ ܒ ܐ ܕ ܐ ܒ ܐ ܕ ܐ ܒ ܐ
ܡ ܗ ܐ ܐ ܟ ܐ ܒ ܐ ܕ ܐ ܒ ܐ ܕ ܐ ܒ ܐ ܕ ܐ ܒ ܐ ܕ ܐ ܒ ܐ ܕ ܐ ܒ ܐ

It is sufficient for the disciple, that he be as his rabbi; and the servant, as his lord. If they call the lord of the house Beelzebub, how much more the children of his family? (*Matthew* 10:25).

In this passage by means of *qal wa-ḥomer* it is proved that the reviling of rabbi (minor) is a reviling of his pupils (major), also. Another example in which there are given reasons, why hatred to Jesus (minor) is no more, than hatred to his community as a whole (major):

ܐܢ ܥܠܡܐ ܣܢܐ ܠܟܘܢ ܝܕܥܝܢ ܕܐܢܐ ܩܕܡ ܠܟܘܢ ܣܢܐ ܗܘܐ ܠܝ ❖
ܐܢ ܗܘܝܬܘܢ ܡܢ ܥܠܡܐ ܥܠܡܐ ܪܚܡ ܗܘܐ ܠܡܕܝܠܗ ܡܢ ܥܠܡܐ ܥܠܡܐ ܡܢ ܐܝܬ ܠܟܘܢ
ܥܠ ܗܕܐ ܥܠܡܐ ܣܢܐ ܠܟܘܢ ܡܛܠ ܕܐܢܐ ܓܒܝܬ ܠܟܘܢ ܡܢ ܥܠܡܐ ܥܠ ܗܕܐ
❖ ܣܢܐ
ܕܠܝܬ ܥܒܕ ܡܪܗ ܡܢ ܕܫܠܝܚ ܗܘ ܠܐ ܪܒ ܗܘ ܥܠ ܡܪܗ ܕܐܢ ܪܕܦܘ ܠܝ ܐܦ ܠܟܘܢ
ܢܪܕܦܘܢ ܐܢ ܛܘܪܬܝ ܢܛܪܘ ܐܦ ܠܟܘܢ ܢܛܪܘܢ ܐܢ ܡܠܬܝ ܩܝܡܘ ܐܦ ܐܢܘܢ

> And if the world hate you, know ye, that it hated me before you. And if ye were of the world, the world would love what is of it. But ye are not of the world, for I have chosen you out of the world; for this cause, the world hateth you. Remember the word that I spoke to you, That there is no servant, who is greater than his lord. If they have persecuted me, they will also persecute you; and if they have observed my teaching, they will also observe yours (*John* 15:18—20).

In order to show the status of the Messiah, Jesus builds his sermon so that the hearer could apply the rule *gezerah šawah* by comparing sayings of Jesus with a number of sayings of prophets.

"I am the bread of life [ܐܢܐ ܗܘ ܠܚܡܐ ܕܚܝܐ]" (*John* 6:48), Jesus speaks about himself, referring to Isaiah (28:8) who says: "For all the tables are full with vomit and filthiness, so there is no place clean". Prophet's phrase means that all the tables, which are places for eating, contains fouled food, not wholesome 'bread' (the truth). Thus, it is supposed that no doubts Jesus is the Messiah for whom all waited so long.

Another self-estimation: "I am a good shepherd. A good shepherd exposeth his life for the sheep [ܐܢܐ ܗܘ ܪܥܝܐ ܛܒܐ ܪܥܝܐ ܛܒܐ ܗܘ ܕܢܦܫܗ ܡܫܠܡ ܥܠ ܐܪܥܝܐ]" (*John* 10:11) suggests the comparison with Isaiah (40:11) by the rule *gezerah šawah*:

> He shall feed his flock like a shepherd: he shall gather the lambs with his arm, and carry them in his bosom, and shall gently lead those that are with young.

The next self-estimation: "I am the true vine; and my Father is the cultivator [ܐܠܗܐ ܐܡ ܕܟܪܡܐ ܫܪܝܪܐ ܐܒܝ ܗܘ ܕܐܬ]" (*John* 15:1) refers to Isaiah (5:1—2):

> Now will I sing to my wellbeloved a song of my beloved touching his vineyard. My wellbeloved hath a vineyard in a very fruitful hill: And he fenced it, and gathered out the stones thereof, and planted it with the choicest vine, and built a tower in the midst of it, and also made a winepress therein: and he looked that it should bring forth grapes, and it brought forth wild grapes.

As we see, Jesus' argumentation assumes quite good knowledge of Judaic hermeneutics. His argumentation is difficult enough, it is addressed to people with Judaic education, for example, it suggests hearers to apply *gezerah šawah*.

Judaic rules of exegesis were widely used also by apostles. More often than other apostles Paul has applied them, which is not surprising in the context of that before he became Christian he had been the Pharisee, i.e. he had rather good preparation for the treatment of the Holy Writ.

In the beginning let us consider Paul's letter to the Romans. Here many theses are supported by *qal wa-ḥomer*. For instance, Paul asserts that God loves us, believers (major), since Messiah died for us when we were sinners (minor). Furthermore, God will be reconciled with us by the life of his Son (major), because He was reconciled with us by Messiah's death (minor):

ܗܳܟܰܢܳܐ ܦܰܣܩ ܠܰܢ ܐܰܒܽܘܗܝ ܕܰܐܠܳܗܳܐ ܕܰܚܽܘܒܬܶܗ ܥܰܠ ܠܰܢ ܒܰܕ ܟܰܕ ܚܰܛܝܳܐ ܗܘܳܐ ܡܶܢ ܩܕܳܡ ܗܳܕܶܐ ❖
ܕܰܡܶܬ ܒܰܥܕ ܠܰܢ ❖ ܘܰܐܦ ܟܰܕ ܐܶܬܪܰܥܝܢ ܗܘܝܢ ܒܰܕܡܘܗܝ ܕܒܪܗ ܐܬܪܥܝܢ ܗܘܝܢ ܠܘܬ ܐܠܗܐ
ܘܟܰܕ ܐܶܬܪܰܥܝܢ ܗܘܝܢ ܠܘܬ ܐܠܗܐ ܒܰܕܡܘܗܝ ܕܒܪܗ ܐܬܪܥܝܢ ܗܘܝܢ ,ܡܰܫܟܰܚ ܠܰܢ ܒܰܚܝܶܘܗܝ ܕܒܪܗ ܕܐܠܗܐ ܚܰܝ ܗܘ ܒܰܕ ܟܰܕ ܕܰܚܽܘܒܬܶܗ ܥܰܠ ܠܰܢ ܐܰܒܽܘܗܝ ܕܰܐܠܳܗܳܐ ܦܰܣܩ ܗܳܟܰܢܳܐ

> God hath here manifested his love towards us. Because, if when we were sinners, Messiah died for us; how much more, shall we

now be justified by his blood and be rescued from wrath by him? For if when we were enemies, God was reconciled with us by the death of his Son; how much more shall we, in his reconciliation, lived by his life? (*Romans* 5:8—10).

According to the Apostle, the significance of Jesus for mankind is not less, than Adam's significance, the forefather for all of us. The given thesis is proved by means of *qal wa-ḥomer*: on the one hand, because of Adam's fault, many died (minor), on the other hand, Lord's free gift, on account of Jesus Messiah, abounds unto many (major):

ܠܐ ܐܝܟ ܕܥܠ ܚܛܝܬܗ ܕܚܕ ܥܠ ܣܓܝܐܝ ܡܝܬܘ ܐܢܫܐ ܠܐ ܐܦ
ܥܠ ܚܢܢܐ ܕܚܝܠܗ ܕܐܠܗܐ ܘܡܬܢܬܗ ܕܒܚܕ ܐܢܫܐ ܝܫܘܥ ܡܫܝܚܐ
ܣܓܝܐܝ ܢܬܝܬܪ ܠܣܓܝܐ ܒܚܝܐ ܠܟܠ ܐܢܫܐ ܡܬܚܝܢ

But not, as the fault, so also the free gift. For if, on account of the fault of one, many died; how much more, will the grace of God and his free gift, on account of one man, Jesus Messiah, abound unto many? (*Romans* 5:15).

The enough difficult argumentation is proposed by Paul the Apostle to analyze the thesis what the outcome is, when Israel rejects the Gospel. It would seem that Israel did not accept the good news. However, God has always provided a remnant in Israel. There are always examples of Jewish believers in God's redemptive plan:

But I say: Hath God cast off his people? Far be it. For I also am of Israel, of the seed of Abraham, of the tribe of Benjamin. God hath not cast off those his people whom he before knew. Do ye not know, what, in the scripture of God, he said to Elijah? When he had complained to God against Israel, and said: My Lord, they have slain thy prophets, and have thrown down thy altars; and I am left alone; and they seek my life. And it was said to him, by revelation: Behold, I have reserved for myself seven thousand men, who have not bowed their knees, and have not worshipped Baal. So also at the present time, a remnant is preserved, by the election of grace (*Romans* 11:1—5).

In these passages Paul refers to the prophet Elijah who was assured that he became a unique true believer though it was not so:

> Yet I have left me seven thousand in Israel, all the knees which have not bowed unto Baal, and every mouth which hath not kissed him (1 *Kings* 19:18).

For all that the Jewish people, according to the Apostle, have saved the status of the sacred people. It is proved by a number of reasoning on *qal wa-ḥomer*. Jews did not accept the good news yet. It is their stumbling. However, due that the riches of God's salvation become available to the whole world, and Gentiles are saved (minor). Then "how much more their completeness," i.e. their acceptance of the good news, the ultimate fulfillment of God's promises to the Jewish people will be riches to the world (major):

ܥܘܬܪ ܕܡܣܟܢܘܬܗܘܢ ܕܥܠܡܐ ܥܘܬܪ ܕܚܛܝܬܗܘܢ ܕܐܡܡܐ ܘܐܢ
ܕܡܫܠܡܢܘܬܗܘܢ ܟܡܐ ܥܕ ܕܢܗܘܐ

> And if their stumbling was riches to the world, and their condemnation riches to the Gentiles; how much more their completeness? (*Romans* 11:12).

If the rejection of the Gospel was a reconciliation of the world (minor), then how much more good news will be for Gentiles when finally Israel is fulfilled in its accepting God's premised Messiah (major) so that there will be life from the dead (resurrection):

ܟܡܐ ܕܢܗܘܐ ܕܩܒܠܗ ܕܥܠܡܐ ܕܫܪܝܐ ܕܡܫܬܘܬܦܘܬܗ ܐܢ ܐܢ
ܕܚܝܐ ܒܝ ܕܡܝ ܐܝܟ ܕܡܐ ܕܡܬܐ

> For if the rejection of them, was a reconciliation of the world; what will their conversion be, but life from the dead? (*Romans* 11:15).

In the letter to the Jews Paul the Apostle gives different reasons for a doctrinal foundation of Jesus' supremacy in every area pertaining to God and His divine program: revelation, priesthood, law, hope, covenant, promise, sacrifice, etc. Paul's central point of claim is as follows:

> Now the sum of the whole is this, we have a High Priest, who is seated on the right hand of the throne of the Majesty in heaven: And he is the minister of the sanctuary, and of the true tabernacle, which God hath pitched, and not man (*Hebrews* 8:1-2).

For the author of Hebrews, Jesus is prior to the prophetic revelation (1:1—3), angels (1:4—2:18), Moses (3:1—6), the Levitical priesthood (4:14—5:10; 7:11—28; 9:1—10), Abraham (7:1—10), the Torah (7:12, 19, 28; 10:1), the Mosaic covenant (8:1—13), Levitical sacrifices (9:11—10:18), etc. These affirmations are needed by the Apostle in order to use *qal wa-ḥomer* in defining the nature of Messiah. This nature is unknown. According to Christians, the Messiah visited us for the first time and just it is necessary to understand the meaning of his coming.

The Apostle is looking for genus-species relations of basic notions of the Torah, assuming that "the inferior is blest by his superior [dmytr] [ܡܢ ܕܒܪܟ ܥܠ ܕܡܝܬܪ ܠܗ]" (7:7). He finds out subalterations between some notions and others by using comparative degrees, for example:

"lower [qlyl mn]" (2:7, 9);

"more [ṭv mn] glory [ܫܒܚܐ ܕܥܠ ܫܡܐ ܡܢ ܫܘܒܚܐ]" (3:3);

"more [ṭv mn] honor" (3:3);

"sharper [ṭv mn] than" (4:12);

"one greater [drv mnh] [ܡܢ ܕܪܒ]" (6:13, 16);

"exalted higher than heaven [ܪܡ ܥܠ ܠܥܠ]" (7:26).

The overall picture of genus-species relations arises, according to the Apostle, thanks to assuming the higher notion, the higher genus, i.e. the notion of the Messiah. For instance, Jesus has become

> altogether superior [yrb mn] to the angels [ܥܠ ܕܪ ܡܢ ܠܐ ܫܡܐ ܕܫܠܡ], as he hath also inherited a name which excelleth theirs (1:4).

The Messiah is superior in all phenomena. He is:

A greater glory: "For much greater is the glory [ṭv myn] of this man, than that of Moses" (3:1—6).

A better hope: "A better [dmytr] hope [ܣܒܪܐ ܕܡܛܪ]" (7:19).

A better covenant: "A better [mytr'] covenant [ܩܝܡܐ ܕܡܛܪ]" (7:22; 8:6).

A greater and more perfect tabernacle: "But the Messiah who came, was a High Priest of the good things which he wrought: and he entered into the great and perfect tabernacle [ܒܝܬܐ ܕܝ ܒܬܪ ܘܡܫܠܡ], which was not made with hands and was not of these created things" (9:11).

A better possession: "and ye were grieved for those who were imprisoned; and ye cheerfully endured the plundering of your goods, because ye knew that ye had a possession in heaven [ܒܬܪ ܕܐܝܬ ܠܟܘܢ], superior [dmytr] and not transitory" (10:34).

A better country: "A better [dldṭv'] country [mdin']" (11:16).

A greater treasure: "and he esteemed the reproach of the Messiah a greater [ṭv mn] treasure than the hoarded riches of Egypt [ܡܢ ܟܠ ܐܘܨܪܐ ܕܡܨܪܝܢ ܕܚܫܒ ܠܗ ܡܫܝܚܐ]; for he looked upon the recompense of reward" (11:26).

A better resurrection: "A better [mytrt'] resurrection [ܩܝܡܬܐ ܕܡܛܪ]" (11:35).

A better blood: "Better [ṭv mn] blood [ܡܢ ܕܡܐ ܕܡܫܬܐ]" (12:24).

There are connected with him:

Better things: "we are persuaded better things ['ylyn dšfyrn] [ܕܫܦܝܪ ܠܢ ܥܒܕܝܢ ܠܚܝܐ], and things pertaining to life" (6:9).

Better promises: "But now, Jesus the Messiah hath received a ministry which is better than that: as also the covenant, of which he is made the Mediator, is better, and is given with better promises [dmytryn] than the former" (8:6).

Better sacrifices: "Better [dmytryn mn] sacrifices [ܕܒܚ̈ܐ ܝܬ̈ܝܪ̈ܐ]" (9:23). "By faith, Abel offered to God a better [dmitr' ṭv mn] sacrifice than that of Cain; and on account of it, he is testified of that he was righteous, and God bore testimony to his offering; and in consequence thereof, though dead he yet speaketh" (11:4).

The assumption of that Jesus is the higher notion, allows us to define it as major, using already existing notions of the Torah as minor. The main grammar construction of such definitions by *qal wa-ḥomer* is presented by '*how much more* [ܟܡ ܕܠܐ ܐܦ ܟܕ]:'

ܐܢ ܗܘ ܕܐܡܝܪ ܒܝܕ ܡܠܐܟ̈ܐ ܩܝܡ ܗܘܐ ܘܟܠ ܡܕ ܕܫܡܥ ܠܗ ܩܒܠ ܓܙܪ̈ܐ ܫܪܝܪ̈ܐ
ܘܟܝܢ ܟܡ ܢܗܘܐ ܠܢ ܕܐܦ ܢܫܠܡ ܐܢ ܢܫܠܚ ܠܚܝܐ ܕܢܦܫܢ ܗܢ ܕܐܡܪ̈ܝܢ ❖
ܗܢ ܕܒܪ̈ܝܫܐ ܫܪܝ ܠܢ ܘܕܒܪ̈ܝ ܡܢ ܦܘܡܗ ܕܡܪܢ ܫܡܥ̈ܘ ܠܢ ܗܢ ܕܫܡܥ̈ܘ ܡܢܝܗ
ܕܐܝܟ ܕܐܢ ܢܗܘ ܠܢ ܐܦ ܗܠܝܢ ܕܫܪ̈ܝ ܠܢ ܒܚ̈ܝܐ ܗܢ ܕܐܡܪ̈ܝܢ ܡܢܝܗ

For if the word uttered by the medium of angels was confirmed, and every one who heard it, and transgressed it, received a just retribution [*A. Sch.*: *minor* of *qal wa-ḥomer*]; how shall we escape, if we despise the things which are our life, things which began to be spoken by our Lord, and were confirmed to us by them who heard from him [*A. Sch.*: *major* of *qal wa-ḥomer*] (*Hebrews* 2:2—3).

ܐܢ ܗܘ ܕܕܡ ܥܬ̈ܘܕ̈ܐ ܘܬܠ̈ܝܢ ܕܪ̈ܡܝ ܥܓ̈ܠܬܐ ܕܬܟ̈ܣܝܬ ܥܠ ܐܢ̈ܫ̈ܐ ܕܬܡ̈ܐ
ܘܩܕ̈ܫ ܠܛܗܪ̈ܘܬ ܒܣܪ̈ܗܘܢ ❖
ܟܡ ܠܐ ܢܗ̈ܘܐ ܕܕܡ ܡܫܝܚܐ ܕܒܪܘܚ ܩܕ̈ܡܝܐ ܢܦܫܗ ܩܪ̈ܒ ܠܐ ܡܘܡ ܠܐܠܗܐ
ܢܕܟ̈ܐ ܠܡܨܒܬܢ̈ ܡܢ ܥܒ̈ܕܐ ܕܡܝܬ̈ܐ ܕܢܦܠܚ ܠܐܠܗܐ ܚܝ̈ܐ

For if the blood of goats and calves, with the ashes of a heifer, was sprinkled upon them that were defiled, and sanctified them as to the purification of their flesh [*A. Sch.*: *minor* of *qal wa-ḥomer*]; then how much more will the blood of the Messiah, who by the eternal Spirit offered himself without blemish to God, purge our conscience from dead works, so that we may serve the living God [*A. Sch.*: *major* of *qal wa-ḥomer*]? (*Hebrews* 9:13—14).

ܟܠ ܡܢ ܕܒܛܠ ܢܡܘܣܐ ܒܠܚܘܕ ܥܠ ܦܘܡ ܬܪܝܢ ܐܘ ܬܠܬܐ ܣܗܕܐ ܡܝܬ ܗܘ
ܟ ܫܪܝ ܠܗ ܕܢܬܚܫܒ ܠܗ ܠܡܐ ܕܪܡܐ ܒܪܗ ܕܐܠܗܐ ܘܕܚܫܒ ܕܕܡ ܩܝܡܗ ܕܐܬܩܕܫ
ܒܗ ܐܦ ܕܕܡ ܟܠ ܒܢܝ ܐܢܫܐ ܘܕܥܒܕ ܒܪܘܚ ܛܝܒܘܬܐ ܒܣܒܪ ܦܢܝ ܠܗ ܘܠܡܐ
ܢܬܚܝܒ ܐܢܫ ܕܐܦ ܐܢܚܢ ܐܢ ܟܕ ܢܣܒܪ ܦܢܝ ܠܗ ܘܢܚܝܐ ܐܒܗܬܢ ܕܒܣܪܐ ܐܢ
ܟܕ ܢܗܘܐ ܠܢ ܠܐܒܗܬ ܪܘܚܢ ܐܦ ܠܚܝܐ ܢܗܘܐ ܠܢ ܠܐܒܗܬ ܪܘܚܢ ܐܦ ܠܚܝܐ

For if he, who transgressed the law of Moses, died without mercies, at the mouth of two or three witnesses [*A. Sch.*: *minor* of *qal wa-ḥomer*]; how much more, think ye, will he receive capital punishment, who hath trodden upon the Son of God, and hath accounted the blood of his covenant, by which he is sanctified, as the blood of all men, and hath treated the Spirit of grace with contumely [*A. Sch.*: *major* of *qal wa-ḥomer*]? (*Hebrews* 10:28—29).

ܐܢ ܐܒܗܬܢ ܕܒܣܪܐ ܝܣܪܘܢ ܐܦ ܚܢܢ ܐܢ ܟܕ ܢܗܘܐ ܠܢ ܠܐܒܗܬ ܪܘܚܢ ܐܦ ܠܚܝܐ
ܢܗܘܐ ܠܢ ܠܐܒܗܬ ܪܘܚܢ ܐܦ ܠܚܝܐ ܢܗܘܐ ܠܢ ܠܐܒܗܬ ܪܘܚܢ ܐܦ ܠܚܝܐ

And if our fathers of the flesh chastened us, and we revered them [*A. Sch.*: *minor* of *qal wa-ḥomer*], how much more ought we to be submissive to our spiritual fathers, and live [*A. Sch.*: *major* of *qal wa-ḥomer*]? (*Hebrews* 12:9).

ܚܙܘ ܕܠܐ ܬܬܚܫܟܘܢ ܠܡܐ ܕ ܐܢ ܠܐ ܐܫܬܟܚܘ ܐܢܘܢ ܕܐܡܪܝܢ ܠܟܘܢ
ܐܢ ܠܐ ܐܫܬܟܚܘ ܐܢܘܢ ܕܐܡܪܝܢ ܠܗܘܢ ܥܠ ܐܪܥܐ ܐܦ ܠܐ ܢܫܬܟܚ ܠܢ ܐܢ
ܐܢ ܐܢܚܢ ܠܐ ܢܫܬܟܚ ܠܡܢ ܕܡܡܠܠ ܥܡܢ ܡܢ ܫܡܝܐ ܕܩܠܗ ܙܥ ܐܪܥܐ
ܗܫܐ ܕܐܡܪ ܐܡܪ ܕܥܕ ܚܕ ܙܒܢ ܐܢܐ ܙܥ ܠܐ ܐܪܥܐ ܒܠܚܘܕ ܐܠܐ ܐܦ
ܫܡܝܐ ܐܦ ܗܠܝܢ ܐܢܐ ܐܫܬܟܚ ܠܟ ܕܐܬܐ

Beware, therefore, lest ye refuse [to hear] him who speaketh with you. For if they escaped not, who refused [to hear] him who spake with them on the earth [*A. Sch.*: *minor* of *qal wa-ḥomer*], how much more shall we not, if we refuse [to hear] him who speaketh with us from heaven [*A. Sch.*: *major* of *qal wa-ḥomer*]? Whose voice [then] shook the earth; but now he hath promised, and said, yet again once more, I will shake not the earth only, but also heaven (*Hebrews* 12:25—26).

As a result of such reasoning, Jesus is estimated by the number of definitions by means of transition of properties from

minor to major. However, the majority of definitions is offered to be constructed by readers on the basis of following dependences: Jesus' new covenant (major) is superior to the Mosaic covenant (minor) (7:22; 8:6—7); God's guarantee of the new covenant (major) is superior to that of the Mosaic covenant (minor) (7:22—24); the New covenant's promises (major) are superior to those of the Mosaic covenant (minor) (8:6—7); the law of Messiah (major) is superior to the Torah (minor) (7:11—19, 28); the Melchizedekian high priesthood (major) is superior to the Aaronic high priesthood (minor) (7:11—19, 28); the Messiah (major) is superior to the Levitical high priests (minor) (7:11—8:6); the Messiah's sacrifice (major) is superior to the Levitical system's sacrifices (minor) (9:23—28); the Messiah's heavenly sanctuary (major) is superior to the Tabernacle (8:2—5; 9:11—12, 24); the hope incited through the Messiah (major) is superior to the hope incited through the Levitical system (7:18—19); the Messiah's new means (major) are superior to the way to relate to God through the Levitical system (minor) (7:25); the Messiah (major) is superior to angels in both identity and ministry (minor) (1:4—2:18); the Messiah (major) is superior to Moses as human mediator of the Torah in both identity and ministry (minor) (3:1—6); the Messiah (major) is superior to Abraham (minor) (7:1—10); God's new revelation in His Son (major) is superior to all prior prophetic revelation (minor) (1:1—2).

Thus, the Christian teaching about the Messiah in the way how it is given in letters of apostles arose due to the attempts to apply Judaic hermeneutic rules to the texts of the Gospels.

3. The hermeneutic jump. The birth of Christian exegetics

The development of Christianity as autonomic religious community has caused an appreciable delimitation with Judaic hermeneutics. In Judaism the basic principle of exegesis consists in

maintaining that it is prohibited to completely eliminate a literal sense of the text of the Torah [פְּשָׁט] from consideration. Conclusions following from the literal sense cannot be discarded. In the Talmud this principle is mentioned several times. Here are some quotes:

> רב כהנא למר בריה דרב הונא האי בדברי תורה כתיב א"ל אין מקרא יוצא מידי פשוטו א"ר כהנא כד הוינא בר תמני סרי שנין והוה גמירנא ליה לכוליה הש"ס ולא הוה ידענא דאין מקרא יוצא מידי פשוטו עד השתא מאי קמ"ל דליגמר איניש והדר ליסבר

> R. Kahana objected to Mar son of R. Huna: But this refers to the words of the Torah?—A verse cannot depart from its plain meaning [פשוטו], he replied. R. Kahana said: By the time I was eighteen years old I had studied the whole Talmud, yet I did not know that a verse cannot depart from its plain meaning until today. What does he inform us?—That a man should study and subsequently understand (*Shabbat* 63a).

> אמר רבא אע"ג דבכל התורה כולה אין מקרא יוצא מידי פשוטו הכא אתאי גזרה שוה אפיקתיה מפשטיה

> Said Raba: Although throughout the Torah no text loses its ordinary meaning [פשוטו], here the gezerah shawah has come and entirely deprived the text of its ordinary meaning [פשטיה] (*Yevamot* 24a).

In Christianity the given principle was replaced by an allegorical expounding of the Holy Writ. One of the early Church Fathers, Origen, began to assert that finding out allegorical (spiritual), instead of direct (corporal), meaning should take place in true exegetics:

> This point, indeed, is not to be passed by without notice, viz., that there are certain passages of Scripture where this body, as we termed it, i.e., this inferential historical sense, is not always found, as we shall prove to be the case in the following pages,

but where that which we termed soul or spirit can only be understood. And this, I think, is indicated in the Gospels, where there are said to be placed, according to the manner of purification among the Jews, six water-vessels, containing two or three firkins a-piece; by which, as I have said, the language of the Gospel seems to indicate, with respect to those who are secretly called by the apostle Jews, that they are purified by the word of Scripture,—receiving indeed sometimes two firkins, i.e., the understanding of the soul or spirit, according to our statement as above; sometimes even three (firkins), when in the reading (of Scripture) the bodily sense, which is the historical, may be preserved for the edification of the people. Now six water-vessels are appropriately spoken of, with regard to those persons who are purified by being placed in the world; for we read that in six days—which is the perfect number—this world and all things in it were finished. How great, then, is the utility of this first historical sense which we have mentioned, is attested by the multitude of all believers, who believe with adequate faith and simplicity, and does not need much argument, because it is openly manifest to all; whereas of that sense which we have called above the soul, as it were, of Scripture, the Apostle Paul has given us numerous examples in the first Epistle to the Corinthians. For we find the expression, You shall not muzzle the mouth of the ox that treads out the grain. And afterwards, when explaining what precept ought to be understood by this, he adds the words: Does God take care for oxen? Or says He it altogether for our sakes? For our sakes, no doubt, this is written; that he who ploughs should plough in hope, and he that threshes, in hope of partaking. Very many other passages also of this nature, which are in this way explained of the law, contribute extensive information to the hearers (Origen. *De Principiis*. Book IV).

One of the first families of hermeneutic rules of Christian exegetics belongs to Tichonius. His corpus has just seven rules:

> The first relates to the Lord and His body, the second to the twofold division of the Lord's body, the third to the promises and the law, the fourth to species and genus, the fifth to times,

> the sixth to recapitulation, the seventh to the devil and his body (Saint Augustine. *On Christian Doctrine*).

Due to Saint Augustine who followed these rules, they have got abroad the Christian world over. We will consider these rules in detail.

T1. According to the first rule, Christ or His Church is meant in the majority of metaphors of the Holy Scripture.

> The first is about the Lord and His body, and it is this, that, knowing as we do that the head and the body—that is, Christ and His Church—are sometimes indicated to us under one person (for it is not in vain that it is said to believers, "Ye then are Abraham's seed," when there is but one seed of Abraham, and that is Christ), we need not be in a difficulty when a transition is made from the head to the body or from the body to the head, and yet no change made in the person spoken of. For a single person is represented as saying, "He has decked me as a bridegroom with ornaments, and adorned me as a bride with jewels;" and yet it is, of course, a matter for interpretation which of these two refers to the head and which to the body, that is, which to Christ and which to the Church (Saint Augustine. *On Christian Doctrine*).

T2. According to the second rule, in the Holy Scripture there can be a narration on one person, which actually is a narration on two and more persons or different states of one person:

> The second rule is about … the true and the mixed body of the Lord, or the true and the counterfeit, or some such name; because, not to speak of eternity, hypocrites cannot even now be said to be in Him, although they seem to be in His Church. And hence this rule might be designated thus: Concerning the mixed Church. Now this rule requires the reader to be on his guard when Scripture, although it has now come to address or speak of a different set of persons, seems to be addressing or speaking of the same persons as before, just as if both sets constituted one body in consequence of their being for the time

> united in a common participation of the sacraments. An example of this is that passage in the Song of Solomon, "I am black, but comely, as the tents of Cedar, as the curtains of Solomon." For it is not said, I *was* black as the tents of Cedar, but am *now* comely as the curtains of Solomon. The Church declares itself to be at present both; and this because the good fish and the bad are for the time mixed up in the one net. For the tents of Cedar pertain to Ishmael, who "shall not be heir with the son of the free woman" (Saint Augustine. *On Christian Doctrine*).

T3. According to the third rule, the allegorical meaning of the Holy Scripture is most important, because even our belief is not ours, but it is the gift of God. The allegorical meaning should be consonant with Lord's grace also:

> The third rule relates to the promises and the law, and may be designated in other terms as relating to the spirit and the letter … It may be also named, of grace and the law. … The grace of God … is through our Lord Jesus Christ and … has made us much more watchful and diligent to discover in Scripture what escaped Tichonius, who, having no enemy to guard against, was less attentive and anxious on this point, namely, that even faith itself is the gift of Him who "has dealt to every man the measure of faith." Whence it is said to certain believers: "Unto you it is given, in the behalf of Christ, not only to believe on Him, but also to suffer for His sake." Who, then, can doubt that each of these is the gift of God, when he learns from this passage, and believes, that each of them is given? (Saint Augustine. *On Christian Doctrine*).

T4. According to the fourth rule, for the correct understanding of verses of the Holy Scripture it is necessary to know how to distinguish species and genus. In other words, in certain cases we should apply Aristotelian logic and move from the general to the particular or from the particular to the general.

The fourth rule of Tichonius is about species and genus. For so he calls it, intending that by species should be understood a part, by genus the whole of which that which he calls species is a part: as, for example, every single city is a part of the great society of nations: the city he calls a species, all nations constitute the genus. There is no necessity for here applying that subtilty of distinction which is in use among logicians, who discuss with great acuteness the difference between a part and a species … And hence these words have come to be commonly known, so that even uneducated people understand what is laid down specially, and what generally, in any given Imperial command. The same thing occurs in the case of men: things are said of Solomon, for example, the scope of which reaches far beyond him, and which are only properly understood when applied to Christ and His Church, of which Solomon is a part.

… Take, for example, what the prophet Ezekiel says: "When the house of Israel dwelt in their own land, they defiled it by their own way, and by their doings: their way was before me as the uncleanness of a removed woman. Wherefore I poured my fury upon them for the blood that they had shed upon the land, and for their idols wherewith they had polluted it: and I scattered them among the heathen, and they were dispersed through the countries: according to their way, and according to their doings, I judged them." Now it is easy to understand that this applies to that house of Israel of which the apostle says "Behold Israel after the flesh;" because the people of Israel after the flesh did both perform and endure all that is here referred to. What immediately follows, too, may be understood as applying to the same people. But when the prophet begins to say, "And I will sanctify my great name, which was profaned among the heathen, which ye have profaned in the midst of them; and the heathen shall know that I am the Lord," the reader ought now carefully to observe the way in which the species is overstepped and the genus taken in. For he goes on to say: "And I shall be sanctified in you before their eyes. For I will take you from among the heathen, and gather you out of all countries, and will bring you into your own land. Then will I sprinkle clean water upon you, and ye shall be clean: from all your filthiness, and from all your idols, will I cleanse you. A new heart also will I give you, and a new spirit will I put within

> you; and I will take away the stony heart out of your flesh and I will give you a heart of flesh. And I will put my Spirit within you, and cause you to walk in my statutes, and ye shall keep my commandments, and do them. And ye shall dwell in the land that I gave to your fathers; and ye shall be my people, and I will be your God. I will also save you from all your uncleannesses." Now that this is a prophecy of the New Testament, to which pertain not only the remnant of that one nation of which it is elsewhere said, "For though the number of the children of Israel be as the sand of the sea, yet a remnant of them shall be saved," but also the other nations which were promised to their fathers and our fathers; and that there is here a promise of that washing of regeneration which, as we see, is now imparted to all nations, no one who looks into the matter can doubt. And that saying of the apostle, when he is commending the grace of the New Testament and its excellence in comparison with the Old, "Ye are our epistle ... written not with ink, but with the Spirit of the living God; not in tables of stone, but in fleshy tables of the heart," has an evident reference to this place where the prophet says, "A new heart also will I give you, and a new spirit will I put within you; and I will take away the stony heart out of your flesh, and I will give you an heart of flesh." Now the heart of flesh from which the apostle's expression, "the fleshy tables of the heart," is drawn, the prophet intended to point out as distinguished from the stony heart by the possession of sentient life; and by sentient he understood intelligent life. And thus the spiritual Israel is made up, not of one nation, but of all the nations which were promised to the fathers in their seed, that is, in Christ (Saint Augustine. *On Christian Doctrine*).

T5. According to the fifth rule, first, the time characteristics of events in the way how they are described in the Holy Scripture, do not contradict each other, and, second, numbers often have an allegorical meaning and designate the indefinite time period, for example, "Seven times in the day do I praise Thee," means just the same as "His praise shall continually be in my mouth:"

> The fifth rule Tichonius lays down is one he designates of times,—a rule by which we can frequently discover or

> conjecture quantities of time which are not expressly mentioned in Scripture. And he says that this rule applies in two ways: either to the figure of speech called synecdoche, or to legitimate numbers. The figure synecdoche either puts the part for the whole, or the whole for the part. As, for example, in reference to the time when, in the presence of only three of His disciples, our Lord was transfigured on the mount, so that His face shone as the sun, and His raiment was white as snow, one evangelist says that this event occurred "after eight days," while another says that it occurred "after six days." Now both of these statements about the number of days cannot be true, unless we suppose that the writer who says "after eight days," counted the latter part of the day on which Christ uttered the prediction and the first part of the day on which he showed its fulfillment as two whole days; while the writer who says "after six days," counted only the whole unbroken days between these two (Saint Augustine. *On Christian Doctrine*).

T6. According to the sixth rule, the Holy Scripture sometimes repeatedly describes events which already took place in the past:

> The sixth rule Tichonius calls the recapitulation, which, with sufficient watchfulness, is discovered in difficult parts of Scripture. For certain occurrences are so related, that the narrative appears to be following the order of time, or the continuity of events, when it really goes back without mentioning it to previous occurrences, which had been passed over in their proper place. And we make mistakes if we do not understand this, from applying the rule here spoken of (Saint Augustine. *On Christian Doctrine*).

T7. According to the seventh rule, when in the Holy Scripture it is spoken about the dark force it is necessary to distinguish the devil and his body:

> The seventh rule of Tichonius and the last, is about the devil and his body. For he is the head of the wicked, who are in a sense his body, and destined to go with him into the

> punishment of everlasting fire, just as Christ is the head of the Church, which is His body, destined to be with Him in His eternal kingdom and glory … For example, what is said in Isaiah, "How he is fallen from heaven, Lucifer, son of the morning!" and the other statements of the context which, under the figure of the king of Babylon, are made about the same person, are of course to be understood of the devil; and yet the statement which is made in the same place, "He is ground down on the earth, who sendeth to all nations," does not altogether fitly apply to the head himself. For, although the devil sends his angels to all nations, yet it is his body, not himself, that is ground down on the earth, except that he himself is in his body, which is beaten small like the dust which the wind blows from the face of the earth (Saint Augustine. *On Christian Doctrine*).

Hermeneutic rules of Tichonius resolutely break off relations with Judaic hermeneutics. Non-return points are as follows: 1) the personified interpretation when behind verses of the Holy Scripture I see living figures, first of all Jesus and the devil in their antagonistic opposition (T1, T2, T7), 2a) the preference of allegorical expounding of verses, the recession from their literal understanding in most cases (T3, T5), 2б) the claim that literal perusal concerns only historical narrations (the reduction of literal perusal only to historical hermeneutics) (T3, T5, T6), 3) the application of Aristotle's logic to expounding the Holy Scripture (T4).

Due to these rules the new theology in which Aristotelian logic has played an appreciable role was formed. It is a theology of total unity. It prefers the whole, instead of parts. It is a theology of Platonism. In it there is no place for debates, disputes, dissents. It is holistic and gives a uniform picture of the world, drawing a science. With this theology the mankind became what it is now. But it is already another story.

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The Talmudic exegesis is constructed on special hermeneutic rules which have the logical meaning in fact. On the basis of this circumstance it is possible to speak about a special logical culture of the Talmud and to call the logic used there Judaic logic. The contemporary interest in Judaic logic is related to the possibility of creating new logical approaches in modern computer science. Currently, computer science has been dealing with logical systems formalizing multi-agent and interactive computing. Talmudic reasoning is just multi-agent and interactive, since it has been constructed by taking dissonant opinions and conflicting authorities into account. Due to the multi-agentness of Judaic reasoning we can find out an original approach in Judaic logic to pure pragmatics and praxeology. In the book the author considers logic of Jews from the standpoint of analytic philosophy and symbolical logic. He shows that philosophical and logical ideas of the Talmud are really actual and after interpretation they can be considered as popular logical introduction to the theory of pure pragmatics and massive-parallel reasoning. As a result, the author proposes a logical formalization of Judaic reasoning.

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